# An Economic Response to Unsolicited Communication
# **Appendices**

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## A Increased Information

There exist two ways to increase participant information: the sender can know $r$, and the recipient can know $s$. Given the sequence of communication, we first analyze informed senders, then add informed recipients. In the latter, private values become common knowledge. In both models, increasing information increases efficiency relative to a perfect filter.

Note that perfect correlation of sender and receiver values (assuming invertibility) is a subset of the common knowledge case. As before, the recipient need not know sender value *ex ante* but she can infer it *ex post*. This follows from the fact that knowing either private value and knowing the correlation function is sufficient to compute the other value.

In the analysis that follows, we make one additional assumption, that the recipient can credibly commit *ex ante* to a policy. Since information is known, commitment can be enforced via external reputation systems or a trusted third party. It does not matter how it is enforced as long as the recipient has the proper incentives to execute the policy to which she committed. As in (Rasmusen, 2001) precommitment and reputations improve welfare. We also relax our assumptions on the distribution of communications values. The following mechanisms work regardless of the magnitude of value, the density function

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on value, and the functional form on correlated value (assuming invertibility), and require only finite maximum and minimum values.

## A.1 Informed Senders

**Proposition 1** *If recipients can commit* ex ante *to a policy and the sender* ex ante *knows the recipient's value, the recipient surplus is at least as great under the ABM as under the perfect filter, always:*

$$\mathrm{RS}_{ABM} \geq \mathrm{RS}_{PF}$$

*Proof:* 1 We offer an existence proof that a Nash equilibrium choices in filtering strategies can do no better than those in ABM strategies. Consider a recipient request for a bond of size $b = c_r - \underline{r}$ coupled with the recipient's *ex ante* commitment to a policy of refunding $\rho(r)$, where

$$\rho(r) = \begin{cases} r - \underline{r} & \text{if } r < c_r \\ c_r - \underline{r} & \text{otherwise.} \end{cases}$$

First consider the case in which $r < c_r$. This represents communications that are unwanted by the recipient and would be filtered by the perfect filter. Here, the recipient keeps $b - \rho(r) = c_r - \underline{r} - (r - \underline{r}) = c_r - r$ of the bond. Adding this to the value of the email leaves her with surplus $r - c_r + c_r - r = 0$. Participation is Individually Rational for the recipient and provides the same surplus as the perfect filter. The sender, however, receives surplus $s - c_s - b + \rho(r) = s - c_s - (c_r - \underline{r}) + (r - \underline{r}) = s - c_s + r - c_r$. Whenever social surplus is positive, the sender rationally sends.

Consider now $r \geq c_r$. Here the recipient returns the entire bond ($\rho(r) = b$) and both parties keep their normal baseline surplus. This reflects the same surplus for both parties as under the perfect filter.

The strategy for the sender depends on knowing both $r$ and $s$, while the recipient strategy depends only on knowing $r$. Since this strategy works regardless of sender strategy (it is a dominant strategy), any recipient chosen strategy must do at least this well. This strategy recreates the perfect filter strategy from the recipient viewpoint, but leaves the sender with the surplus of the communications that would have been filtered. This equivalence was not possible before because the private value of $r$ did not allow senders to compute their own best response. ■

## A.2 Common Information

Here we consider what happens when both the sender and recipient know each other's value. The sender knows the value to the recipient *ex ante*, but the recipient does not learn the sender's value until she learns her own private $r$ after receiving and incurring costs $c_r$.

**Proposition 2** *If the recipient can commit* ex ante *to a policy, senders know* ex ante *the value to the recipient, and the recipient learns or can infer* s *after learning* r*, then recipient surplus under the ABM is at least as great as that under the perfect filter:* $\mathrm{RS}_{ABM} \geq \mathrm{RS}_{PF}$. *In addition, maximum social surplus is achieved, with all surplus going to the recipient, meaning* $\mathrm{W}_{ABM} = \max W$, $\mathrm{SS}_{ABM} = 0$, *and* $\mathrm{RS}_{ABM} = \mathrm{W}_{ABM}$.

*Proof:* 2 Consider the following mechanism. The recipient requests a bond of size $b = \max\{\overline{s} - c_s, c_r - \underline{r}\}$. She commits *ex ante* to refund $\rho(s,r)$ to the sender, where

$$\rho(s,r) = b - \max\{s - c_s - \epsilon, c_r - r\}$$

and where $\epsilon \geq 0$. The sender's strategy is to send if and only if $s - c_s - b + \rho(s,r) \geq 0$.

This mechanism is Budget Balanced, Individually Rational for both parties, Efficient, and is a dominant strategy equilibrium. Since this mechanism uses no outside funds, it is trivially Budget Balanced. Next we show that it is Individually Rational.

The bond is added to the recipient's net value, but the refund is then removed. This yields her total surplus:

$$\begin{aligned}
\mathrm{RS}_{ABM} &= r - c_r + b - \rho(r,s) \\
&= r - c_r + b - (b - \max\{s - c_s - \epsilon, c_r - r\}) \\
&= r - c_r + \max\{s - c_s - \epsilon, c_r - r\} \\
&= \max\{r - c_r + s - c_s - \epsilon, 0\}
\end{aligned}$$

Since $\mathrm{RS}_{ABM} \geq 0$, it is *ex ante* Individually Rational for the recipient to participate. As before, the sender's interim Individual Rationality allows him to choose per-message whether to send.

Next we show efficiency. First we must determine sender willingness to send messages. The sender loses the value of the bond $b$ and receives his net value plus the refund from the recipient:

$$\begin{aligned}
\mathrm{SS}_{ABM} &= s - c_s - b + \rho(s,r) \\
&= s - c_s - b + b - \max\{s - c_s - \epsilon, c_r - r\} \\
&= s - c_s - \max\{s - c_s - \epsilon, c_r - r\} \\
&= \min\{(s - c_s) - (s - c_s - \epsilon), (s - c_s) - (c_r - r)\} \\
&= \min\{\epsilon, s - c_s + r - c_r\}
\end{aligned}$$

Since $\epsilon \geq 0$, the sender will choose *not* to send messages only when $s - c_s + r - c_r < 0$. This is precisely the condition for positive welfare. Therefore, all positive welfare messages are sent, yielding the maximum possible total surplus.

Finally, we show that this mechanism is a dominant strategy equilibrium. Let $\epsilon \to 0$. This provides the recipient with all available surplus, and leaves zero surplus for the sender while still remaining Individually Rational. Since the sender has interim Individual Rationality, it is impossible for any mechanism to provide the recipient with greater surplus. Therefore, it is a dominant strategy for the recipient. Given this dominant strategy, the best response for the sender is to participate, as at worst he is indifferent between participation and nonparticipation. ■

Achieving maximum surplus is consistent with the Myerson and Satterthwaite (1983) claim on the impossibility of designing a mechanism for which Individual Rationality, Budget Balance, and Efficiency all hold when there also exists the possibility of inefficient trade, here $s + r < c_s + c_r$. Their theorem hinges on the fact that private information introduces inefficiency. The mechanism must provide both participation incentive and cover information rents. Because full information eliminates information rents, we can achieve all three criteria simultaneously. As a special case, consider the situation in which the recipient's value is any invertible function of the sender's value ($r = f(s)$).

Since communication itself is the negotiation problem, the ABM takes advantage of the fact that senders initiate contact. It avoids negotiation and instead has the recipient implement a take-it-or-leave-it contact policy that shifts power to the recipient. In this case, it is possible both for these transactions to be efficient and for the recipient to take all surplus from the sender.

Two other conditions can lead to first-best levels of welfare. One is a recipient capacity constraint. If recipients are boundedly rational and can process no more than $m$ messages, sender and recipient values are private information, and senders want at most one unit of attention, then as van Zandt (2004) observes, scarce attention is efficiently allocated by a Vickrey auction. The communications network delivers messages with the $m$ highest positive bids, transferring the value of the $(m+1)^{st}$ highest bid or 0 if senders place $\leq m$ bids. Because of the practical complexity of the first-best mechanism, van Zandt develops a second best alternative in the form of a tax. This reduces "information overload" caused by over-exploitation of scarce attention.

Section 6 in the main paper illustrates a second mechanism that can produce first-best welfare in the context of heterogeneous senders and recipients. In certain cases, a separating equilibrium is possible that supports perfect matching of senders and recipients. The choice of bond serves as both screen for unwanted contact and signal of recipient type.

# B Proof of Lemma 5

The proof has two steps. We show that at least one sender always remains in the market, and of those that remain, at least one has zero surplus. We show this in reverse order.

*Proof:* Consider the choice of a bond for a recipient. We proceed with a proof by contradiction. Assume that a recipient pool chooses a bond that leaves all its matching senders with positive surplus. This is not incentive compatible because the recipient could increase the bond by $\epsilon$, receive higher surplus, and not change the behavior of any sender in the pool. Therefore at least one sender will have zero surplus.

Next assume that the recipient chooses a bond higher than maximum expected surplus of any sender. This will cause all senders to stop sending to this recipient type causing the recipient to receive zero surplus. Reducing the bond enough so that at least one sender sends to this type increases that recipient type's surplus. Therefore, at least one sender will remain in the market. ■

# C Proof of Lemma 6

*Proof:* To establish a contradiction, assume that a bond choice grants any recipient *less* surplus than under Open Access. Then, choosing a bond value of 0 reproduces surplus of the baseline for that recipient type and all matching senders. Note that any equilibrium choice but 0 by other recipient types improves sender targeting, strictly increasing welfare for recipients who choose 0 bonds. Thus recipient surplus can be equal to or greater than under Open Access, but it cannot be less. ■

# D Proof of Lemma 7

*Proof:* Consider the recipient choice of the bond size, as this is the first strategic move. If a recipient chooses a pooling bond that causes her to stop receiving matched messages, then the value she receives from mismatched sender bonds must exceed the maximum value of receiving matching bonds and messages. Shown is the proof for $\mathcal{G}$ type recipients. For type $\mathcal{B}$ recipients, the proof is similar.

$$\gamma(s_G - c_s + r_G - c_r) \leq (1-\gamma)(s_B - c_s - c_r) \tag{1}$$

If this is true, then

$$\gamma(s_G - c_S) \leq \gamma(s_G - c_s + r_G - c_r) \leq (1-\gamma)(s_B - c_s - c_r) \leq (1-\gamma)(s_B - c_s) \tag{2}$$

so

$$\gamma(s_G - c_S) \leq (1-\gamma)(s_B - c_s) \tag{3}$$

So, if a recipient is willing to forgo receipt of messages she likes, then the other sender type has greater aggregate surplus. Or, the contrapositive of this is that if a sender has greater aggregate surplus, then no recipient who likes those messages is willing to forgo them. ■

# E Proof of Lemma 8

*Proof:* If all three recipient types have separate bond values, then senders will be able to distinguish recipients by type. In this situation, senders prefer to send to only those recipients who receive value from their messages, and thus no losses are created due to mistargeting. At the same time, all messages that are valuable communications are sent. This leads to the maximum social surplus. This corresponds to Case 1 in Table 1.

If all three recipient types choose separate bonds, then senders will be able to distinguish recipients by type. In this situation, senders prefer to send to only those recipients who value their messages, and thus no losses are created as a result of mistargeting. At the same time, all messages that are valuable communications are sent. This leads to the maximum social surplus, corresponding to Case 1 above.

First, we define conditions that must be true for type $\mathcal{G}$ recipients to separate. If type $\mathbb{G}$ senders are willing to send to a pool including the type $\mathcal{G}$

recipients, then the constraint for type $\mathcal{G}$ recipients to separate is

$$(1-\gamma)c_r \geq ((1-\epsilon_G)s_B - c_s) - \gamma(s_G - c_s) \tag{4}$$

However, if the $\mathbb{G}$ senders forgo any value from type $\mathcal{G}$ recipients, then the constraint is

$$c_r \geq (1-\epsilon_G)s_B - c_s \tag{5}$$

Next, we have similar constraints for type $\mathcal{B}$ recipients:

$$\gamma c_r \geq ((1-\epsilon_B)s_G - c_s) - (1-\gamma)(s_B - c_s) \tag{6}$$

$$c_r \geq (1-\epsilon_B)s_G - c_s \tag{7}$$

Finally, we need to determine when type $\mathcal{U}$ recipients will separate. They have two logical strategies: pool with the recipients who collect a higher bond, forgoing any value from the other sender type, or collect a bond $\delta$ below the smaller bond, uniquely identifying themselves and receiving *all* messages. If the type $\mathbb{G}$ senders have a higher surplus, then the constraint for being unique is

$$\gamma(s_G - s_B) \leq (1-\gamma)(r_B - c_r + s_B - c_s) - \delta \tag{8}$$

Likewise, if type $\mathbb{B}$ senders have a higher surplus, then the constraint is

$$(1-\gamma)(s_B - s_G) \leq \gamma(r_G - c_r + s_G - c_s) - \delta \tag{9}$$

If all six of these inequalities hold, then no recipient will have an incentive to pool, and the three recipient types will all have distinct bond values. This situation produces a dominant strategy equilibrium.

Finally, we provide example values that show the intersection of these inequalities is non-empty:

$$c_s = c_r = \frac{3}{2}$$
$$s_G = s_B = 2$$
$$r_G = r_B = 3$$
$$\gamma = \frac{1}{2}$$
$$\epsilon_G = \epsilon_B = \frac{1}{3}$$

Note that if we let $c_s = c_r = 1$, then this first-best property of the ABM still holds, but a flat tax actually harms welfare. ■

| Case | Pooled | Separate |
|---|---|---|
| 1 | $\{\}$ | $\{U\},\{G\},\{B\}$ |
| 2 | $\{U,G\}$ | $\{B\}$ |
| 3 | $\{U,B\}$ | $\{G\}$ |
| 4 | $\{G,B\}$ | $\{U\}$ |
| 5 | $\{U,B,G\}$ | $\{\}$ |

Table 1: Feasible Equilibria

# F Proof of Proposition 5

What follows is an analysis and comparison with a tax for each of the five equilibrium cases of the ABM.

**$\mathcal{U}$, $\mathcal{G}$, and $\mathcal{B}$ are all separate** By Lemma 8 above, the ABM is a first-best solution in this case. Therefore, it is at least as good as any other solution, including the flat tax.

**$\mathcal{U}$ and $\mathcal{G}$ pool, $\mathcal{B}$ is separate** For this situation, we must consider the behavior of the senders. A type $\mathbb{G}$ sender will always just send to the pooled $\mathcal{U}$ and $\mathcal{G}$ recipients, as they are exactly who he wants to reach. However, a type $\mathbb{B}$ sender has a choice. He can send only to $\mathcal{B}$ recipients, giving up any value from the $\mathcal{U}$ type recipients. Or he can send to everyone, incurring the costs of mistargeting the $\mathcal{G}$ types. Note that these costs also include the cost of paying a bond to the $\mathcal{G}$ types, who have no incentive to do anything other than collect the bond for mistargeted messages.

**Subcase A:** First let us consider the case where type $\mathbb{B}$ senders only send to type $\mathcal{B}$ recipients, and do not send to the pool. In this case, no costs arise from mistargeted messages. As such, regardless of bond values and collection policies, the total welfare for the system will be

$$\begin{aligned} \mathrm{W}^{\mathrm{ABM}} = {} & \gamma(1-\epsilon_B)(s_G - c_s + r_G - c_r) \\ & + (1-\gamma)(\epsilon_B)(s_B - c_s + r_B - c_r) \end{aligned} \tag{10}$$

which is strictly greater than $\mathrm{W}^t$, the welfare from a tax.

**Subcase B:** Now, we must consider the case where type $\mathbb{B}$ senders send to everyone. In this case, the total welfare of the system will be

$$\begin{aligned} \mathrm{W}^{\mathrm{ABM}} = {} & \gamma(1-\epsilon_B)(s_G - c_s + r_G - c_r) \\ & + (1-\gamma)(1-\epsilon_G)(s_B - c_s + r_B - c_r) \\ & - (1-\gamma)(\epsilon_G)(c_s + c_r) \end{aligned} \tag{11}$$

$$\begin{aligned} \mathrm{W}^{\mathrm{ABM}} = {} & \gamma(1-\epsilon_B)(s_G - c_s + r_G - c_r) \\ & + (1-\gamma)\epsilon_B(s_B - c_s + r_B - c_r) \\ & + (1-\gamma)(1-\epsilon_G-\epsilon_B)(s_B - c_s + r_B - c_r) - (1-\gamma)(\epsilon_G)(c_s + c_r) \end{aligned} \tag{12}$$

In this equation, the first line represents all transactions between type $\mathbb{G}$ senders and the pool. This is the surplus under a tax. The second line represents all transactions between type $\mathbb{B}$ senders and type $\mathcal{B}$ recipients. This is positive by assumption. The final line represents the transactions between type $\mathbb{B}$ senders and the pool, including both messages to type $\mathcal{U}$ recipients and the mistargeted messages to type $\mathcal{G}$ recipients.

Considering bonds paid and received, the final line can be divided into three parts, the surpluses for the pooled recipients and the spamming sender, where $b - c_s$ is the bond charged by the pool:

| | |
|---|---|
| $\mathcal{U}$ Recipients | $(1-\gamma)(1-\epsilon_G-\epsilon_B)(b - c_s + r_B - c_r)$ |
| $\mathcal{G}$ Recipients | $(1-\gamma)\epsilon_G(b - c_s - c_r)$ |
| $\mathcal{B}$ Senders | $(1-\gamma)\left[(1-\epsilon_G-\epsilon_B)s_B - (1-\epsilon_B)b\right]$ |

All three of these equations must be positive by Individual Rationality. The second equation shows the $\mathcal{G}$ recipient bearing all mistargeting costs in the form of reduced bonds and reading costs she pays directly. If the bond $b$ is not enough to cover those costs, then $\mathcal{G}$ recipients could lower their bond by $\delta$ to distinguish themselves from $\mathcal{U}$ recipients. Therefore, the surplus generated by having these transactions in the market is positive, and the ABM is the more socially beneficial mechanism in this situation.

**$\mathcal{U}$ and $\mathcal{B}$ pool, $\mathcal{G}$ is separate** This situation is very similar to the previous case. It too has two subcases, that in which type $\mathbb{G}$ senders send to only $\mathcal{G}$ recipients, and the other in which they send to everyone.

**Subcase A:** When the $\mathbb{G}$ senders do not send to the pool, then the total social welfare is

$$\begin{aligned} W &= \gamma\epsilon_G(s_G - c_s + r_G - c_r) + \\ &\quad (1-\gamma)(1-\epsilon_G)(s_B - c_s + r_B - c_r) \\ &= \gamma(1-\epsilon_B)(s_G - c_s + r_G - c_r) + \\ &\quad (1-\gamma)(1-\epsilon_G)(s_B - c_s + r_B - c_r) + \\ &\quad - \gamma(1-\epsilon_G-\epsilon_B)(s_G - c_s + r_G - c_r) \end{aligned}$$

The universals pool only if

$$\begin{aligned} (1-\gamma)(s_B - c_s + r_B - c_r) &\geq \gamma(s_G - c_s + r_G - c_r) + (1-\gamma)(s_G - c_s + r_B - c_r) \\ &\geq \gamma(s_G - c_s + r_G - c_r) \end{aligned}$$

Multiplying both sides by $(1-\epsilon_B-\epsilon_G)$, we get

$$\begin{aligned} \gamma(1-\epsilon_B-\epsilon_G)(s_G - c_s + r_G - c_r) &\leq (1-\gamma)(1-\epsilon_B-\epsilon_G)(s_B - c_s + r_B - c_r) \\ &\leq (1-\gamma)(1-\epsilon_B)(s_B - c_s + r_B - c_r) \end{aligned}$$

which implies that the welfare from the ABM is greater than that of the tax.

**Subcase B:** The welfare in this subcase is

$$\begin{aligned} W &= \gamma(1-\epsilon_B)(s_G - c_s + r_G - c_r) + \\ &\quad (1-\gamma)(1-\epsilon_G)(s_B - c_s + r_B - c_r) + \\ &\quad - \gamma\epsilon_B(c_s + c_r) \\ &= (1-\gamma)(1-\epsilon_G)(s_B - c_s + r_B - c_r) + \\ &\quad \gamma\epsilon_G(s_G - c_s + r_G - c_r) + \\ &\quad \gamma(1-\epsilon_G-\epsilon_B)(s_G - c_s + r_G - c_r) - \gamma\epsilon_B(c_s + c_r) \end{aligned}$$

As before, this last line represents the total surplus from type $\mathbb{G}$ senders when sending to the pool, and can be divided to the surplus retained by each party involved (given a bond $b - c_s$):

| | |
|---|---|
| $\mathcal{U}$ Recipients | $\gamma(1-\epsilon_G-\epsilon_B)(b - c_s + r_G - c_s)$ |
| $\mathcal{B}$ Recipients | $\gamma\epsilon_B(b - c_s - c_r)$ |
| $\mathcal{G}$ Senders | $\gamma\left[(1-\epsilon_B-\epsilon_G)s_G - (1-\epsilon_G)b\right]$ |

All three of these surpluses must be non-negative by Individual Rationality. In addition to this, type $\mathcal{U}$ recipients had to have chosen to be in the pool:

$$\gamma(b - c_s + r_G - c_s) + (1-\gamma)(b - c_s + r_B - c_s) \geq \gamma(s_G - c_s + r_G - c_s) \quad (13)$$

An extended proof follows.

$$(1 - \epsilon_B - \epsilon_G)s_B - (1-\epsilon_G)b \geq 0 \quad (14)$$

Senders afford to spam

$$(1-\epsilon_G)s_G - \epsilon_B \geq (1-\epsilon_G)b \quad (15)$$

$$b \geq c_s + c_r \quad (16)$$

since $\mathcal{B}$ recipients are willing to pool

$$s_G - c_s \geq b - c_S \quad (17)$$

$\mathbb{G}$ senders can afford to send to the pool

$$s_G \geq c_s + c_r \quad (18)$$

$$(1-\epsilon_G)s_G - \epsilon_B(c_s + c_r) \geq (1-\epsilon_G)s_G - \epsilon_B s_G \quad (19)$$

$$(1-\epsilon_G)s_G - (1-\epsilon_B)(c_s + c_r) \geq (1-\epsilon_G)b \quad (20)$$

from Eq. 15 and Eq. 19

$$(1-\epsilon_G)s_G - (1-\epsilon_G)b \geq (1-\epsilon_B)(c_s + c_r) \quad (21)$$

$$\gamma(b - c_s + r_G - c_s) + (1-\gamma)(b - c_s + r_B - c_s) \geq \gamma(s_G - c_s + r_G - c_s) \quad (22)$$

since $\mathcal{U}$ is willing to be in pool

$$(1-\gamma)(b - c_s + r_B - c_r) \geq \gamma(s_G - b) \quad (23)$$

$$(1-\gamma)(1-\epsilon_G)(b - c_s + r_B - c_r) \geq \gamma(1-\epsilon_G)(s_G - b) \quad (24)$$

$$\gamma(1-\epsilon_G)(s_G - b) \geq \gamma(1-\epsilon_B)(c_s + c_r) \quad (25)$$

from Eq. 21

$$(1-\gamma)(1-\epsilon_G)(b - c_s + r_B - c_r) \geq \gamma(1-\epsilon_B)(c_s + c_r) \quad (26)$$

$$s_B \geq b \quad (27)$$

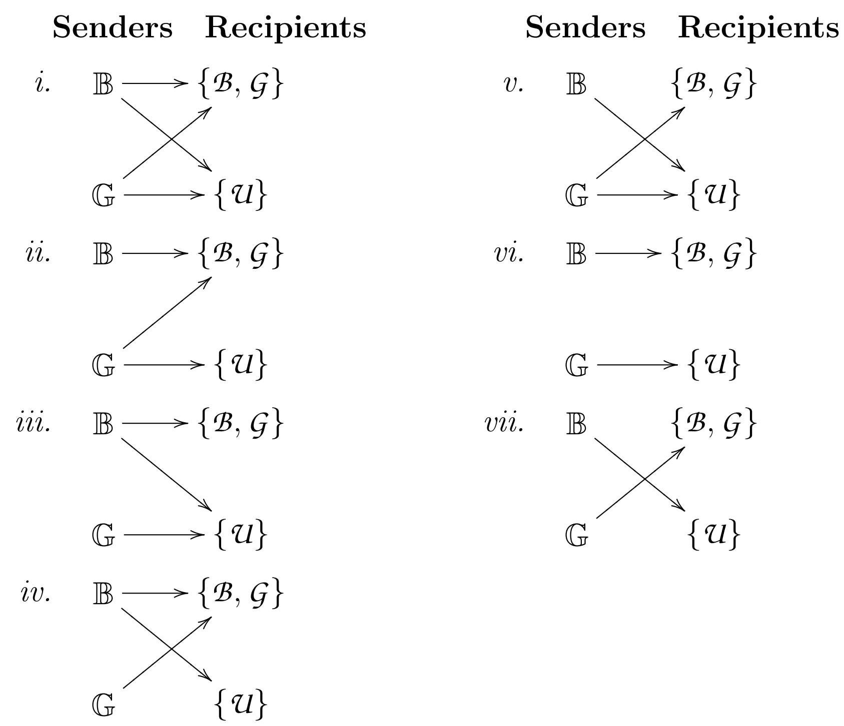


Figure 1: Subcases where $\mathcal{G}$ and $\mathcal{B}$ recipients pool together

$\mathbb{B}$ senders can afford it

$$(1-\gamma)(1-\epsilon_G)(s_B - c_s + r_B - c_r) \geq \gamma(1-\epsilon_B)(c_s + c_r) \tag{28}$$

This last line shows that the benefits of the ABM over the tax, and allowing type $\mathbb{B}$ to remain in the market, exceed any costs it imposes due to mistargeting.

**$\mathcal{G}$ and $\mathcal{B}$ pool, $\mathcal{U}$ is separate** This situation can be divided into five distinct subcases. These are graphically depicted in Figure 1. The first subcase involves all senders sending to all recipients, but universal recipients have a bond value distinct from the other recipient types. The second and fourth subcases both involve the situation where $\mathcal{B}$ and $\mathcal{G}$ recipients pool together on a bond low enough that both sender types send to them. The third and fifth subcases have the pool on a higher bond such that only one type of sender sends to the pool.

**Subcase *i*** This situation cannot happen. To see this, ask what (distinct) bond values the pool and type $\mathcal{U}$ recipients will have.

**Lemma 1** *There cannot exist two distinct bond values such that all sender types send their messages to recipients with those values.*

*Proof:* A quick proof by contradiction will show this. Assume that there exist two distinct bonds such that both senders send and pay both bond values. At least one party on the lower bond has an incentive to increase her bond value to that of the higher bond, since that will not cause any sender to stop sending. Therefore, in equilibrium, two distinct bond values are not stable. ■

In this subcase, all sender types send to two distinct pools of recipients. In order to have two different pools, there must be two distinct bond values. By the lemma above, this cannot happen.

**Subcase *ii*** In this subcase, type $\mathcal{U}$ recipients have chosen to receive only type $\mathbb{B}$ messages (giving up the bond and message from type $\mathbb{G}$ senders), but type $\mathcal{B}$ recipients have chosen to give up the messages they like to get the bonds from type $\mathbb{G}$ senders. These choices are inconsistent.

The type $\mathcal{U}$ recipient's choice here is

$$(1-\gamma)(s_B-c_s+r_B-c_r) \geq \gamma((1-\epsilon_B)s_G-c_s+r_G-c_r)+(1-\gamma)((1-\epsilon_B)s_G-c_s+r_B-c_r) \tag{29}$$

The type $\mathcal{B}$ recipient's choice here is

$$\gamma\left(\frac{\epsilon_G}{\epsilon_G+\epsilon_B}s_G - c_s - c_r\right)+(1-\gamma)\left(\frac{\epsilon_G}{\epsilon_G+\epsilon_B}s_G - c_s + r_B - c_r\right) \geq (1-\gamma)(s_B-c_s+r_B-c_r) \tag{30}$$

These two equations together form a contradiction, so this situation cannot happen.

**Subcase *iii*** In this subcase, there exist times when a flat tax provides greater welfare than the ABM. To see this, let the fraction of type $\mathcal{U}$ recipients $(1-\epsilon_G-\epsilon_B)$ become arbitrarily small. However, this is consistent with the proposition, since in this case type $\mathcal{G}$ recipients are willing to not receive any message they like.

**Subcase *iv*** This situation is very similar to Subcase *ii*. There is an inherent contradiction in the choices of type $\mathcal{U}$ and type $\mathcal{G}$ recipients.

The type $\mathcal{U}$ recipient's choice here is

$$\gamma(s_G-c_s+r_G-c_r) \geq (1-\gamma)((1-\epsilon_G)s_B-c_s+r_B-c_r)+\gamma((1-\epsilon_G)s_B-c_s+r_G-c_r) \tag{31}$$

The type $\mathcal{G}$ recipient's choice here is

$$(1-\gamma)\left(\frac{\epsilon_B}{\epsilon_G+\epsilon_B}s_B-c_s-c_r\right)+\gamma\left(\frac{\epsilon_B}{\epsilon_G+\epsilon_B}s_B-c_s+r_G-c_r\right) \geq \gamma(s_G-c_s+r_G-c_r) \tag{32}$$

These two equations together form a contradiction. Therefore this situation cannot happen.

**Subcase *v*** In this subcase, as in Subcase *iii*, there exist times when a flat tax provides greater welfare than the ABM. However, this is consistent with the proposition, since in this case type $\mathcal{B}$ recipients are willing to not receive matching messages.

**Subcases *vi* and *vii*** These two subcases cannot happen. By comparing the rationality constraints for this case, it can be seen that it is irrational for universals to choose to receive messages from only one sender type when the recipients who like only that type are willing to forgo those same messages.

**$\mathcal{U}$, $\mathcal{G}$, and $\mathcal{B}$ all pool** This can again be divided into three distinct subcases, graphically depicted in Figure 2. Subcase *i* is the situation where all three recipients choose a common bond size, and that bond size is low enough that both senders are willing to send to everyone in the pool. Subcase *ii* is the situation where all recipients pool on a high bond, such that only type B senders can afford to send to the pool. Finally, Subcase *iii* is the analogous situation, but only type $\mathbb{G}$ senders can afford to send to the pool.

**Subcase *i*** clearly has welfare equal to $\mathrm{W}^0$, the welfare in the baseline situation. This is because all possible transactions occur, both helpful and harmful. However, in this case, welfare under the ABM is greater than welfare under a flat tax because this case will only occur when a flat tax lowers welfare.

Let us assume that $(1-\epsilon_G)s_B-c_s \leq (1-\epsilon_B)s_G-c_s$. This implies that a flat tax would target and eliminate type $\mathbb{B}$ senders. In order for type $\mathcal{G}$ recipients to rationally choose this case, their surplus must be greater than that which they would get by choosing a high bond such that only type $\mathbb{G}$ senders could

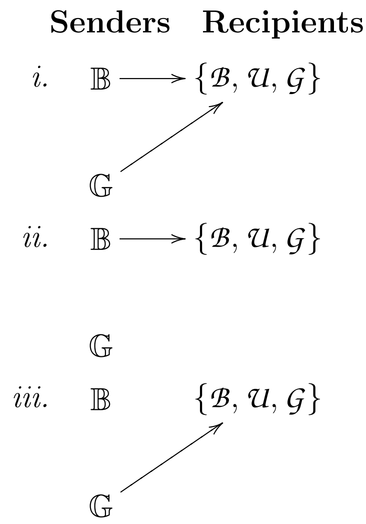


Figure 2: Subcases where all recipients pool together

reach them:

$$\gamma\left((1-\epsilon_G)s_B - c_s + r_G - c_r\right) + (1-\gamma)\left((1-\epsilon_G)s_B - c_s - c_r\right) > \gamma(s_G - c_s + r_G - c_r) \tag{33}$$

This equation and the assumption above together imply that

$$(1-\gamma)(1-\epsilon_G)(s_B - c_s + r_B - c_r) > ((1-\gamma)\epsilon_G + \gamma\epsilon_B)(c_s + c_r) \tag{34}$$

which implies that the tax harms welfare.

If we reverse the assumption, then the tax would target type $\mathbb{G}$ senders. Through an analogous derivation (starting with the $\mathcal{B}$ recipient's rationality constraint) a similar result (tax harms welfare) can be shown.

The intuition here is that in order for everyone to rationally choose a bond that keeps the type $\mathbb{B}$ senders in the market, the surplus gained from them must be enough to cover the costs of mistargeting caused by this.

**Subcase *ii*** cannot happen in equilibrium. The intuition here is that since type $\mathbb{G}$ senders cause more surplus by their transactions than type $\mathbb{B}$ senders do (by definition) and all recipients split the sender's surplus, at least one recipient type will prefer to set a bond such that type $\mathbb{G}$ senders can still receive them, since they have more surplus overall.

**Subcase *iii*** is one case where welfare from a flat tax clearly exceeds welfare from the ABM. In this case, recipients choose a bond equal to the flat tax, causing the type $\mathbb{B}$ senders to leave the market. However, unlike the tax, type $\mathcal{B}$ recipients remain in the market in order to extract some surplus in the form of bond payments. By staying in the market, they cause some mistargeting costs that do not exist under a tax, leaving lower total welfare.

However, note that in this situation, type $\mathcal{B}$ recipients are willing to forgo all messages they like in order to collect bonds from type $\mathbb{G}$ senders. This is consistent with the proposition.

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