Appendices

This appendix will not be included in the main text. It will be provided through the webpage of the Journal.)

1 Proof of Proposition 1

The Lagrangian is:

$$L = \int_1^{i^*} v^u(i) n_i di + \int_{i^*}^2 v^s(i) n_i di + \int_1^{i^*} \mu_i^u \{\dot{v^u} - a^u h_i^u (g'_{ui}/g_{ui})\} di + \int_{i^*}^2 \mu_i^s \{\dot{v^s} - a^s h_i^s (g'_{si}/g_{si})\} di$$
$$+\beta_1 \{v^s_{i^*} - v^u_{i^*}\} + \beta_2 \{R^s_{i^*} - R^u_{i^*}\}$$
$$+\lambda \int_1^{i^*} \{R_i^u - x(R_i^u, v_i^u, w_i^s, w_i^u, p_1 + t)\} n_i di + \lambda \int_{i^*}^2 \{R_i^s - x(R_i^s, v_i^s, w_i^s, w_i^u, p_1 + t)\} n_i di$$
$$+\lambda t \int_1^2 n_i c_{1i} di + \beta_s \{\int_{i^*}^2 g_{si} h_i^s di - H^s\} + \beta_u \{\int_1^{i^*} g_{ui} h_i^u di - H^u\}.$$

By using the integration by parts, we can obtain:

$$L = \int_1^{i^*} v^u(i) n_i di + \int_{i^*}^2 v^s(i) n_i di + \mu^u_{i^*} v^u_{i^*} - \mu_1^u v_1^u - \int_1^{i^*} \dot{\mu}_i^u v_i^u di - \int_1^{i^*} \mu_i^u a^u h_i^u (g'_{ui}/g_{ui}) di$$
$$\mu_2^s v_2^s - \mu^s_{i^*} v^s_{i^*} - \int_{i^*}^2 \dot{\mu}_i^s v_i^s di - \int_{i^*}^2 \mu_i^s a^s h_i^s (g'_{si}/g_{si}) di + \beta_1 \{v^s_{i^*} - v^u_{i^*}\} + \beta_2 \{R^s_{i^*} - R^u_{i^*}\}$$
$$+\lambda \int_1^{i^*} \{R_i^u - x(R_i^u, v_i^u, q, w_i^s, w_i^u, p_1 + t)\} n_i di + \lambda \int_{i^*}^2 \{R_i^s - x(R_i^s, v_i^s, w_i^s, w_i^u, p_1 + t)\} n_i di$$
$$+\lambda t \int_1^2 n_i c_{1i} di + +\beta_s \{\int_{i^*}^2 g_{si} h_i^s di - H^s\} + \beta_u \{\int_1^{i^*} g_{ui} h_i^u di - H^u\}.$$

Denote $x(R_i^j, v_i^j, w_i^s, w_i^j, p_1 + t)$ as x_i. The first order condition for v_i^s, v_2^s, $v^s_{i^*}$, R_i^s, $R^s_{i^*}$, v_i^u, $v^u_{i^*}$, v_1^u, R_i^u , $R^u_{i^*}$, H^s and H^u are:

$$v_i^s : n_i - \dot{\mu}_i^s - \lambda n_i \frac{\partial x_i}{\partial v_i^s} + \lambda t n_i \frac{\partial c_{1i}}{\partial x_i} \frac{\partial x_i}{\partial v_i^s} = 0$$
$$v_2^s : \mu_2^s = 0$$
$$v^s_{i^*} : -\mu^s_{i^*} + \beta_1 = 0$$
$$R_i^s : [-\mu_i^s \times a^s \frac{g'_{si}}{g_{si}} + \beta_s g_{si}] \frac{\partial h_i^s}{\partial R_i^s} + \lambda n_i - \lambda n_i \frac{\partial x_i}{\partial R_i^s} + \lambda t n_i \frac{\partial c_{1i}}{\partial x_i} \frac{\partial x_i}{\partial R_i} = 0$$
$$R^s_{i^*} : \beta_2 = 0$$
$$v_i^u : n_i - \dot{\mu}_i^s - \lambda n_i \frac{\partial x_i}{\partial v_i^s} + \lambda t n_i \frac{\partial c_{1i}}{\partial x_i} \frac{\partial x_i}{\partial v_i^s} = 0$$
$$v^u_{i^*} : \mu^u_{i^*} - \beta_1 = 0$$

$$v_1^u : \mu_1^u = 0$$

$$R_i^u : [-\mu_i^u \times a^u \frac{g'_{ui}}{g_{ui}} + \beta_u g_{ui}] \frac{\partial h_i^u}{\partial R_i^u} + \lambda n_i - \lambda n_i \frac{\partial x_i}{\partial R_i^u} + \lambda t n_i \frac{\partial c_{1i}}{\partial x_i} \frac{\partial x_i}{\partial R_i^u} = 0$$

$$R_{i^*}^u : \beta_2 = 0$$

$$H^s : \frac{\partial L}{\partial p_1} \frac{\partial p_1}{\partial H^s} = \beta_s$$

$$H^u : \frac{\partial L}{\partial p_1} \frac{\partial p_1}{\partial H^u} = \beta_u$$

Now we characterize those first order conditions. First, note that:

$$\mu_i^s = \mu_{i^*}^s + \int_1^{i^*} n_j (1 - \lambda \frac{\partial x_j}{\partial v_j}) dj \text{ and } \mu_{i^*}^u = \mu_1^u + \int_1^{i^*} n_j (1 - \lambda \frac{\partial x_j}{\partial v_j}) dj \tag{1}$$

Since $\mu_{i^*}^s = \mu_{i^*}^u$, $\mu_i^s = \int_1^i n_j (1 - \lambda \frac{\partial x_j}{\partial v_j}) dj$ for $i \in (i^*, 2)$ and $\mu_i^u = \int_1^i n_j (1 - \lambda \frac{\partial x_j}{\partial v_j}) dj$ for $i \in (1, i^*)$. Note that $\frac{\partial x_j}{\partial v_j} = 1/(U_x)$. A single crossing property and the monotonicity of R_i^s and R_i^u guarantee that x_i is increasing. This implies that $\frac{\partial x_j}{\partial v_j}$ is increasing and the inside of the integral is a decreasing function of i. Since $\mu_2^s = 0$ and $\mu_1^u = 0$, the only way that these conditions are satisfied is that initially $n_i (1 - \lambda \frac{\partial x_j}{\partial v_j})$ is positive and after some i^{**},it becomes negative. In this case, for all $i_s \in [i^*, 2)$ and $i_u \in (1, i^*]$, $\mu_{i_s}^s$ and $\mu_{i_s}^u$ are strictly positive.

Now we examine dW/dt and evaluate at $t = 0$. From the envelope theorem,

$$\frac{dW}{dt} = \frac{\partial L}{\partial p_1} \frac{\partial p_1}{\partial t} + \lambda \int_1^2 n_i c_{1i} di - \lambda \int_{i^*}^2 \frac{\partial x_i}{\partial (p_1 + t)} n_i di - \lambda \int_1^{i^*} \frac{\partial x_i}{\partial (p_1 + t)} n_i di$$

where

$$\begin{aligned}
\frac{\partial L}{\partial p_1} = \frac{di^*}{dp} \Big\{ & v_{i^*}^u n_{i^*} - v_{i^*}^s n_{i^*} + \dot{\mu}^u{}_{i^*} v_{i^*}^u + \mu_{i^*}^u \dot{v}_{i^*}^u - \dot{\mu}^u{}_{i^*} v_{i*}^u - \mu_{i^*}^u a^u h_{i^*}^u \frac{g'_{ui^*}}{g_{ui^*}} \\
& - \dot{\mu}^s{}_{i^*} v_{i^*}^s - \mu_{i^*}^s \dot{v}_{i^*}^s + \dot{\mu}_{i^*}^s v(i^*) + \mu_{i^*}^s a^s h_{i^*}^s \frac{g'_{si^*}}{g_{si^*}} + \beta_1 \{ \dot{v}^s{}_{i^*} - \dot{v}_{i^*}^u \} + \beta_2 \dot{R}^u{}_{i^*} - \beta_2 \dot{R}^s{}_{i^*} \\
& - \lambda \{ R_{i^*}^s - x_{i^*} \} n_{i^*} + \lambda \{ R_{i^*}^u - x_{i^*} \} n_{i^*} \\
& - \beta_s g_{si^*} h_{i^*}^s + \beta_u g_{ui^*} h_{i^*} \Big\} \\
& + \Big\{ \int_{i^*}^2 [-\mu_i^s a^s (g'_{si}/g_{si}) + \beta_s g_{si}] \frac{\partial h_i^s}{\partial w_i^s} \frac{\partial w_i^s}{\partial p_1} di - \lambda \int_{i^*}^2 \frac{\partial x_i}{\partial w_i^s} \frac{\partial w_i^s}{\partial p_1} n_i di \Big\} \\
& + \Big\{ \int_{i^*}^2 [-\mu_i^u a^u (g'_{ui}/g_{ui}) + \beta_u g_{ui}] \frac{\partial h_i^u}{\partial w_i^u} \frac{\partial w_i^u}{\partial p_1} - \lambda \int_1^{i^*} \frac{\partial x_i}{\partial w_i^u} \frac{\partial w_i^u}{\partial p_1} n_i di \Big\} \\
& - \lambda \int_{i^*}^2 \frac{\partial x_i}{\partial (p_1 + t)} n_i di - \lambda \int_1^{i^*} \frac{\partial x_i}{\partial (p_1 + t)} n_i di
\end{aligned}$$

By using the above first order conditions, we have:

$$\begin{aligned}\frac{\partial L}{\partial p_1} = \frac{di^*}{dp_1} &\left\{\mu_{i^*}^s a^s h_i^s \frac{g_s'}{g_s} - \mu_{i^*}^u a^u h_i^u \frac{g_u'}{g_u} - \beta_s g_{si^*} h_{i^*}^s + \beta_s g_{si^*} h_{i^*}^s \right\} \\ &+ \left\{ \int_{i^*}^2 [-\mu_i^s a^s (g_{si}'/g_{si}) + \beta_s g_{si}] \frac{\partial h_i^s}{\partial w_i^s} \frac{\partial w_i^s}{\partial p_1} di - \lambda \int_{i^*}^2 \frac{\partial x_i}{\partial w_i^s} \frac{\partial w_i^s}{\partial p_1} n_i di \right\} \\ &+ \left\{ \int_{i^*}^2 [-\mu_i^u a^u (g_{ui}'/g_{ui}) + \beta_u g_{ui}] \frac{\partial h_i^u}{\partial w_i^u} \frac{\partial w_i^u}{\partial p_1} - \lambda \int_1^{i^*} \frac{\partial x_i}{\partial w_i^u} \frac{\partial w_i^u}{\partial p_1} n_i di \right\} \\ &- \lambda \int_{i^*}^2 \frac{\partial x_i}{\partial (p_1 + t)} n_i di - \lambda \int_1^{i^*} \frac{\partial x_i}{\partial (p_1 + t)} n_i di\end{aligned}$$

Now we need to calculate the inside of the integral. Note that from the definition of h_i^s and h_i^u, we have:

$$\frac{\partial h_i^s}{\partial w_i^s} = -h_i^s \frac{\partial h_i^s}{\partial R_i^s} \text{ and } \frac{\partial h_i^u}{\partial w_i^u} = -h_i^u \frac{\partial h_i^u}{\partial R_i^u}$$

This implies that:

$$\begin{aligned} [-\mu_i^s a^s (g_{si}'/g_{si}) + \beta_s g_{si}] \frac{\partial h_i^s}{\partial w_i^s} &= [\mu_i^s a^{ss} (g_{si}'/g_{si}) - \beta_s g_{si}] h_i^s \frac{\partial h_i^s}{\partial R_i^s} \\ \text{and } [-\mu_i^u a^u (g_{ui}'/g_{ui}) + \beta_u g_{ui}] \frac{\partial h_i^u}{\partial w_i^u} &= [\mu_i^u a^u (g_{ui}'/g_{ui}) - \beta_u g_{ui}] h_i^u \frac{\partial h_i^u}{\partial R_i^u} \end{aligned}$$

By using the FOC of R_i^s and R_i^u,

$$\begin{aligned} [\mu_i^s a^{ss} (g_{si}'/g_{si}) - \beta_s g_{si}] h_i^s \frac{\partial h_i^s}{\partial R_i^s} &= h_i^s \{\lambda n_i - \lambda n_i \frac{\partial x}{\partial R_i^s}\} \\ [\mu_i^s a^{ss} (g_{si}'/g_{si}) - \beta_u g_{si}] h_i^u \frac{\partial h_i^u}{\partial R_i^u} &= h_i^u \{\lambda n_i - \lambda n_i \frac{\partial x}{\partial R_i^u}\} \end{aligned}$$

Thus, $\frac{\partial L}{\partial p_1}$ is:

$$\begin{aligned}\frac{\partial L}{\partial p_1} = \frac{di^*}{dt} &\left\{\mu_{i^*}^s h_i^s \frac{g_{si^*}'}{g_{si^*}} - \mu_{i^*}^u h_i^u \frac{g_{ui^*}'}{g_{ui^*}} - \beta_s g_{si^*} h_{i^*}^s + \beta_u g_{ui^*} h_{i^*}^u \right\} \\ &+ \left\{ \int_{i^*}^2 h_i^s \{\lambda n_i - \lambda n_i \frac{\partial x_i}{\partial R_i^s}\} \frac{\partial w_i^s}{\partial p_1} di - \lambda \int_{i^*}^2 \frac{\partial x_i}{\partial w_i^s} \frac{\partial w_i^s}{\partial p_1} n_i di \right\} \\ &+ \left\{ \int_{i^*}^2 h_i^u \{\lambda n_i - \lambda n_i \frac{\partial x_i}{\partial R_i^u}\} \frac{\partial w_i^u}{\partial p_1} di - \lambda \int_1^{i^*} \frac{\partial x_i}{\partial w_i^u} \frac{\partial w_i^u}{\partial p_1} n_i di \right\} \\ &- \lambda \int_{i^*}^2 \frac{\partial x_i}{\partial (p_1 + t)} n_i di - \lambda \int_1^{i^*} \frac{\partial x_i}{\partial (p_1 + t)} n_i di\end{aligned}$$

Next, we need to calculate $\lambda \int_{i^*}^2 h_i^s n_i \frac{\partial w_i^s}{\partial p_1} + \lambda \int_1^{i^*} h_i^u n_i \frac{\partial w_i^u}{\partial p_1} di$. Note that $\lambda \int_{i^*}^2 h_i^s n_i \frac{\partial w_i^s}{\partial p_1} + \lambda \int_1^{i^*} h_i^u n_i \frac{\partial w_i^u}{\partial p_1} di = \lambda \int_{i^*}^2 h_i^s g_{sii} \frac{\partial w^s}{\partial p_1} + \lambda \int_1^{i^*} h_i^u g_{ui} n_i \frac{\partial w^u}{\partial p_1} di$. $\lambda \int_{i^*}^2 h_i^s g_{sii} \frac{\partial w^s}{\partial p_1} + \lambda \int_1^{i^*} h_i^u g_{ui} n_i \frac{\partial w^u}{\partial p_1} di$

is a change in total earnings due to a change in the price of good 1 when levels of human capital of all individuals are fixed. On the other hand, in perfect competition, for given levels of human capital of all individuals, the total revenue of a firm should be equal to the total payments to factor owners. Thus, $p_1 y_1 + y_2 = w^s \int_{i^*}^{2} n_i h_i^s g_{si} di + w^u \int_1^{i^*} n_i h_i^u g_{ui} di$ always holds true. Let $Q(p_1)$ be the total revenue of firms when all human capital levels of all individuals are fixed. Then, $dQ/dp_1 = \lambda \int_{i^*}^{2} h_i^s g_{sii} \frac{\partial w^s}{\partial p_1} + \lambda \int_1^{i^*} h_i^u g_{ui} n_i \frac{\partial w^u}{\partial p_1} di$. By definition of $Q(p_1)$

$$\begin{aligned} Q(p_1) \equiv \max \ p_1 y_1 + y_2 \quad \text{s.t.} \ \ (y_1, y_2) \in \Gamma(H^s, H^u) \\ H^s \text{ and } H^u \text{ are fixed.} \end{aligned}$$

From the envelope theorem, $\frac{dQ}{dp_1} = y_1$. Therefore, we have:

$$\lambda y_1 = \lambda \frac{\partial w^s}{\partial \sigma} \int_{i^*}^{2} h_i^s g_{si} n_i + \lambda \frac{\partial w^u}{\partial \sigma} \int_1^{i^*} h_i^u g_{ui} n_i di.$$

Third, we will show that $h_i^s \frac{\partial x}{\partial R_i^s} = -\frac{\partial x}{\partial w_i^s}$ and $h_i^u \frac{\partial x}{\partial R_i^u} = -\frac{\partial x}{\partial w_i^u}$. From the definition of Z, we have:

$$\begin{aligned} \frac{\partial Z}{\partial R_i^s} &= a^s / w_i^s \text{ and } \frac{\partial Z}{\partial w_i^s} = -h_i^s (1/w_i^s) \text{ for } i \in (i^*, 2) \\ \frac{\partial Z}{\partial R_i^u} &= a^u / w_i^u \text{and } \frac{\partial Z}{\partial w_i^u} = -h_i^u (1/w_i^s) \text{ for } i \in (1, i^*) \end{aligned}$$

Thus, by using the definition of $\frac{\partial x}{\partial R_i^s}, \frac{\partial x}{\partial w^s}, \frac{\partial x}{\partial R_i^s}, \frac{\partial x}{\partial w^s}$, we can confirm that $h_i^s \frac{\partial x}{\partial R_i^s} = -\frac{\partial x}{\partial w_i^s}$ and $h_i^u \frac{\partial x}{\partial R_i^u} = -\frac{\partial x}{\partial w_i^u}$.

Therefore, $\partial L / \partial p_1$ is:

$$\left. \frac{\partial L}{\partial p_1} \right|_{\sigma=0} = \frac{di^*}{dp_1} \left\{ \mu_{i^*}^s h_{i^*}^s \frac{g_s'}{g_s} - \mu_{i^*}^u h_{i^*}^u \frac{g_u'}{g_u} - \frac{\partial L}{\partial p_1} \frac{\partial p_1}{\partial H^s} g_{si^*} h_{i^*}^s + \frac{\partial L}{\partial p_1} \frac{\partial p_1}{\partial H^u} g_{ui^*} h_{i^*}^u \right\}$$

From the FOC of H^s and H^u, we have:

$$\frac{\partial L}{\partial p_1} \{ 1 + \frac{di^*}{dp_1} [\frac{\partial p_1}{\partial H^s} g_{si^*} h_{i^*}^s - \frac{\partial p_1}{\partial H^u} g_{ui^*} h_{i^*}^u] \} = \frac{di^*}{dp_1} \left\{ \mu_{i^*}^s a^s h_{i^*}^s \frac{g_s'}{g_s} - \mu_{i^*}^u a^u h_{i^*}^u \frac{g_u'}{g_u} \right\}$$

Therefore, this implies that

$$\frac{\partial L}{\partial p_1} = \frac{\frac{di^*}{dp_1}}{\triangle'} \left\{ \mu_{i^*}^s a^s h_{i^*}^s \frac{g_s'}{g_s} - \mu_{i^*}^u a^u h_{i^*}^u \frac{g_u'}{g_u} \right\}$$

where $\triangle' = 1 + \frac{di^*}{dp_1} [\frac{\partial p_1}{\partial H^s} g_{si^*} h_{i^*}^s - \frac{\partial p_1}{\partial H^u} g_{ui^*} h_{i^*}^u] > 0$. By using the definition of $\partial p_1 / \partial t, \partial p_1 / \partial H^s$ and $\partial p_1 / \partial H$, we have:

$$\left. \frac{dW}{dt} \right|_{t=0} = \Psi_1 \frac{di^*}{dp_1} \left\{ \mu_{i^*}^s a^s h_{i^*}^s \frac{g_s'}{g_s} - \mu_{i^*}^u a^u h_{i^*}^u \frac{g_u'}{g_u} \right\}$$

where

$$\Psi_1 = \frac{-RD_p}{RD_p - RS_p + RS_{H^s}\frac{\partial i^*}{\partial p_1} g_{si^*} h^s_{i^*} - RS_{H^u}\frac{\partial i^*}{\partial p_1} g_{ui^*} h^u_{i^*}}$$

From the FOC of $v^s_{i^*}$ and $v^u_{i^*}$, we have $\mu^s_{i^*} = \mu^u_{i^*}$. In addition, $a^s h^s_i \frac{g'_s}{g_s}$ and $a^u h^u_i \frac{g'_u}{g_u}$ are the right side slope of v^s_i and the left side slope v^u_i at i^*. From Lemma 1, the slope of v^s_i is steeper than the slope of v^u_i at i^*. Since $\frac{di^*}{dp_1} < 0$, $\frac{dW}{dt} > 0$.Also for equation (12), notice that $R^s_{i^*} = R^u_{i^*}$. Then, we obtain equation (12).

2 A Case of Imperfect Substitutes

In the main text, I considered a case where two types of human capital accumulation are perfect substitutes in the disutility function. As a result, each person accumulates only one type of human capital. In reality, however, individuals might accumulate both types of human capital. It is important to check the robustness of the proposition when two types of human capital are imperfect substitutes.

In order to make two types of human capital accumulation process imperfect substitutes, we assume that the utility function of the type i agent has the following form:

$$u(c_{1i}, c_{2i}) - f_s(h^s_i) - f_u(h^u_i).$$

Regarding $u(c_{1i}, c_{2i})$, I make the same assumption as in the previous section. As for $f_j(h^s_i)$ (j=s,u), $f_j(h^j_i)$ is strictly increasing and strictly convex. The labor supply is fixed. In addition, to simplify the analysis, it is assumed that $f_s(h^s_i)$ and $f(h^u_i)$ have the following functional forms:[1]

$$f_s(h^s_i) = (h^s_i)^{\gamma_s} \text{ and } f_s(h^u_i) = (h^u_i)^{\gamma_u}$$

where γ_s and γ_u measure the curvature of the disutility functions of skilled and unskilled human capital accumulation respectively and are strictly greater than one. Given the amount of skilled human capital and unskilled human capital of individual i, I assume that the earnings of individual i is determined in the same way as in the previous subsection. Agents who have greater ability have more comparative advantage in accumulating skilled human capital than unskilled human capital. More specifically, I assume that:

$$\frac{g'_{si}}{g_{si}} > \frac{g'_{ui}}{g_{ui}}\frac{\gamma_u}{\gamma_s}. \tag{2}$$

(2) implies that when the disutility of accumulating human capital is not constant, the comparative advantage condition needs to be adjusted by the curvature of the disutility function.[2]

---

[1] Our main results can hold in more general functional forms.

[2] The economic interpretation of (2) is as follows. Consider a condition that type i and type $i+\varepsilon$ agents have the same degree of comparative advantage. Equation (2) says that if the marginal disutility of accumulating skilled human capital grows faster than the marginal disutility of accumulating unskilled human capital ($\gamma_s > \gamma_u$), then the increase of the return from skilled human capital, g'_{si}/g_{si}, can be lower than the increase of the return from unskilled human capital, g'_{ui}/g_{ui}, in order to have the same comparative advantage. This is because accumulating skilled human capital accompanies larger disutility from the first place.

As for the government policy, the prices and production side of the economy, I make the same assumptions as in the previous sub-section.

When designing an income tax system, it is useful to analyze it in two steps. The first step is to know how an individual i will choose skilled human capital and unskilled human capital to generate pre-tax income, R. The second step is to know, given an after-tax-income schedule of $X = R - T(R)$, how each individual chooses pre-tax income.

The first stage of the problem can be solved by considering the following programming problem:

$$\begin{aligned} &\min f_s(h_i^s) + f_u(h_i^u) \\ &\text{s.t. } R = w_i^s \times h_i^s + w_i^u \times h_i^u \\ &\text{where } w_i^s = g_{si} \times w^s \text{ and } w_i^u = g_{ui} \times w^u \end{aligned} \tag{3}$$

Let the minimized value of the above problem be $Z(w_i^s, w_i^u, R)$. Denote the solution of the above problem as $h_i^s(w_i^s, w_i^u, R)$ and $h_i^u(w_i^s, w_i^u, R)$. For later analysis it is useful to calculate compensated human capital supply. Consider the following dual problem of (3):

$$\begin{aligned} E(w_i^s, w_i^u, V) &\equiv \max \ w_i^s h_i^s + w_i^u h_i^u \\ \text{st. } \ f_s(h_i^s) + f_u(h_i^u) &\leq V \end{aligned}$$

Let the solution of the above problem be $\widetilde{h}_i^j(w_i^s, w_i^u, V)$ where $j = s, u$. Then, from the dual relationship, we will have:

$$h_i^j(w_i^s, w_i^u, E(w_i^s, w_i^u, V)) \equiv \widetilde{h}_i^j(w_i^s, w_i^u, V) \text{ ; j=s,u.}$$

By taking derivatives from both sides, we will have the Slutsky equation for h_i^s and h_i^u:

$$\frac{\partial h_i^j}{\partial w_i^s} + \frac{\partial h_i^j}{\partial R} h_i^s = \frac{\partial \widetilde{h}_i^j}{\partial w_i^s} \text{ and } \frac{\partial h_i^j}{\partial w_i^u} + \frac{\partial h_i^j}{\partial R} h_i^u = \frac{\partial \widetilde{h}_i^j}{\partial w_i^u} \text{ ; j=s,u.}$$

Note that the indifference curve of $f_s(h_i^s) + f_u(h_i^u)$ is strictly concave. Therefore, $\partial \widetilde{h}_i^s / \partial w_i^s > 0$, $\partial \widetilde{h}_i^u / \partial w^u < 0$, $\partial \widetilde{h}_i^u / \partial w_i^u > 0$ and $\partial \widetilde{h}_i^u / \partial w_i^s < 0$. This relationship means that if an individual maximizes his earnings holding the total disutility constant, an increase in the net return from skilled human capital, will increase the supply of skilled human capital and an increase in the return of unskilled human capital will decrease the supply of skilled human capital.

Let $X(R)$ be the government-designed after-tax income schedule. Then, at the second stage of the problem, given $Z(w_i^s, w^u, R)$ and $X(R)$, each individual i will maximize his utility:

$$\max_{\{R\}} \ U(p_1 + t, X(R)) - Z(w_i^s, w_i^u, R) \ .$$

The objective of the social planner is to design a schedule of $X(R)$ to maximize the social welfare. By using the same technique as in the previous section, we can calculate dv/dvi: $dv/di = -\sum_{j=s,u} Z_{w_i^j} \times (dw_i^j/di)$. Let α_i be the Lagrangian multiplier of the required income constraint in the disutility minimization problem (3). From the FOC of the minimization problem for $Z(w_i^s, w_i^u, R)$, we obtain:

$$\frac{dv}{di} = \alpha_i R_i \{ \frac{g_{si}^{'}}{g_{si}} \theta_{si} + \frac{g_{ui}^{'}}{g_{ui}} \theta_{ui} \} \text{ where } \theta_{ji} = \frac{w_i^j h_i^j}{R_i} \text{ ; } j = s,u. \tag{4}$$

Because of the assumption of an absolute advantage, $dv/di > 0$. (4) has a clear economic significance. It means that the slope of the value function $v(i)$ is proportional to the weighted average of the absolute advantage of skilled human capital accumulation and unskilled human capital accumulation. For analytical reasons, it is useful to eliminate α_i in the above equation. Using the first order condition for h_i^s and h_i^u, we can rewrite (4) as follows:

$$\frac{dv}{di} = \frac{g_{si}^{'}}{g_{si}} f_s'(h_i^s) h_i^s + \frac{g_{ui}^{'}}{g_{ui}} f'(h_i^u) h_i^u. \tag{5}$$

Given (5), as in the previous section, it is more useful to assume that the social planner controls v_i and R_i and x_i are defined by the following relationship:[3]

$$v(i) = U(p_1 + t, X_i) - Z(w_i^s, w_i^u, R_i).$$

The problem for the social planner is to solve the following constrained optimization program:

$$\begin{gathered} W(t) = \max_{\{R_i, v_i\}} \int_1^2 v(i) n_i di \\ \text{st. } \frac{dv}{di} = \frac{g_{si}^{'}}{g_{si}} f_s'(h_i^s) h_i^s + \frac{g_{ui}^{'}}{g_{ui}} f'(h_i^u) h_i^u \\ \int_1^2 n_i \{R_i - x_i\} di + t \int_1^2 c_{1i} n_i di = 0 \\ H^s = \int g_{si} h_i^s di \text{ , } H^u = \int g_{ui} h_i^u di, \\ \text{where } p_1 = p_1(t, H^s, H^u) \\ \text{and } t \text{ is given.} \end{gathered}$$

In the above programming problem, $W(t)$ is the maximized social welfare for given t. Also note that h_i^s and h_i^u are functions of (R_i, w_i^s, w_i^u) and that w_i^s and w_i^u are functions of p_1.

After several calculations, we can obtain the following equation (See the Appendix):

$$\left. \frac{dW}{dt} \right|_{t=0} = -\Psi_2 \left\{ \int_1^2 \mu_i [\gamma^s (g_{si}^{'}/g_{si}) - \gamma^u (g_{ui}^{'}/g_{ui})] f_s^i [\frac{\partial \widetilde{h}_i^s}{\partial w_i^s} \frac{dw_i^s}{dp_1} + f_s' \frac{\partial \widetilde{h}_i^s}{\partial w_i^u} \frac{dw_i^u}{dp_1}] di \right\} \tag{6}$$

---

[3] As for SCP, we can check it by examining $\frac{\partial^2 Z}{\partial R \partial i} > 0$. This is true as long as $\frac{\partial h_i^j}{\partial R} > 0$ for j=s,u.

where

$$\Psi_2 = \frac{RD_p}{RS_p - RD_p + RS_{H^s}\int_1^2 g_i^s[\frac{\partial \widetilde{h}_i^s}{\partial w_i^s}\frac{dw_i^s}{dp_1} + \frac{\partial \widetilde{h}_i^s}{\partial w_i^u}\frac{dw_i^u}{dp_1}]di + RS_{H^u}\int_1^2 g_i^u[\frac{\partial \widetilde{h}_i^u}{\partial w_i^s}\frac{dw_i^s}{dp_1} + \frac{\partial \widetilde{h}_i^u}{\partial w_i^u}\frac{dw_i^u}{dp_1}]di}$$

and μ_i is the Lagrangian multiplier of the incentive compatibility constraint. Because of the properties of the compensated supply function of h_i^s, $\partial\widetilde{h}_i^s/\partial w_i^s > 0$ and $\partial\widetilde{h}_i^s/\partial w_i^u < 0$. From the Stolper-Samuelson theorem, $\partial w_i^s/\partial p_1 > 0$ and $\partial w_i^u/\partial p_1 < 0$. From the Rybczynski theorem, $RS_{H^s} > 0$ and $RS_{H^u} < 0$. From the assumption on comparative advantage, $\gamma_s(g_{si}^{'}/g_{si}) - \gamma_u(g_{ui}^{'}/g_{ui}) > 0$. As for the sign of the Lagrangian multiplier of the incentive compatibility constraint, the standard argument shows that $\mu_i > 0$ for all $i \in (1,2)$ (See Section 3 of this appendices). Thus, we obtain $dW/dt > 0$.

Proposition2

Suppose that the social planner sets the income tax structure to maximize the social welfare function in an endogenous skill accumulation model at a zero commodity tax . Then, an introduction of a commodity tax on skilled-labor-intensive goods will increase the social welfare.

Equation (6) has several implications. For an illustration, consider a situation where the disutility functions of skilled and unskilled human capital accumulation have the same degree of curvature, i.e. $\gamma_s = \gamma_u \equiv \gamma$. Then, (6) shows that if $(g_{si}^{'}/g_{si}) = (g_{ui}^{'}/g_{ui})$, $dW/dt = 0$. In other words, if there is no comparative advantage and if greater ability individuals are as good at accumulating skilled and unskilled human capital as lower ability individuals, then there is no gain in social welfare from changing the returns of skilled and unskilled human capital. Second, $(\partial\widetilde{h}_i^s/\partial w_i^s)(\partial w_i^s/\partial p_1)$ and $(\partial\widetilde{h}_i^s/\partial w_i^u)(\partial w_i^u/\partial p_1)$ measure how changes of returns from each type of human capital change the compensated supply of skilled human capital. Third, Ψ_2 measures how a change in the commodity tax t will change the relative price of good 1, taking the effect of changes in the supply of human capital into consideration. [4] Also note that $\gamma \times f_s'(h_i^s) = f_s''(h_i^s)h_i^s + f_s'(h_i^s)$ and that $f_s''(h_i^s)h_i^s + f_s'(h_i^s)$ is related to a change in $\dot{v}$. In addition, note that μ_i measures how social welfare increases when the incentive compatibility is relaxed. This implies that the term after integration measures how a compensated change in returns from skilled and unskilled human capital changes the slope of $\dot{v}$ and increases the social welfare.

The intuition of the above proposition is as follows: In a situation where individuals with greater ability have comparative advantage in accumulating skilled human capital and individuals with lesser ability have comparative advantage in accumulating unskilled human capital, a decrease in the return from skilled human capital and an increase in the return from unskilled human capital will hurt individuals with greater

---

[4] First note that the total skilled and unskilled human capital are functions of the price of good1. Thus, for a given level of the commodity tax, the equilibrium price can be determined from $RS(t + p_1) = RS(p_1, H^s(p_1), H^u(p_1))$. Second, $\int_1^2 g_i^s[\frac{\partial\widetilde{h}_i^s}{\partial w_i^s}\frac{\partial w_i^s}{\partial p_1} + \frac{\partial\widetilde{h}_i^s}{\partial w_i^u}\frac{\partial w_i^u}{\partial p_1}]di$ and $\int_1^2 g_i^u[\frac{\partial\widetilde{h}_i^u}{\partial w_i^s}\frac{\partial w_i^s}{\partial p_1} + \frac{\partial\widetilde{h}_i^u}{\partial w_i^u}\frac{\partial w_i^u}{\partial p_1}]di$ are the compensated change of supply of skilled and unskilled human capital when the price of good 1 increases. Thus, we obtain $dp_1/dt = \Psi_2$

ability and benefit individuals with lesser ability. If the social planner is interested in redistributing income from high ability individuals to low ability individuals, such changes in the returns from skilled and unskilled capital can indirectly redistribute income. On the other hand, starting from zero distortion, the deadweight loss of the commodity tax is of the second-order but the social welfare gain of relaxing the incentive problem has a first-order effect. As a result, introducing production distortion increases social welfare.

3 Proof of the Proposition 2

Let μ_i and λ be the Lagrangian multiplier of the incentive constraint and the resource constraint. Denote $x(R_i^j, v_i, w_i^s, w_i^u, p_1+t)$ as x^i. Then, the Lagrangian function is:

$$\begin{aligned} W(t) = \int_1^2 v_i n_i di + \int_1^2 \mu_i [\frac{dv}{di} - f_s'(h_i^s) h_i^s (g_{si}'/g_{si}) - f_u'(h_i^u) h_i^u (g_{ui}'/g_{ui}) di] + \\ +\lambda \int_1^2 n_i \{R_i - x_i\} di + t \int_1^2 n_i c_{1i} di \\ +\beta_s \{\int_1^2 g_{si} h_i^s di - H^s\} + \beta_u \{\int_1^2 g_{ui} h_i^u di - H^u\} \end{aligned}$$

By using the integration by parts, we can obtain:

$$\begin{aligned} W(t) &= \int_1^2 v_i n_i di + \int_1^2 \mu_i \frac{dv}{di} di - \int_1^2 \mu_i f_s'(h_i^s) h_i^s (g_{si}'/g_{si}) di - \int_1^2 \mu_i f_u'(h_i^u) h_i^u (g_{ui}'/g_{ui}) di \\ &+ \lambda \int_1^2 n_i \{R_i - x_i\} di + \lambda t \int_1^2 n_i c_{1i} di + \beta_s \{\int_1^2 g_{si} h_i^s di - H^s\} + \beta_u \{\int_1^2 g_{ui} h_i^u di - H^u\} \\ &= \int_1^2 v_i n_i di + \mu_2 v_2 - \mu_1 v_1 - \int_1^2 \dot{u}_i v_i di - \int_1^2 \mu_i f_s'(h_i^s) h_i^s (g_{si}'/g_{si}) di - \int_1^2 \mu_i f_u'(h_i^u) h_i^u (g_{ui}'/g_{ui}) di \\ &+ \lambda \int_1^2 n_i \{R_i - x_i\} di + \lambda t \int_1^2 n_i c_{1i} di + \beta_s \{\int_1^2 g_{si} h_i^s di - H^s\} + \beta_u \{\int_1^2 g_{ui} h_i^u di - H^u\} \end{aligned}$$

the first-order-conditions are:

$$\begin{gathered} v_i : n_i - \dot{u}_i - \lambda n_i \frac{\partial x_i}{\partial v_i} + \lambda t n_i \frac{\partial c_{1i}}{\partial x} \frac{\partial x_i}{\partial v_i} = 0 \\ R_i : -\mu_i \frac{d[f_s'(h_i^s) h_i^s (g_{si}^{'}/g_{si})]}{dh_i^s} \frac{\partial h_i^s}{\partial R_i} - \mu_i \frac{d[f_u'(h_i^u) h_i^u (g_{ui}^{'}/g_{ui})]}{dh_i^u} \frac{\partial h_i^u}{\partial R_i} + \lambda n_i \\ +\beta_s g_{si} \frac{\partial h_i^s}{\partial R_i} + \beta_u g_{ui} \frac{\partial h_i^u}{\partial R_i} - \lambda n_i \frac{\partial x_i}{\partial R_i} + \lambda t n_i \frac{\partial c_{1i}}{\partial x} \frac{\partial x}{\partial R_i} = 0 \\ \mu_1 = 0 \text{ and } \mu_2 = 0 \end{gathered}$$

From the FOC of v_i, we will have $n_i - \dot{u}_i - \lambda n_i \frac{\partial x_i}{\partial v_i} + \lambda t n_i \frac{\partial c_{1i}}{\partial x_i} \frac{\partial x_i}{\partial v_i} = 0$

$$n_i - \lambda n_i \frac{\partial x_i}{\partial v_i} = \dot{\mu}_i$$

at $t = 0$. By integrating both sides and using the definitions of $\frac{\partial x_i}{\partial v_i}$ and $\mu_1 = 0$, we will have

$$\int_1^i n_i \{1 - \frac{\lambda}{U_x}\} = \mu_i$$

From the first order condition of the revelation problem, $U_x(p_1, X)X'(i) = Z_R R'(i)$. This means that the sign of $X'(i)$ and $R'(i)$ are the same. Since $v(i)$ is strictly increasing, $X'(i)$ and $R'(i)$ must be increasing. When $X'(i)$ is increasing, $\frac{\lambda}{U_x}$ is increasing. This implies that if at some i^{**}, $1 - \lambda/U_x = 0$, then for any $i > i^{**}$, $1 - \lambda/U_x < 0$. However, $\mu_2 = 0$ from the FOC of v_2. This implies that μ_1 is initially strictly positive until i^{**} and then it begins to decrease and reaches zero at $i = 2$. Therefore, $\mu_i > 0$ for all $1 < i < 2$.

Now, we calculate the effect of increasing the commodity tax from $t = 0$. By using the envelope theorem, we have:

$$\frac{dW}{dt}\bigg|_{t=0} = \frac{\partial L}{\partial p_1}\frac{\partial p_1}{\partial t} + \lambda \int_1^2 c_{1i} n_i di \lambda + \int_1^2 [-\frac{\partial x_i}{\partial (p_1 + t)}] di$$

where $\partial L / \partial p_1$ is:

$$\begin{aligned}\frac{\partial L}{\partial p_1} = & \int_1^2 \{-\mu_i \frac{d[f'(h_i^s)h_i^s(g'_{si}/g_{si})]}{dh_i^s} + \beta_s g_{si}\}\{\frac{dh_i^s}{dw_i^s}\frac{dw_i^s}{dp_1} + \frac{dh_i^s}{dw^u}\frac{dw_i^u}{dp_1}\} di \\ & + \int_1^2 \{-\mu_i \frac{d[f'(h_i^u)h_i^u(g'_{ui}/g_{ui})]}{dh_i^u} + \beta_u g_{ui}\}\{\frac{dh_i^u}{dw_i^u}\frac{dw_i^u}{dp_1} + \frac{dh_i^u}{dw_i^u}\frac{dw_i^u}{dp_1}\} di \\ & + \lambda \int_1^2 [-\frac{\partial x_i}{\partial (p_1 + t)} - \frac{\partial x_i}{\partial w_i^s}\frac{\partial w_i^s}{\partial p_1} - \frac{\partial x_i}{\partial w_i^u}\frac{\partial w_i^u}{\partial p_1}] n_i di\end{aligned}$$

Note that $\partial x_i / \partial (p_1 + t) = -(U_{p_1})/(U_x)$. From the Roy's identity, $-(U_{p_1})/(U_x) = c_{1i}$. Therefore, $\lambda \int_1^2 c_{1i} n_i di = \lambda \int_1^2 (\frac{\partial x_i}{\partial p_1}) n_i di$. In addition, $\frac{\partial x_i}{\partial w_i^s} = z_{w_i^s}/U_x$ and $\frac{\partial x_i}{\partial w^u} = z_{w^u}/U_x$ and $\frac{\partial x_i}{\partial R_i} = Z_{R_i}/U_x$. Using the definition of $Z_{w_i^s}$ and Z_{w^u}, $\frac{\partial x_i}{\partial w_i^s} = -\alpha_i h_i^s / U_x$, $\frac{\partial x_i}{\partial w^u} = -\alpha_i h_i^u / U_x$ and $\frac{\partial x_i}{\partial R_i} = \alpha_i / U_x$. Thus, $\int_1^2 c_{1i} n_i di = \int_1^2 [-\frac{\partial x_i}{\partial (p_1 + t)} n_i di$. Therefore, on the other hand, the FOC of R_i at $t = 0$ is:

$$\begin{gathered}\{-\mu_i \frac{d[f'_s(h_i^s)h_i^s(g'_{si}/g_{si})]}{dh_i^s} + \beta_s g_{si}\}\frac{\partial h_i^s}{\partial R_i} + \{-\mu_i \frac{d[f'_u(h_i^u)h_i^u(g'_{ui}/g_{ui})]}{dh_i^u} + \beta_u g_{ui}\}\frac{\partial h_i^u}{\partial R_i} \\ + \lambda n_i = \lambda n_i \alpha_i / U_x\end{gathered}$$

Now, we will calculate $\lambda \int_1^2 [-\frac{\partial x_i}{\partial w_i^s}\frac{\partial w_i^s}{\partial p_1} - \frac{\partial x_i}{\partial w_i^u}\frac{\partial w_i^u}{\partial p_1}] n_i di$. Note that $-\lambda \frac{\partial x_i}{\partial w_i^s} n_i = \lambda \alpha_i n_i h_i^s / U_x$ and $-\lambda \frac{\partial x_i}{\partial w_i^u} n_i = \lambda \alpha_i n_i h_i^u / U_x$. From the FOC of R_i, $\lambda \int_1^2 [-\frac{\partial x_i}{\partial w_i^s}\frac{\partial w_i^s}{\partial p_1} - \frac{\partial x_i}{\partial w_i^u}\frac{\partial w_i^u}{\partial p_1}] n_i di$ is equal to:

$$\int_1^2 \left\{ \left(-\mu_i \frac{d[f_s'(h_i^s)h_i^s(g_{si}^{'}/g_{si})]}{dh_i^s} + \beta_s g_{si} \right) \frac{\partial h_i^s}{\partial R_i} \right.$$
$$\left. + \left(-\mu_i \frac{d[f_u'(h_i^u)h_i^u(g_{ui}^{'}/g_{ui})]}{dh_i^u} + \beta_u g_{ui} \right) \frac{\partial h_i^u}{\partial R_i} + \lambda n_i \right\} h_i^s \frac{\partial w_i^s}{\partial p_1} di$$
$$+ \left\{ \int_1^2 \left(-\mu_i \frac{d[f_s'(h_i^s)h_i^s(g_{si}/g_{si})]}{dh_i^s} + \beta_s g_{si} \right) \frac{\partial h_i^s}{\partial R_i} \right.$$
$$\left. + \left(-\mu_i \frac{d[f_u'(h_i^u)h_i^u(g_{ui}^{'}/g_{ui})]}{dh_i^u} + \beta_u g_{ui} \right) \frac{\partial h_i^u}{\partial R_i} + \lambda n_i \right\} h_i^u \frac{\partial w_i^u}{\partial p_1} di$$

Therefore, $\partial L/\partial p_1$ becomes

$$\left. \frac{\partial L}{\partial p_1} \right|_{t=0} = \left\{ \int_1^2 \left(-\mu_i \frac{d[f_s'(h_i^s)h_i^s(g_s^{'}/g_s)]}{dh_i^s} + \beta_s g_i^s \right) [\{ \frac{dh_i^s}{dw_i^s} + h_i^s \frac{\partial h_i^s}{\partial R_i} \} \frac{dw_i^s}{dp_1} + \{ \frac{dh_i^s}{dw_i^u} + h_i^u \frac{\partial h_i^s}{\partial R_i} \} \frac{dw_i^u}{dp_1}] di \right.$$
$$- \int_1^2 \left(-\mu_i \frac{d[f_u'(h_i^u)h_i^u(g_u^{'}/g_u)]}{dh_i^u} + \beta_u g_i^u \right) [\{ \frac{dh_i^u}{dw_i^s} + \frac{\partial h_i^u}{\partial R_i} h_i^s \} \frac{dw_i^s}{dp_1} + \{ \frac{dh_i^u}{dw_i^u} + \frac{\partial h_i^u}{\partial R_i} h_i^u \} \frac{dw_i^u}{dp_1}] di$$
$$\left. + \int_1^2 [-\lambda \frac{\partial x_i}{\partial (p_1 + t)} + \int_1^2 \lambda n_i h_i^s \frac{\partial w_i^s}{\partial p_1} di + \int_1^2 \lambda n_i h_i^u \frac{\partial w_i^u}{\partial p_i} di \right\}$$

Note that $\int_1^2 \lambda n_i h_i^s \frac{\partial w_i^s}{\partial \sigma} di + \int_1^2 \lambda n_i h_i^u \frac{\partial w^u}{\partial \sigma} di = \lambda y^1$ from the argument in the previous sub-section. Therefore, $+\int_1^2 [-\lambda \frac{\partial x_i}{\partial (p_1+t)} + \int_1^2 \lambda n_i h_i^s \frac{\partial w_i^s}{\partial p_1} di + \int_1^2 \lambda n_i h_i^u \frac{\partial w_i^u}{\partial p_i} di = 0$. We have:

$$\frac{\partial L}{\partial p_1} = \left\{ \int_1^2 \left(-\mu_i \frac{d[f_s'(h_i^s)h_i^s(g_s^{'}/g_s)]}{dh_i^s} + \beta_s g_i^s \right) [\frac{\partial \widetilde{h}_i^s}{\partial w_i^s} \frac{dw_i^s}{dp_1} + \frac{\partial \widetilde{h}_i^s}{\partial w_i^u} \frac{dw_i^u}{dp_1}] di \right.$$
$$\left. + \int_1^2 \left(-\mu_i \frac{d[f_u'(h_i^u)h_i^u(g_u^{'}/g_u)]}{dh_i^u} + \beta_u g_i^u \right) [\frac{\partial \widetilde{h}_i^u}{\partial w_i^s} \frac{dw_i^s}{dp_1} + \frac{\partial \widetilde{h}_i^u}{\partial w_i^u} \frac{dw_i^u}{dp_1}] di \right\}$$

From the FOC of H^s and H^u, $\beta_s = \frac{\partial W}{\partial p_1} \frac{\partial p}{\partial H^s}$ and $\beta_u = \frac{\partial W}{\partial p_1} \frac{\partial p}{\partial H^u}$. Thus,

$$\frac{\partial L}{\partial p_1} = \int_1^2 \left(-\mu_i \frac{d[f_s'(h_i^s)h_i^s(g_s^{'}/g_s)]}{dh_i^s} \right) [\frac{\partial \widetilde{h}_i^s}{\partial w_i^s} \frac{dw_i^s}{dp_1} + \frac{\partial \widetilde{h}_i^s}{\partial w_i^u} \frac{dw_i^u}{dp_1}] di$$
$$+ \int_1^2 \left(-\mu_i \frac{d[f_u'(h_i^u)h_i^u(g_u^{'}/g_u)]}{dh_i^u} \right) [\frac{\partial \widetilde{h}_i^u}{\partial w_i^s} \frac{dw_i^s}{dp_1} + \frac{\partial \widetilde{h}_i^u}{\partial w_i^u} \frac{dw_i^u}{dp_1}] di$$
$$+ \frac{\partial L}{\partial p_1} \left(\int_1^2 \frac{\partial p}{\partial H^s} g_i^s [\frac{\partial \widetilde{h}_i^s}{\partial w_i^s} \frac{dw_i^s}{dp_1} + \frac{\partial \widetilde{h}_i^s}{\partial w_i^u} \frac{dw_i^u}{dp_1}] di + \int_1^2 \frac{\partial p}{\partial H^u} g_i^u [\frac{\partial \widetilde{h}_i^u}{\partial w_i^s} \frac{dw_i^s}{dp_1} + \frac{\partial \widetilde{h}_i^u}{\partial w_i^u} \frac{dw_i^u}{dp_1}] di \right)$$

Solving $\frac{\partial L}{\partial p_1}$, we have:

$$\frac{\partial L}{\partial p_1} = \frac{1}{\triangle}\left\{\int_1^2 \left(-\mu_i \frac{d[f_s'(h_i^s)h_i^s(g_s^{'}/g_s)]}{dh_i^s}\right)[\frac{\partial \widetilde{h}_i^s}{\partial w_i^s}\frac{dw_i^s}{dp_1} + \frac{\partial \widetilde{h}_i^s}{\partial w_i^u}\frac{dw_i^u}{dp_1}]di \right.$$
$$\left. \int_1^2 \left(-\mu_i \frac{d[f_u'(h_i^u)h_i^u(g_u^{'}/g_u)]}{dh_i^u}\right)[\frac{\partial \widetilde{h}_i^u}{\partial w_i^s}\frac{dw_i^s}{dp_1} + \frac{\partial \widetilde{h}_i^u}{\partial w_i^u}\frac{dw_i^u}{dp_1}]di\right\}$$

where $\triangle = 1 - \left(\int_1^2 \frac{\partial p}{\partial H^s} g_i^s[\frac{\partial \widetilde{h}_i^s}{\partial w_i^s}\frac{dw_i^s}{dp_1} + \frac{\partial \widetilde{h}_i^s}{\partial w_i^u}\frac{dw_i^u}{dp_1}]di + \int_1^2 \frac{\partial p}{\partial H^u} g_i^u[\frac{\partial \widetilde{h}_i^u}{\partial w_i^s}\frac{dw_i^s}{dp_1} + \frac{\partial \widetilde{h}_i^u}{\partial w_i^u}\frac{dw_i^u}{dp_1}]di\right) > 0.$

$$\frac{\partial L}{\partial p_1} = \frac{1}{\triangle}\left\{\int_1^2 \left(-\mu_i[\frac{f_s''(h_i^s)h_i^s}{f_s'(h_i^s)} + 1](g_s^{'}/g_s)\right)[f_s'\frac{\partial \widetilde{h}_i^s}{\partial w_i^s}\frac{dw_i^s}{dp_1} + f_s'\frac{\partial \widetilde{h}_i^s}{\partial w_i^u}\frac{dw_i^u}{dp_1}]di \right.$$
$$\left. \int_1^2 \left(-\mu_i[\frac{f_u''(h_i^u)h_i^u}{f_u'(h_i^u)} + 1](g_u^{'}/g_u)\right)[f_u'\frac{\partial \widetilde{h}_i^u}{\partial w_i^s}\frac{dw_i^s}{dp_1} + f_u'\frac{\partial \widetilde{h}_i^u}{\partial w_i^u}\frac{dw_i^u}{dp_1}]di\right\}$$

From the definition of $\widetilde{h}_i^s$ and $\widetilde{h}_i^u$, we have:

$$f_s'(\widetilde{h}_i^s)\frac{\partial \widetilde{h}_i^s}{\partial w_i^s} + f_u'(\widetilde{h}_i^u)\frac{\partial \widetilde{h}_i^u}{\partial w_i^s} = 0 \text{ and } f_u'(\widetilde{h}_i^s)\frac{\partial \widetilde{h}_i^s}{\partial w_i^u} + f_u'(\widetilde{h}_i^u)\frac{\partial \widetilde{h}_i^u}{\partial w_i^u} = 0$$

Therefore, we have:

$$\left.\frac{\partial L}{\partial p_1}\right|_{t=0} = -\frac{1}{\triangle}\left\{\int_1^2 \mu_i \left([\frac{f_s''(h_i^s)h_i^s}{f_s'(h_i^s)} + 1](g_s^{'}/g_s) - [\frac{f_u''(h_i^u)h_i^u}{f_u'(h_i^u)} + 1](g_u^{'}/g_u)\right) f_s^i[\frac{\partial \widetilde{h}_i^s}{\partial w_i^s}\frac{dw_i^s}{dp_1} + f_s'\frac{\partial \widetilde{h}_i^s}{\partial w_i^u}\frac{dw_i^u}{dp_1}]di\right\}$$
$$= -\frac{1}{\triangle}\left\{\int_1^2 \mu_i[\gamma^s(g_s^{'}/g_s) - \gamma^u(g_u^{'}/g_u)]f_s^i[\frac{\partial \widetilde{h}_i^s}{\partial w_i^s}\frac{dw_i^s}{dp_1} + f_s'\frac{\partial \widetilde{h}_i^s}{\partial w_i^u}\frac{dw_i^u}{dp_1}]di\right\}$$

Using the definition of $\partial p_1/\partial H^s$ and $\partial p_1/\partial H^u$, This implies that dW/dt is equal to

$$\frac{dW}{dt_1} = -\Psi_2\left\{\int_1^2 \mu_i[\gamma^s(g_s^{'}/g_s) - \gamma^u(g_u^{'}/g_u)]f_s^i[\frac{\partial \widetilde{h}_i^s}{\partial w_i^s}\frac{dw_i^s}{dp_1} + f_s'\frac{\partial \widetilde{h}_i^s}{\partial w_i^u}\frac{dw_i^u}{dp_1}]di\right\}$$

where

$$\Psi_2 = \frac{-RD_p}{RD_{p_1} - RS_{p_1} - RS_{H^s}\int_1^2 g_i^s[\frac{\partial \widetilde{h}_i^s}{\partial w_i^s}\frac{dw_i^s}{dp_1} + \frac{\partial \widetilde{h}_i^s}{\partial w_i^u}\frac{dw_i^u}{dp_1}]di - RS_{H^u}\int_1^2 g_i^u[\frac{\partial \widetilde{h}_i^u}{\partial w_i^s}\frac{dw_i^s}{dp_1} + \frac{\partial \widetilde{h}_i^u}{\partial w_i^u}\frac{dw_i^u}{dp_1}]di}$$

From the condition of the comparative advantage, $\gamma^s(g_s^{'}/g_s) - \gamma^u(g_u^{'}/g_u) > 0$. In addition, $\Psi_2 < 0$. Thus, $\frac{dW}{dt} > 0$.