Proofs for “Income Insecurity and Youth Emancipation: A Theoretical Approach”*

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1 Proofs of the results in Section 2

Proof of Lemma 1

We first note that:

$$\Delta_2 > 0 \Leftrightarrow c_{i2} > c_{p2}^n.$$

In words, the child prefers to move out if and only if her consumption while independent exceeds the consumption she would enjoy if she stayed. When parental income is such that $\tilde{y}_{c2} > \gamma_c$ (that is, the parent would provide positive transfers to a child who moved out), the difference in the child's consumption across residential states is:

$$c_{i2} - c_{p2}^n = \frac{y_{p2} + y_{c2} - \gamma_p - \gamma_c}{(\Gamma(n-1)+1)} - \frac{y_{p2} + y_{c2} - \gamma_p}{n},$$

which is negative, since $\Gamma(n-1)+1 > n$. This shows that $\Delta_2(y_{c2}) < 0$ for $y_{c2} \in [\gamma_c, \tilde{y}_{c2}]$. For $y_{c2} \in (\tilde{y}_{c2}, \bar{y}_{c2}]$, the child would not receive any transfers if she moved out. In this case,

$$c_{i2} - c_{p2}^n = y_{c2} - \gamma_c - \frac{y_{p2} + y_{c2} - \gamma_p}{n}.$$

It is straightforward to show that this difference is negative for income values y_{c2} such that $y_{c2} < \bar{y}_{c2}$, and positive for $y_{c2} > \bar{y}_{c2}$. This proves that $\Delta_2(y_{c2}) < 0$ for $y_{c2} \in (\tilde{y}_{c2}, \bar{y}_{c2})$ and $\Delta_2(y_{c2}) > 0$ for $y_{c2} > \bar{y}_{c2}$.

The derivative of Δ_2 with respect to y_{c2} is:

$$\frac{\partial u(c_{i2})}{\partial c_{i2}} \frac{\partial c_{i2}}{\partial y_{c2}} - \frac{\partial u\left(c_{p2}^n\right)}{\partial c_{p2}^n} \frac{\partial c_{p2}^n}{\partial y_{c2}}.$$

For $y_{c2} \in (\tilde{y}_{c2}, \bar{y}_{c2})$, $\partial c_{i2}/\partial y_{c2} = 1$, and $\partial c_{p2}^n/\partial y_{c2} = 1/n$. Also, since $\Delta_2 < 0$ in this range, $\partial u(c_{i2})/\partial c_{i2}$ exceeds $\partial u\left(c_{p2}^n\right)/\partial c_{p2}^n$. This implies that $\partial \Delta_2/\partial y_{c2} > 0$ in this interval.

The expression for $\partial \Delta_2/\partial y_{c2}$, for $y_{c2} \in (\gamma_c, \tilde{y}_{c2})$ is given by:

$$\frac{\partial \Delta_2}{\partial y_{c2}} = (c_{i2})^{-\alpha} \frac{1}{(\Gamma(n-1)+1)} - \left(c_{p2}^n\right)^{-\alpha} \frac{1}{n},$$

and

$$\begin{aligned} \frac{\partial \Delta_2}{\partial y_{c2}} &> 0 \Leftrightarrow \left(\frac{c_{p2}^n}{c_{i2}}\right)^{\alpha} > \frac{\Gamma(n-1)+1}{n} \\ &\Leftrightarrow \left(\underbrace{\frac{y_{p2} + y_{c2} - \gamma_p}{y_{p2} + y_{c2} - \gamma_p - \gamma_c}}_{>1}\right)^{\alpha} > \left(\underbrace{\frac{(\Gamma(n-1)+1)}{n}}_{>1}\right)^{1-\alpha}. \end{aligned}$$

Since $\Gamma > 1$, the term in braces on the right-hand side exceeds unity. When $\alpha > 1$, the right-hand side will be smaller than 1 and, since the left-hand side of the inequality is greater than 1, the inequality will be satisfied for all values of y_{p2} and y_{c2}. This proves that $\Delta(y_{c2})$ is strictly increasing for $y_{c2} \in (\gamma_c, \tilde{y}_{c2})$ when $\alpha > 1$.

The expression for $\partial\Delta_2/\partial y_{c2}$, for $y_{c2} > \bar{y}_{c2}$ is given by:

$$\frac{\partial \Delta_2}{\partial y_{c2}} = (c_{i2})^{-\alpha} - \left(c_{p2}^n\right)^{-\alpha}\frac{1}{n},$$

and

$$\frac{\partial \Delta_2}{\partial y_{c2}} > 0 \Leftrightarrow \left(\frac{c_{p2}^n}{c_{i2}}\right)^{\alpha} > \frac{1}{n} \Leftrightarrow \left(\underbrace{\frac{y_{p2} + y_{c2} - \gamma_p}{y_{c2} - \gamma_c}}_{>1}\right)^{\alpha} > \left(\underbrace{\frac{1}{n}}_{<1}\right)^{1-\alpha}.$$

Since $y_{p2} \geq \gamma_p$, the fraction on the left-hand side of this inequality exceeds unity and, since families have at least two persons, the fraction on the right-hand side is smaller than 1. For $\alpha < 1$, therefore, the inequality above is satisfied for all values of y_{c2} and y_{p2}. This proves that $\Delta_2(y_{c2})$ is strictly increasing for $y_{c2} > \bar{y}_{c2}$ when $\alpha < 1$. ■

Proof of Lemma 2

The proof is identical to the previous one. ■

Proof of Proposition 3

If $F(R) > 0$, the right-hand side of (6) is strictly positive. The moving-out income threshold $\bar{y}_{c1}$, such that $\Delta_1(\bar{y}_{c1}) = 0$, solves:

$$u\left(c_{i1}(\bar{y}_{c1})\right) - u\left(c_{p1}^n(\bar{y}_{c1})\right) = \int_R \left(u\left(c_{p2}^n\right) - u(c_{i2})\right) dF(y_{c2}, y_{p2}) > 0$$

$$\Leftrightarrow \quad \Delta_2(\bar{y}_{c1}) = \int_R \left(u\left(c_{p2}^n\right) - u(c_{i2})\right) dF(y_{c2}, y_{p2}) > 0.$$

Applying Lemma 1 to Δ_2, we know that it is strictly negative for $y_{c1} < \bar{y}_{c2}$, and strictly positive for $y_{c1} > \bar{y}_{c2}$. Further, from the properties of the utility function $u(\cdot)$, Δ_2 is continuous. Since $\Delta_2(\bar{y}_{c2}) = 0$, it follows that, for identical values of parental income across periods, $y_{p1} = y_{p2}$, the value of $\bar{y}_{c1}(y_{p1})$ that solves the previous equation must strictly exceed $\bar{y}_{c2}(y_{p1})$. If $\Delta_2(y_{c2})$ has a decreasing range for $y_{c1} > \bar{y}_{c2}$, there could be multiple solutions to the previous equation. If, however, $\alpha < 1$, $\Delta_2(y_{c1})$ has a strictly positive slope for $y_{c1} > \bar{y}_{c2}$ as shown in Lemma 1. In this case, the value of $\bar{y}_{c1}$ that solves the previous equation is unique. ■

Proof of Lemma 4

First, we show that the derivative of $\Delta_2(\cdot)$ with respect to y_{c2} in the range $y_{c2} \in [\gamma_c, \tilde{y}_{c2})$ is identical to the derivative of $\Delta_2(\cdot)$ with respect to y_{p2} in the range $y_{p2} > \tilde{y}_{p2}$. Notice that in both income intervals, an independent child is receiving transfers. We have that:

$$\frac{\partial \Delta_2}{\partial y_{c2}}|_{y_{c2}\in[\gamma_c,\tilde{y}_{c2})} = \frac{\partial}{\partial y_{c2}}\left[\frac{1}{1-\alpha}\left(\frac{y_{p2}+y_{c2}-\gamma_p-\gamma_c}{\Gamma(n-1)+1}\right)^{1-\alpha} - \frac{\left(y_{p2}+y_{c2}-\gamma_p\right)^{1-\alpha}}{1-\alpha}\right],$$

whereas

$$\frac{\partial \Delta_2}{\partial y_{p2}}|_{y_{p2}>\tilde{y}_{p2}} = \frac{\partial}{\partial y_{p2}}\left[\frac{1}{1-\alpha}\left(\frac{y_{p2}+y_{c2}-\gamma_p-\gamma_c}{\Gamma(n-1)+1}\right)^{1-\alpha} - \frac{\left(y_{p2}+y_{c2}-\gamma_p\right)^{1-\alpha}}{1-\alpha}\right],$$

the same as above. From Lemma 1, we have that $\Delta_2(\cdot, y_{p2})$ is strictly increasing for $y_{c2} \in (\tilde{y}_{c2}, \bar{y}_{c2})$, whereas Lemma 2 shows that $\Delta_2(y_{c2}, \cdot)$ is strictly decreasing for $y_{p2} \in (\bar{y}_{p2}, \tilde{y}_{p2})$. Say that $\partial\Delta_2/\partial y_{c2}$ in the range $y_{c2} \in [\gamma_c, \tilde{y}_{c2})$ is positive. Then, $\Delta_2(\cdot, y_{p2})$ will be monotonically increasing for $y_{c2} \leq \bar{y}_{c2}$. However, this implies that $\Delta_2(y_{c2}, \cdot)$ will be decreasing for $y_{p2} \in (\bar{y}_{p2}, \tilde{y}_{p2})$ and increasing for values of y_{p2} that exceed $\tilde{y}_{p2}$. Conversely, if $\partial\Delta_2/\partial y_{c2}$ in the range $y_{c2} \in [\gamma_c, \tilde{y}_{c2})$ were to be negative, then monotonicity of $\Delta_2(\cdot, y_{p2})$ would fail, whereas $\Delta_2(y_{c2}, \cdot)$ would be strictly decreasing for $y_{p2} \geq \bar{y}_{p2}$.

Proof of Proposition 5

$\bar{y}_{c1}\left(F_p^1\right)$ is the solution to the first line of the following equation:

$$\begin{aligned}
& \Delta_2\left(\bar{y}_{c1}\left(F_p^1\right), y_{p1}\right) = \int_{\gamma_c} \int_{\bar{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} \left[-\Delta_2\left(y_{c2}, y_{p2}\right)\right] dF_{p2}^1 dF_{c2} \\
= & -\int_{\gamma_c} \left[\begin{array}{c} \Delta_2\left(y_{c2}, y_{p2}\right) F_p^1\left(y_{p2}\right) |_{\bar{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} \\ -\int_{\bar{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} F_p^1\left(y_{p2}\right)\left(\partial\Delta_2\left(y_{c2}, y_{p2}\right)/\partial y_{p2}\right) dy_{p2} \end{array}\right] dF_{c2} \\
= & -\int_{\gamma_c} \left[\begin{array}{c} \Delta_2\left(y_{c2}, \tilde{y}_{p2}\left(y_{c2}\right)\right) F_p^1\left(\tilde{y}_{p2}\left(y_{c2}\right)\right) \\ -\int_{\bar{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} F_p^1\left(y_{p2}\right)\left(\partial\Delta_2\left(y_{c2}, y_{p2}\right)/\partial y_{p2}\right) dy_{p2} \end{array}\right] dF_{c2} \\
= & -\int_{\gamma_c} \left[\Delta_2\left(y_{c2}, \tilde{y}_{p2}\left(y_{c2}\right)\right) - \int_{\bar{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} F_p^1\left(y_{p2}\right)\left(\partial\Delta_2\left(y_{c2}, y_{p2}\right)/\partial y_{p2}\right) dy_{p2}\right] dF_{c2} \\
\geq & -\int_{\gamma_c} \left[\Delta_2\left(y_{c2}, \tilde{y}_{p2}\left(y_{c2}\right)\right) - \int_{\bar{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} F_p^2\left(y_{p2}\right)\left(\partial\Delta_2\left(y_{c2}, y_{p2}\right)/\partial y_{p2}\right) dy_{p2}\right] dF_{c2} \\
= & \Delta_2\left(\bar{y}_{c1}\left(F_p^2\right), y_{p1}\right)
\end{aligned}$$

where we are using $F_p^1\left(\tilde{y}_{p2}\left(y_{c2}\right)\right) = F_p^2\left(\tilde{y}_{p2}\left(y_{c2}\right)\right) = 1$, $\Delta_2\left(y_{c2}, \bar{y}_{p2}\left(y_{c2}\right)\right) = 0$, with the inequality following from the first-order stochastic dominance of F_p^1 over F_p^2 and the fact that Δ_2 is a decreasing function of y_{c2} in the range $\left[\bar{y}_{p2}\left(y_{c2}\right), \tilde{y}_{p2}\left(y_{c2}\right)\right)$, as shown in Lemma 2. Given $\alpha < 1$ and the corresponding strict monotonicity of $\Delta_2\left(\cdot, y_{p1}\right)$ for $y_{c1} \geq \bar{y}_{c2}$, it follows that:

$$\Delta_2\left(\bar{y}_{c1}\left(F_p^1\right), y_{p1}\right) \geq \Delta_2\left(\bar{y}_{c1}\left(F_p^2\right), y_{p1}\right) \Leftrightarrow \bar{y}_{c1}\left(F_p^1\right) \geq \bar{y}_{c1}\left(F_p^2\right). \blacksquare$$

Proof of Proposition 6

$\bar{y}_{c1}\left(F_c^1\right)$ is the solution to the first line of the following equation:

$$
\begin{aligned}
\Delta_2\left(\bar{y}_{c1}\left(F_c^1\right), y_{p1}\right) &= \int_{\gamma_p}\left[\int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})}\left(u\left(c_{pn2}\right)-u\left(c_{i2}\right)\right) dF_c^1\left(y_{c2}\right)\right] dF_p\left(y_{p2}\right) \\
&= -\int_{\gamma_p}\left[\int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} \Delta_2\left(y_{c2}, y_{p2}\right) dF_c^1\left(y_{c2}\right)\right] dF_p\left(y_{p2}\right) \\
&= -\int_{\gamma_p}\left(\begin{array}{c} \Delta_2\left(y_{c2}, y_{p2}\right) F_c^1\big|_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} \\ -\int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} F_c^1\left(y_{c2}\right)\left(\partial \Delta_2\left(y_{c2}, y_{p2}\right)/\partial y_{c2}\right) dy_{c2} \end{array}\right) dF_p\left(y_{p2}\right) \\
&= \int_{\gamma_p}\int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} F_c^1\left(y_{c2}\right)\left(\partial \Delta_2\left(y_{c2}, y_{p2}\right)/\partial y_{c2}\right) dy_{c2} dF_p\left(y_{p2}\right) \\
&\leq \int_{\gamma_p}\int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} F_c^2\left(y_{c2}\right)\left(\partial \Delta_2\left(y_{c2}, y_{p2}\right)/\partial y_{c2}\right) dy_{c2} dF_p\left(y_{p2}\right) \\
&= \Delta_2\left(\bar{y}_{c1}\left(F_c^2\right), y_{p1}\right)
\end{aligned}
$$

where we are using $F_c^1\left(\tilde{y}_{c2}\left(y_p\right)\right) = F_c^2\left(\tilde{y}_{c2}\left(y_p\right)\right) = 0$, $\Delta_2\left(\bar{y}_{c2}\left(y_{p2}\right), y_{p2}\right) = 0$, $\Delta_2\left(\cdot, y_{p2}\right)$ is an increasing function of the child's income in the range $\left(\tilde{y}_{c2}\left(y_{p2}\right), \bar{y}_{c2}\left(y_{p2}\right)\right)$, as shown in Lemma 1; the inequality follows finally from the assumed first-order stochastic dominance of F_1^c over F_c^2.

When $\alpha < 1$, it follows that:

$$
\Delta_2\left(\bar{y}_{c1}\left(F_c^1\right), y_{p1}\right) \leq \Delta_2\left(\bar{y}_{c1}\left(F_c^2\right), y_{p1}\right) \Leftrightarrow \bar{y}_{c1}\left(F_c^1\right) \leq \bar{y}_{c1}\left(F_c^2\right). \blacksquare
$$

Proof of Proposition 7

Recall that expected regret is computed according to:

$$
\bar{R} = \int_{\gamma_p}\left[\int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_p)} -\Delta_2\left(y_{c2}, y_{p2}\right) dF_c\right] dF_p.
$$

We begin by showing that $\Delta_2\left(\cdot, y_{p2}\right)$ is an increasing and concave function in the range $y_{c2} \in \left(\tilde{y}_{c2}, \bar{y}_{c2}\right)$. Lemma 1 establishes that Δ_2 is increasing in this range. As for concavity,

$$
\frac{\partial \Delta_2\left(y_{c2}\right)}{\partial y_{c2}} = \left(y_{c2}-\gamma_c\right)^{-\alpha} - \frac{1}{n}\left(\frac{y_{c2}+y_{p2}-\gamma_p}{n}\right)^{-\alpha}
$$

and

$$\frac{\partial^2 \Delta_2 \left(y_{c2}\right)}{\partial y_{c2}^2} = -\alpha \left(y_{c2} - \gamma_c\right)^{-\alpha-1} + \frac{\alpha}{n^2} \left(\frac{y_{c2} + y_{p2} - \gamma_p}{n}\right)^{-\alpha-1} < 0,$$

where the inequality follows from the fact that, in this range,

$$y_{c2} - \gamma_c < \frac{y_{c2} + y_{p2} - \gamma_p}{n}.$$

Define:

$$G\left(x\right) \equiv \int_{\tilde{y}_c}^{x} \left[F_c^1 - F_c^2\right] dy_{c2},$$

with $G\left(x\right) \leq 0$ from second-order stochastic dominance, and $G\left(\tilde{y}_{c2}\right) = 0$. Further,

$$dG\left(x\right) = F_c^1\left(x\right) - F_c^2\left(x\right).$$

We have then:

$$\begin{aligned}
\bar{R}\left(F_c^1\right) - \bar{R}\left(F_c^2\right) &= -\int_{\gamma_p} \left[\int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} \Delta_2\left(y_{c2}, y_{p2}\right) d\left(F_c^1 - F_c^2\right)\right] dF_p \\
&= -\int_{\gamma_p} \left(\begin{array}{c} \Delta_2\left(y_{c2}, y_{p2}\right)\left[F_c^1 - F_c^2\right]\Big|_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}\left(y_{p_2}\right)} \\ -\int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} \left(\partial\Delta_2\left(y_{c2}, y_{p2}\right)/\partial y_{c2}\right)\left(F_c^1 - F_c^2\right) dy_{c2} \end{array}\right) dF_p \\
&= \int_{\gamma_p}\int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} \left(\partial\Delta_2\left(y_{c2}, y_{p2}\right)/\partial y_{c2}\right) dG\left(y_{c2}\right) dy_{c2} dF_p \\
&= \int_{\gamma_p} \left(\partial\Delta_2\left(y_{c2}, y_{p2}\right)/\partial y_{c2} G\left(y_{c2}\right)\Big|_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}\left(y_{p_2}\right)} - \int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} \partial^2\Delta_2/\partial y_{c2}^2 G\left(y_{c2}\right)\right) dF_p \\
&= \int_{\gamma_p} \left(\begin{array}{c} \left(\partial\Delta_2\left(\bar{y}_{c2}\left(y_{p_2}\right), y_{p2}\right)/\partial y_{c2}\right) G\left(\bar{y}_{c2}\right) \\ -\int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} \left(\partial^2\Delta_2/\partial y_{c2}^2\right) G\left(y_{c2}\right) \end{array}\right) dF_p \leq 0,
\end{aligned}$$

where we are using $\Delta_2\left(\bar{y}_{c2}, y_{p2}\right) = 0$ and $F_c^1\left(\tilde{y}_{c2}\right) = F_c^2\left(\tilde{y}_{c2}\right) = 0$.

Then, under $\alpha < 1$, $\bar{y}_{c1}\left(F_c^1\right)$ which solves:

$$\Delta_1\left(\bar{y}_{c1}\right) = \bar{R}\left(F_c^1\right),$$

must be smaller than $\bar{y}_{c1}\left(F_c^2\right)$, the solution to

$$\Delta_1\left(\bar{y}_{c1}\right) = \bar{R}\left(F_c^2\right). \blacksquare$$

Proof of Proposition 8

We begin by showing that $\Delta_2\left(y_{p2}\right)$ is convex in the range $y_{p2} \in \left(\bar{y}_{p2}, \tilde{y}_{c2}\right)$. We have:

$$\frac{\partial \Delta_2\left(y_{p2}\right)}{\partial y_{p2}} = -\frac{1}{n}\left(\frac{y_{c2}+y_{p2}-\gamma_p}{n}\right)^{-\alpha} < 0$$

and

$$\frac{\partial^2 \Delta_2\left(y_{p2}\right)}{\partial y_{p2}^2} = \frac{\alpha}{n^2}\left(\frac{y_{c2}+y_{p2}-\gamma_p}{n}\right)^{-\alpha-1} > 0.$$

Then, $-\Delta_2\left(y_{p2}\right)$ is increasing and concave.

Define:

$$G\left(x\right) \equiv \int_{\bar{y}_{p2}}^{x}\left(F_p^1 - F_p^2\right) dy_{p2},$$

with $G\left(\bar{y}_{p2}\right) = 0$, by assumption, $G\left(x\right) \leq 0$, from second-order stochastic dominance, and $dG\left(x\right) = F_p^1\left(x\right) - F_p^2\left(x\right)$.

We have:

$$\bar{R}\left(F_p^1\right) - \bar{R}\left(F_p^2\right) = -\int_{\gamma_c}\left[\int_{\bar{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})}\left[\Delta_2\left(y_{c2}, y_{p2}\right)\right] d\left(F_p^1 - F_{p2}\right)\right] dF_c$$

$$= -\int_{\gamma_c}\left[\begin{array}{c} \Delta_2\left(y_{c2}, y_{p2}\right)\left(F_p^1 - F_p^2\right)|_{\bar{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} \\ -\int_{\bar{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})}\left(\partial \Delta_2\left(y_{c2}, y_{p2}\right)/\partial y_{p2}\right)\left(F_p^1 - F_p^2\right) dy_{p2} \end{array}\right] dF_c$$

$$= \int_{\gamma_c}\left(\int_{\bar{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})}\left(\partial \Delta_2\left(y_{c2}, y_{p2}\right)/\partial y_{p2}\right)\left(F_p^1 - F_{p2}\right) dy_{p2}\right) dF_c$$

$$= \int_{\gamma_c}\left(\begin{array}{c} \left(\partial \Delta_2\left(y_{c2}, y_{p2}\right)/\partial y_{p2}\right) G\left(y_{p2}\right)|_{\bar{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} \\ -\int_{\bar{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})}\left(\partial^2 \Delta_2\left(y_{c2}, y_{p2}\right)/\partial y_{p2}^2\right) G\left(y_{p2}\right) dy_{p2} \end{array}\right) dF_c$$

$$= \int_{\gamma_c}\left(\begin{array}{c} \left(\partial \Delta_2\left(y_{c2}, \tilde{y}_{p2}\right)/\partial y_{p2}\right) G\left(\tilde{y}_{p2}\right) \\ -\int_{\bar{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})}\left(\partial^2 \Delta_2\left(y_{c2}, y_{p2}\right)/\partial y_{p2}^2\right) G\left(y_{p2}\right) dy_{p2} \end{array}\right) dF_c \geq 0.$$

Therefore, when $\alpha < 1$, $\bar{y}_{c1}\left(F_p^1\right) \geq \bar{y}_{c1}\left(F_p^2\right)$. ■

Proof of Lemma 9

Since $\bar{R} > 0$, from Proposition 3 we know that $\bar{y}_{c1} > \bar{y}_{c2}$. From Lemma 1, we know that $\partial \Delta_2/\partial y_{c1} > 0$ for $y_{c1} > \bar{y}_{c2}$, when $\alpha < 1$. Further, a pair $\left(\bar{y}_{c1}, y_{p1}\right)$ that solves (7) implies that, when holding $\bar{y}_{c1}$ constant, we are in the decreasing range of $\Delta_2\left(\bar{y}_{c1}, \cdot\right)$, and y_{p1} is smaller than $\bar{y}_{p1}$. From Lemma

2, we know that $\Delta_2(\bar{y}_{c1}, \cdot)$ is strictly decreasing in y_{p1} for $y_{p1} \in [\gamma_p, \tilde{y}_{p2})$. Therefore, fully differentiating (7), we get:

$$\frac{\partial \Delta_2}{\partial \bar{y}_{c1}} d\bar{y}_{c1} + \frac{\partial \Delta_2}{\partial y_{p1}} dy_{p1} = 0 \Leftrightarrow \frac{d\bar{y}_{c1}}{dy_{p1}} = -\frac{\partial \Delta_2 / \partial y_{p1}}{\partial \Delta_2 / \partial \bar{y}_{c1}} > 0,$$

and the result follows. ■

The proof of Lemma 10 builds on auxiliary lemmas A.1 and A.2.

Lemma A.1 *For $n^+ > n$, the transfer and moving-out thresholds decrease with family of size: $\tilde{y}_{c2}^{n+} < \tilde{y}_{c2}^{n}$ and $\bar{y}_{c2}^{n+} < \bar{y}_{c2}^{n}$. Further, for $y_{c2} \geq \tilde{y}_{c2}^{n}$, the differential between the utility under independence and coresidence increases with family size: $\Delta^{n+}(y_{c2}) > \Delta^{n}(y_{c2})$.*

Proof of Lemma A.1

Equation (5) directly implies:

$$\tilde{y}_{c2}^{n+} < \tilde{y}_{c2}^{n} \text{ and } \bar{y}_{c2}^{n+} < \bar{y}_{c2}^{n},$$

and also that $\tilde{y}_{c2}^{n+} < \bar{y}_{c2}^{n+}$ continues to hold. For $y_{c2} \geq \tilde{y}_{c2}^{n} > \tilde{y}_{c2}^{n+}$, since in this range no transfers are given by families with either n or n^+ members, consumption and utility in the state of independence are identical in both cases. Consumption at home, however, is strictly lower in the case of a larger family size: $c_{p2}^{n+} < c_{p2}^{n}$, and:

$$\Delta_2^{n+}(y_{c2}) = u(c_{i2}) - u\left(c_{p2}^{n+}\right) > u(c_{i2}) - u\left(c_{p2}^{n}\right) = \Delta_2^{n}(y_{c2}). \ ■$$

For very altruistic parents and $\alpha < 1$, we can rank the magnitudes of $\Delta_2^{n+}(y_{c2})$ and $\Delta_2^{n}(y_{c2})$ over their entire domain:

Lemma A.2 *When the parent is fully altruistic ($\lambda = 0.5$) and $\alpha < 1$, $\Delta^{n+}(y_{c2}) > \Delta^{n}(y_{c2})$ everywhere.*

Proof of Lemma A.2

Given the previous lemma and the continuity of $\Delta_2^{n}(y_{c2})$ and $\Delta_2^{n+}(y_{c2})$ with respect to y_{c2}, it suffices to prove that

$$\Delta_2^{n+}(y_{c2}) > \Delta_2^{n}(y_{c2})$$

for $y_{c2} \in [\gamma_c, \tilde{y}_{c2}^{n})$.

We consider first the interval $y_{c2} \in \left[\gamma_c, \tilde{y}_{c2}^{n+}\right]$. In this range, the excess utility from independence relative to staying home involves positive transfers for either family size. To show our result in this range, it suffices to show that $\partial\Delta_2^n\left(y_{c2}\right)/\partial n > 0$.

$$\Delta_2^n\left(t_2 > 0\right) = \frac{1}{1-\alpha}\left\{\left(\frac{y_{p2}+y_{c2}-\gamma_p-\gamma_c}{\left(\Gamma\left(n-1\right)+1\right)}\right)^{1-\alpha} - \left(\frac{y_{p2}+y_{c2}-\gamma_p}{n}\right)^{1-\alpha}\right\}$$

and,

$$\begin{aligned} \frac{\partial\Delta_2^n}{\partial n} &> 0 \\ \iff & \left(\frac{y_{p2}+y_{c2}-\gamma_p}{n}\right)^{1-\alpha}\frac{1}{n} - \left(\frac{y_{p2}+y_{c2}-\gamma_p-\gamma_c}{\left(\Gamma\left(n-1\right)+1\right)}\right)^{1-\alpha}\frac{\Gamma}{\left(\Gamma\left(n-1\right)+1\right)} > 0 \\ \iff & \left(1-\alpha\right)\left(u\left(c_{p2}^n\right)\frac{1}{n} - u\left(c_{i2}\right)\frac{\Gamma}{\left(\Gamma\left(n-1\right)+1\right)}\right) > 0. \end{aligned}$$

For $y_{c2} \in \left[\gamma_c, \tilde{y}_{c2}^{n+}\right]$, we know that $u\left(c_{p2}^n\right) > u\left(c_{i2}\right)$. When $\lambda = 0.5$, $\Gamma = 1$, and the result immediately follows. Finally, to show the result for $y_{c2} \in \left(\tilde{y}_{c2}^{n+}, \tilde{y}_{c2}^n\right)$, it suffices to show that the slope of $\Delta\left(y_{c2}\right)$ computed under zero transfers increases with n. Since we have shown that $\Delta_2^{n+}\left(y_{c2}\right)$ is above $\Delta_2^n\left(y_{c2}\right)$, at $y_{c2} = \tilde{y}_{c2}^{n+}$, if the slope of $\Delta_{c2}^{n+}\left(y_{c2}\right)$, which we know is positive, exceeds that of $\Delta_2^n\left(y_{c2}\right)$ for every $y_{c2} \in \left(\tilde{y}_{c2}^{n+}, \tilde{y}_{c2}^n\right)$, then $\Delta_{c2}^{n+}\left(y_{c2}\right)$ must remain above $\Delta_{c2}^n\left(y_{c2}\right)$ for these income values.

We have:

$$\Delta_2^n\left(y_{c2}\right)|_{t=0} = \frac{1}{1-\alpha}\left\{\left(y_{c2}-\gamma_c\right)^{1-\alpha} - \left(\frac{y_{c2}+y_{p2}-\gamma_p}{n}\right)^{1-\alpha}\right\},$$

and

$$\frac{\partial}{\partial y_{c2}}\left[\Delta_2^n\left(y_{c2}\right)|_{t=0}\right] = \left(y_{c2}-\gamma_c\right)^{-\alpha} - \frac{1}{n}\left(\frac{y_{c2}+y_{p2}-\gamma_p}{n}\right)^{-\alpha},$$

$$\frac{\partial^2}{\partial y_{c2}\partial n}\left[\Delta_2^n\left(y_{c2}\right)|_{t=0}\right] = \frac{1}{n^2}\left(\frac{y_{c2}+y_{p2}-\gamma_p}{n}\right)^{-\alpha} - \alpha\frac{1}{n^2}\left(\frac{y_{c2}+y_{p2}-\gamma_p}{n}\right)^{-\alpha},$$

and the derivative will be positive iff $\alpha < 1$. Since

$$\frac{\partial\Delta_2^{n+}\left(y_{c2}\right)|_{t=0}}{\partial y_{c2}} > \frac{\partial\Delta_2^n\left(y_{c2}\right)|_{t=0}}{\partial y_{c2}} > \frac{\partial\Delta_2^n\left(y_{c2}\right)|_{t>0}}{\partial y_{c2}},$$

it follows that $\Delta_2^{n+}\left(y_{c2}\right)$ is steeper than $\Delta_2^n\left(y_{c2}\right)$ for $y_{c2} \in \left(\tilde{y}_{c2}^{n+}, \tilde{y}_{c2}^n\right)$. ■

In (y_{c2}, y_{p2}) space, a larger family size makes the schedules $\tilde{y}_{c2}(y_p)$ and $\bar{y}_{c2}(y_{c2})$ steeper. Further, the schedule $\bar{y}_{c2}^{n+}(y_{p2})$ lies to the left of $\bar{y}_{c2}^{n}(y_{p2})$. Therefore, given parental income, the child will require a lower income in order to move out if she has a larger family.

Proof of Lemma 10

From lemma A.2, we know that the function $\Delta_2^{n+}(y_{c2})$ is everywhere above $\Delta_2^{n}(y_{c2})$. We need to establish how $\bar{R}$ changes with family size. Since $\Delta_2^{n+}(y_{c2}) > \Delta_2^{n}(y_{c2})$,

$$\begin{aligned}\bar{R}^{n+} &= \int_{\gamma_p}\int_{\gamma_c}^{\bar{y}_{c2}^{n+}(y_{p2})} \left[-\Delta_2^{n+}(y_{c2}, y_{p2})\right] dF_{p2}^{1} dF_{c2} \\ &< \int_{\gamma_p}\int_{\gamma_c}^{\bar{y}_{c2}^{n}(y_{p2})} \left[-\Delta_2^{n}(y_{c2}, y_{p2})\right] dF_{p2}^{1} dF_{c2} = \bar{R}^{n},\end{aligned}$$

where the inequality follows since we are integrating only over the range where Δ_2 takes negative values and since $\tilde{y}_2^{n+}(y_p) < \tilde{y}_2^{n}(y_p)$. Since $R^{n+} < R^{n}$, the root of the equation:

$$\Delta_2^{n+}(y_{c2}) = \bar{R}^{n+},$$

which is the value of $\bar{y}_1^{n+}$ has to be smaller than the root of

$$\Delta_2^{n}(y_{c2}) = \bar{R}^{n},$$

which is $\bar{y}_1^{n}$. ■