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On Regulation and Competition: Pros and Cons of a Diversified Monopolist

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Keywords: Regulation, Competition, Asymmetric Information, Conglomerate firms, Multi-utility, Scope economies, Informational externality.

Journal of Economic Literature Classification Numbers: L51, L43, L52.

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1 Introduction

In many sectors the boundaries between regulation and competition are blurred and often go through firms that operate both in regulated and competitive segments. For example, Centrica in the UK operates both in gas and electricity transmission and in some competitive segments of energy sectors as well as in telecommunications and financial services. GDF-Suez in France and other countries, RWE in Germany, and Enel in Italy all operate in regulated as well as unregulated markets in energy, water and other utility sectors. In the US a major utility such as Pepco also offers energy management services. Local telephone operators such as the regional Bell operating companies also provide unregulated broadband Internet services. In Europe hundreds of municipal enterprises (e.g. in Italy, Germany or Scandinavian countries) offer a wide array of services in both regulated and unregulated sectors.[1]

The diversification of regulated firms into competitive sectors as well, thus leading to "conglomerates", has raised substantial objections from regulatory institutions in the EU, in the US, as well as in other countries. The institutional response to utilities' expansion into unregulated sectors has often been very negative, if sometimes differentiated. In the EU several Directives referring to utility services have stated that firms in regulated sectors that want to operate in competitive sectors as well must "unbundle", i.e. separate the assets and the personnel of the two sectors.[2] In the US, some regional Bell operating companies have been prohibited from expanding into unregulated long distance services.

This institutional response has been motivated by the fear that it may imbalance competition in the unregulated sectors (a "level playing field" argument), and affect negatively consumers in the regulated markets due to the increased complexity of the regulatory task.

This paper assesses these concerns, in order to determine whether allowing the regulated firm to operate in the competitive sector increases or decreases social welfare. Our response is positive, as we show that total welfare increases when diversification is allowed. The concern of regulators that allowing diversification may harm consumers in the regulated market is not supported by our model, even if diversification actually entails an informational advantage of the diversified firm relative to both the regulator and the rival firms. On the contrary, we show that the presence of a competitive market where the regulated firm operates often increases the efficiency of the regulatory constraint. Even when this does not happen, the possible regulatory problems have no

[1] This diversification is partly the consequence of utility de-regulation, taking place at different pace in different sectors, some of which have already been deregulated and liberalized, whilst others remain heavily regulated.

[2] The unbundling may be of several types. The Directives require at least separate accounts and sometimes separate companies which may or may not belong to different shareholders.

effect on consumers in the monopoly market. Moreover, the presence of a diversified monopolist in the competitive market needs not distort competition and damage consumers. The overall effect of diversification on total welfare is thus positive.

There are several reasons for regulated firms to expand and diversify into unregulated markets. For example, a multi-market strategy may facilitate collusion. Or regulated activities may leave some free-cash flow that managers invest in unregulated sectors so they can then operate aggressively in those markets or just because they are "empire builders". However, the most commonly cited motivation goes under the heading "synergy", the buzz-word indicating economies of scope in the joint production and supply of horizontally diversified services. In this respect, the main concern that we want to emphasize is that the amount of cost savings is generally unknown both to the regulator and to the competitors in unregulated markets, and it is private information of the conglomerate.[3] Hence, both competitors and the regulator of a complex multi-product firm may suffer a very substantial asymmetry of information.

In this paper we assess the trade-off between the potential benefits and costs of conglomerate firms. First we study how a conglomerate operates in both a regulated and an unregulated sector, where it competes with rivals either in prices or in quantities for possibly differentiated goods. The magnitude of scope economies is the private information of the conglomerate when it is allowed to "bundle" its productions.[4] With this respect, the regulatory process and the conglomerate's activity in the regulated market (e.g. the price) may reveal important information to competitors about the costs of the conglomerate. In other terms, the game in the unregulated market may or may not be one of asymmetric information, depending on the information generated by the regulatory process; this in turn affects the conglomerate's behavior in its regulated market, i.e. its incentives to disclose information to the regulator.

The effects of this informational externality from the regulated to the unregulated market depend on what type of competition obtains in the unregulated market. When firms compete in quantity, the regulator can more easily elicit information on scope economies, reducing distortions in the regulated sector. Acting as if economies of scope were small so as to obtain lenient regulation,

[3] For example, in a joint document issued by British sectoral utility regulators (OFWAT, 1998), it is made clear that for regulators, external auditors, as well as for rival firms, measuring scope economies is complex and often inconclusive, especially when conducted before integration takes place. Event studies of abnormal stock returns in mergers (see Berry, 2000 and Leggio and Lien, 2000) as well as the few direct econometric studies (e.g., Fraquelli, Piacenza and Vannoni, 2004), do not allow one to reach clear cut conclusions. In a survey of multi-utilities' managers ("*EU Multi-utilities,*" Marketline International, 1998, London), respondents showed a great variability in their assessment of the cost savings from diversification.

[4] Our model can be also reinterpreted as one where consumers get higher utility from "one stop shop" (e.g. when joint billing lowers transaction costs): the cross-market effect goes through the utility function rather than economies of scope.

the conglomerate gets the countervailing effect of inducing the rival firms to expand in the unregulated market. With price competition, by contrast, the informational externality complicates the regulatory process since behaving as if scope economies were small prompts an accommodating reaction by the rivals in the unregulated market. The regulator may thus be forced to apply a uniform regulatory policy (regardless of the actual size of the scope economies) so that no information is disclosed to rivals.

After discussing the effects of unregulated activities (and of such features as the size of the unregulated market and the number of competitors) on the optimally regulated price, we compare welfare when the conglomerate is allowed to run joint productions for the two markets with that with compulsory separation preventing a firm from exploiting its economies of scope for the sake of regulatory clarity. On the one hand, if economies of scope are substantial, consumers in the two markets may benefit from the gain in efficiency. On the other hand, since scope economies is the private information of the conglomerate, the lack of information for the regulator and competitors distorts the price in the regulated market and also competition in the unregulated market.[5] We show that, even though horizontal integration causes informational problems for regulation and for competitors, these adverse effects are smaller than the efficiency gains from integration. Letting the conglomerate integrate its operations is desirable unless there are diseconomies of scope, which may be the case, for example, if the managers of the regulated firm expand simply to build their empires.

It is important to stress that the key to our result is not that the regulator is able to internalize some of the efficiency generated by the joint production. Conglomerate integration increases the difficulty of the regulatory contract and may bring about negative spillovers on the unregulated markets. The point in our results is that these issues – usually placed at the centre of the policy debate and used to justify the aforementioned institutional responses – are more than compensated by the greater efficiency of the firm.

Some early papers, such as Braeutigam and Panzar (1989), Brennan (1990), Brennan and Palmer (1994), addressed the problems and the desirability of horizontal diversification mainly in terms of cross-subsidies. This literature analyzes cost shifting, in which an integrated firm attributes to the regulated activity costs that actually pertain to non-regulated ones and thus obtains higher regulated prices, while at the same time behaving more aggressively in the unregulated sectors. Another pertinent analysis is Sappington (2003), in a model where effort can be allocated

[5] It is well known from the literature on information sharing in oligopolies (see Sakai 1985 and Vives 1999, Chapter 8, for a survey) that total welfare may be reduced when firms compete under asymmetric information.

to regulated and unregulated activities. Although the analysis of optimal regulated prices and competition in the unregulated market remains limited by modelling choice, Sappington offers an interesting discussion of optimal regulatory policy towards diversification that complements our analysis. He shows that for diversification to be undesirable two conditions must hold at the same time: the regulator cannot control effort diversion and also the firm can inflate expenditures (by cost padding) on unregulated activities. These papers illustrate the potential risks of diversification in terms of cross-subsidies and effort diversion. Lewis and Sappington (1989) instead study a model where the costs of regulated activities are positively correlated with profitability in the unregulated sector. Within this setting and with a black-boxed description of profitability in the competitive market, they show how "countervailing incentives" may affect regulation.[6] Countervailing incentives have also been discussed in Iossa (1999) who considers the design of a regulated two-product industry with interdependent and unknown demand. She shows that whether an integrated monopolist or two separate firms is desirable depends on the interplay between the demand complementarity/substitutability of the two products.

Our analysis differs from all these papers on several respects. We emphasize that integrated production is both a source of scope economies but also of private information for the conglomerate with respect to the regulators and its rivals. The informational issues arise exactly from joint production in that neither the regulator nor the rivals know the exact magnitude of scope economies. Hence, our analysis complements that on effort diversion in the previous papers. Unlike the papers cited, we explicitly account for the reactions of the unregulated market to the regulatory decisions (of the regulator and the conglomerate). Explicitly describing the unregulated market we can properly study if and when the regulated firm might have an unfair advantage in that market and how the rivals react to the information generated by regulation itself. Furthermore, in assessing the desirability of integrated production we consider welfare in both sectors so that we can take into account the potential negative effects of lack of information on the unregulated market as well.

This paper also complements Vickers (1995) analysis. While that paper specifically addresses the issue of vertically related markets, where the same firm is regulated upstream and competes downstream with others, here we consider horizontally related markets. In our paper, the regulated firm's rivals do not need to purchase the regulated good, but still are affected by the cost saving that the integrated firm enjoys by operating both services.

Finally, this paper also contributes to the contract theory literature by considering an environ-

[6] Chaaban (2004) studies the effects of various cost-apportionment rules for a joint fixed cost that is privately known by the multi-utility.

ment in which (i) the agent (the conglomerate) has private information on the complementarity between a contractible and a non contractible variable (respectively the regulated output and the firm's activity in the unregulated market); (ii) the optimal (regulatory) contract at the same time screens the agent's type and signals this private information (to the firms in the unregulated market). This informational externality that plays such an important role in our analysis of regulated and unregulated markets also arise in different contexts. For example, Calzolari and Pavan (2006) study the optimal disclosure of information between two sellers who contract sequentially with the same privately-informed buyer, showing that the upstream seller may gain by disclosing information downstream when there are countervailing incentives or if the goods of the two sellers are substitutes.

The paper is organized as follows. The next section introduces the model. Section 3 analyzes of benchmark cases, with full information and separation of activities. Section 4 derives optimal regulation when the conglomerate is allowed to integrate. Section 5 uses these results to study the welfare effects of integration in the case of quantity and of price competition. Section 6 concludes. All the proofs are in the Appendix.

2 Model Set-up

We consider a regulated natural monopoly (market R) and an unregulated oligopoly (market U). Demand functions in regulated and unregulated markets are independent, decreasing and (twice) differentiable. Inverse demand in the regulated market R is $p(q)$ where q is output. The unregulated market U consists of n firms indexed by $i = 1, ..., n$, each producing (possibly differentiated) output y_i with price p_i^U. Inverse demand functions are $p_i^U(y_i, Y_{-i})$ $i = 1, ..., n$, where Y_{-i} denotes the vector of the outputs of other firms. The vectors of prices and outputs in the unregulated market are denoted by p^U and Y respectively. Competition in the unregulated sector takes place either in quantities or in prices.

A "conglomerate" firm operates in both markets, respectively producing outputs q and y_1 (index $i = 1$ will denote the conglomerate firm in market U). This firm may be allowed to run productions in the two markets jointly, or may be forced to organize productions in separate units (unbundling). In the latter case, separating productions makes impossible for the conglomerate to share assets and internal resources that may bring about cost savings. Formally, let $C(q, y_1; \theta)$ denote the total production cost of the conglomerate with joint production, where θ is an efficiency parameter.

If instead separation is imposed, the conglomerate's total costs is $C(0, y_1; \theta) + C(q, 0; \theta)$. Joint production thus generates a cost saving corresponding to

$$C(0, y_1; \theta) + C(q, 0; \theta) - C(q, y_1; \theta) \geq 0, \tag{1}$$

which is nil when either $q = 0$ or $y_1 = 0$. The size of scope economies is parametrized by θ so that the expression in (1) is nil if $\theta = 0$ and, for any $\theta'' \geq \theta'$,

$$C\left(0, y_1; \theta''\right) + C\left(q, 0; \theta''\right) - C\left(q, y_1; \theta''\right) \geq C\left(0, y_1; \theta'\right) + C\left(q, 0; \theta'\right) - C\left(q, y_1; \theta'\right),$$

with $C\left(q, y_1; \theta''\right) = C\left(q, y_1; \theta'\right)$ if either $q = 0$ or $y_1 = 0$. Thus, the larger is θ the higher are scope economies and, if separation is imposed, θ has no bite on costs.[7] Assuming that the cost function is twice differentiable with respect to q and y_1, the previous conditions imply that (i) a larger output for one of the two markets induces a marginal cost reduction for the output in the other market, (ii) this cost reduction is larger the higher is θ (and vanishing when $\theta = 0$),

$$\frac{\partial^2 C(q, y_1; \theta'')}{\partial q \partial y_1} \leq \frac{\partial^2 C(q, y_1; \theta')}{\partial q \partial y_1} \leq 0, \text{ for any } \theta'' \geq \theta', \tag{2}$$

and (iii) a higher value of θ (weakly) reduces the marginal cost for both outputs,

$$\frac{\partial C(q, y_1; \theta'')}{\partial z} \leq \frac{\partial C(q, y_1; \theta')}{\partial z} \text{ for } z \in \{q, y_1\} \text{ and any } \theta'' \geq \theta'. \tag{3}$$

The following specification of the cost function, that we will use in some examples, satisfies all the previous properties,

$$C_1(q, y_1; \theta) = c(q + y_1) - \theta q y_1. \tag{4}$$

The technology available to all other firms in market U (i.e. firms with index $i = 2, ..., n$) is simply $C(y_i) \equiv C(0, y_1; \theta)$ for any $y_i = y_1$ and profits are

$$\pi_i(y_i, Y_{-i}) \equiv y_i p_i^U(Y) - C(y_i). \tag{5}$$

When the firm is allowed to integrate production its *total* profit Π is (the apex I will stand for

[7] Although we concentrate on economies of scope related to variable costs, when scope economies are due to common fixed costs some cost-allocation rule is required by the regulator typically allocating fixed costs proportionally to outputs. This may re-introduce variable-cost non separability as in the present setting (see Calzolari, 2001 and Chaaban, 2004).

integration)

$$\Pi^{I}\left(q, y_1, Y_{-1}; \theta\right) \equiv qp\left(q\right) + y_1 p_1^U\left(Y\right) - C\left(q, y_1; \theta\right) - T, \tag{6}$$

where T is a tax/transfer which is part of the regulatory contract in market R (see below). On the contrary, if the conglomerate must keep apart its production for the two markets, its profit becomes $\Pi^S + \pi_1(y_1, Y_{-1})$ (the apex S will stand for separation) where

$$\Pi^S \equiv qp\left(q\right) - C\left(q, 0; \theta\right) - T. \tag{7}$$

The regulator maximizes social welfare W which is a weighted sum of net consumer surplus in the two markets, firms profits and taxes (or transfers). Let V_j denote gross consumer surplus in sector $j = R, U$. The welfare function then is

$$W = V_R\left(q\right) - qp\left(q\right) + V_U\left(Y\right) - Yp^U\left(Y\right) + T + \alpha(\Pi + \sum_{i=2}^{n} \pi_i), \tag{8}$$

where $Yp^U\left(Y\right) = \sum_{i=1}^{n} y_i p_i^U\left(Y\right)$ and the weight to profits is $\alpha < 1$.[8] The regulatory contract contemplates a quantity q and the transfer (T) to the firm. By definition of unregulated market U, the institutional set-up is such that the regulator cannot explicitly control output (of single firms or total output) in that market.

By directly operating joint production, a firm is able to realize much better than the regulator and the rival firms whether there are economies of scopes and, if so, their actual magnitude. Hence, the exact value of θ is private information of the conglomerate and neither the regulator, nor the competitors in the unregulated market know it. For simplicity, we assume there are no other pieces of private information. This is clearly a simplification as regulation of a standard single-product firm is also often affected by informational issues. We employ this assumption to single out the effects of asymmetric information explicitly related to economies of scope and to the complexity of conglomerates. It is common knowledge that scope economies can be either high or low, i.e. $\theta \in \Theta \equiv \left\{\underline{\theta}, \overline{\theta}\right\}$ with $\nu = \Pr(\theta = \overline{\theta}) = 1 - \Pr(\theta = \underline{\theta})$, $\overline{\theta} \geq \underline{\theta}$ and we rule out dis-economies of scope, i.e. $\underline{\theta} \geq 0$ (see the discussion in Section 6 on the possibility of dis-economies).[9]

[8] As usual we assume $\alpha < 1$ to avoid the well-known Loeb-Magat paradox and we will not consider the cost of public funds which in any case would not qualitatively alter our analysis. This would also be the case if the regulator weights the surpluses in the two markets differently. We will also not discuss the possibility that the regulator uses different weights to profits of regulated and unregulated firms. In a different context of regulation, this is analyzed by Calzolari and Scarpa (2009).

[9] The restriction to two types is only for ease of exposition, and an extension to a continuum of types would not qualitatively affect our results. The basic references for regulation under asymmetric information are Baron and

The timing of the game is the following:

1. The regulator decides whether or not to impose separation of productions to the conglomerate firm, then accordingly sets and publicly announces the regulatory policy.
2. The firm learns the size of scope economies, i.e. its type θ, and then decides in which markets to operate. Regulation is enforced.
3. Finally, competition in the unregulated sector takes place.

Stage 2 indicates that the conglomerate is not obliged to participate the regulated market and will do so only if it finds it profitable. As we will discuss, if the firm wants to serve the regulated market, then it always prefers to bundle production in the two sectors, if allowed to do so. Finally, the timing also shows that the execution of a regulatory contract naturally anticipates the determination of the equilibrium in the competitive sector. Indeed, regulation usually follows procedures and activities which are more complicated to modify than price decisions of private firms.

The exact magnitude of scope economies becomes clear to the conglomerate if it is allowed to integrate production and effectively does so. In this respect, we thus regard as impractical the possibility to condition the decision concerning joint or separate production on the realization of θ. This would require letting the conglomerate set up integrated production, learn θ and then subsequently impose separation by splitting productions if scope economies turn out to be low. Notice therefore that the regulator's decision on separation/integration cannot be made conditional on the specific regulatory policy and/or on the actual realization of θ.[10]

Notice that whether or not integration is allowed, the number of firms in the unregulated market is assumed to be constant, equal to n. If integration instead entailed a reduction in the number of firms active in U, then we would have a "trivial" anti-competitive effect of integration.

3 Efficient regulation

In this section we introduce two benchmarks which will help to discuss the pros and cons of joint production in the presence of asymmetric information. We first analyze the case where separation of productions is imposed, and then we study the case with joint productions and full information.

Myerson (1982) and Laffont and Tirole (1986).

[10]In the sequel we will nevertheless argue that even if one considers this possibility, this entails no qualitative change in our main results.

Optimal regulation with separate productions Since asymmetric information matters only in case of joint production, the regulated firm's profit in sector R with separation is simply as in (7). Let firm i's *equilibrium* output in sector U be defined as y^S which depends neither on θ nor on q and the associated profits $\pi^S \equiv \pi_i\left(y_i^S, Y_{-i}^S\right) \geq 0$. The regulator then maximizes (8) with respect to q and T, subject to the participation constraint of the regulated firm, which assures that the conglomerate wants to serve (also) the regulated market, i.e. $\Pi^S + \pi^S \geq \pi^S$. Welfare can be written as follows

$$W^S = V_R(q) + V_U(Y^S) - C\left[q, 0; \theta\right] - \sum_{i=1} C(y^S) - (1-\alpha)(\Pi^S + \sum_{i=1} \pi^S)$$

which shows that, as usual, distributive efficiency would require to reduce as much as possible firms' profits in the two markets. The regulator then optimally sets the transfer at a level T^S so that the participation constraint binds and the conglomerate earns no additional profits with respect to π^S, i.e. $\Pi^S = 0$. Furthermore, the optimal regulated quantity q^S is set efficiently so that the price in the regulated sector is equal to the marginal cost, i.e. $p(q^S) = \partial C(q^S, 0; \theta)/\partial q$. For future reference we indicate with $\mathcal{C}^S \equiv (q^S, T^S)$ this optimal regulatory policy when separation is imposed and with $W^S(\mathcal{C}^S)$ the associated social welfare.

Joint productions and full information Assume now that the conglomerate is allowed to integrate productions and that the public authority and the rivals are fully informed on scope economies θ. Consider a generic strategic variable x_i for firm i in market U so that $x_i = p_i$ if competition takes place in prices and $x_i = y_i$ for quantity competition. The following system of first order conditions

$$\frac{\partial \Pi^I(x_1, X_{-1}, q; \theta)}{\partial x_1} = 0, \quad \frac{\partial \pi_i(x_i, X_{-i})}{\partial x_i} = 0, \text{ for } i = 2, ..., n$$

yields the market equilibrium in sector U.[11] For the sake of convenience, in the following we express profits as functions of equilibrium output levels $y_1(q, \theta)$, $y_i(q, \theta)$ $i = 2, ..., n$:

$$\pi_i^I(q, \theta) = \pi_i\left[y_i(q, \theta), Y_{-i}(q, \theta)\right], \text{ for } i = 2, ..., n$$
$$\Pi^I(q, \theta) = \Pi^I\left[q, y_1(q, \theta), Y_{-1}(q, \theta); \theta\right],$$

[11] We assume that the conditions for an interior unique equilibrium are met.

and similarly for welfare,

$$\begin{aligned} W^{I}\left(q, Y\left(q,\theta\right),\theta\right) = \quad & V_{R}\left(q\right) + V_{U}\left[Y\left(q,\theta\right)\right] - C\left[q, y_{1}\left(q,\theta\right);\theta\right] - \sum_{i\neq 1} C\left[y_{i}\left(q,\theta\right)\right] + \\ & -(1-\alpha)[\Pi^{I}\left(q,\theta\right) + \sum_{i\neq 1} \pi_{i}^{I}\left(q,\theta\right)]. \end{aligned} \tag{9}$$

Anticipating outputs in market U and for a given (and known) θ, the regulator maximizes (9) subject to the conglomerate's participation constraint

$$\Pi^{I}\left(q,\theta\right) \geq Max\{\pi^{S}, \Pi^{S}\} = \pi^{S}$$

where $\Pi^{S} = 0$ as shown above. Knowing θ, the regulator sets the transfer T such that the participation constraint binds for any θ and no extra-profits are given to the conglomerate, i.e. $\Pi^{I}\left(q,\theta\right) = \pi^{S}$. Maximizing (9) with respect to q for any θ, the optimal regulated quantity with full information and integration $q_{FI}^{I}(\theta)$ is such that

$$p(q_{FI}^{I}(\theta)) = SMC[q_{FI}^{I}(\theta), \theta] \tag{10}$$

where the right hand side is the *social marginal cost* of q, i.e.

$$\begin{aligned} SMC\left[q,\theta\right] \quad \equiv \; & \tfrac{\partial C\left[q,y_{1}(q,\theta);\theta\right]}{\partial q} + \\ & -\left(p_{1}^{U} - \tfrac{\partial C\left[q,y_{1}(q,\theta);\theta\right]}{\partial y_{1}}\right)\tfrac{\partial y_{1}(q,\theta)}{\partial q} - \sum_{i\neq 1}^{n}\left(p_{i}^{U} - \tfrac{\partial C\left[y_{i}(q,\theta)\right]}{\partial y_{i}}\right)\tfrac{\partial y_{i}(q,\theta)}{\partial q} + \\ & +(1-\alpha)\sum_{i\neq 1}^{n}\tfrac{\partial \pi_{i}^{I}(q,\theta)}{\partial q}. \end{aligned} \tag{11}$$

The optimality condition (10) shows that the price differs from the simple marginal cost $\partial C/\partial q$ (the first line in (11)) for two reasons. Since q affects firms' decisions in the unregulated market, the regulator internalizes the effect of q on distortions in market U due to market power. This is illustrated in the second line of SMC where the price-cost margin for each firm are weighted by the impact that q has on each firm's equilibrium output in that sector (i.e. $\partial y_{1}\left(q,\theta\right)/\partial q \geq 0$ and $\partial y_{i}\left(q,\theta\right)/\partial q \leq 0$ for $i \neq 1$).[12] The third line in (11) indicates an additional reason to give up standard allocative efficiency in the regulated market, now due to a distributional concern. By inducing the regulated firm to produce more in market R, the regulator reduces the profits of

[12] This departure from marginal cost pricing is typical in the literature on mixed oligopolies (see for example De Fraja and Delbono, 1990), where the firm under public control distorts its choices to boost the efficiency of private firms.

other firms in the unregulated market (since $\partial\pi_i^I/\partial q \leq 0$), thus increasing social welfare through enhanced distributive efficiency. Although these effects may be possibly conflicting (contrary to all other terms, inducing rivals to expand their outputs requires a reduction of q), integration tends to expand regulated output so that $\overline{q}_{FI}^I \geq \underline{q}_{FI}^I \geq q^S$, as illustrated in the explicit model of in Section 4.1.[13]

For future reference we indicate with $\mathcal{C}_{FI}^I = \left\{q_{FI}^I(\theta), T_{FI}^I(\theta)\right\}_{\theta\in\Theta}$ the optimal regulatory contract with integrated production and full information and with $W_{FI}^I = E_\theta[W^I(\mathcal{C}_{FI}^I, \theta)]$ the associated (expected) welfare.

4 Regulation of a privately informed conglomerate

Let us now consider a conglomerate allowed to jointly run productions but also privately informed on the level of scope economies θ. We can rely on the Revelation Principle so that for the case of integration the regulator designs a menu of type-dependent contracts $\mathcal{C} = \{(q(\theta), T(\theta))\}_{\theta\in\Theta}$ which maximizes the (expected) social welfare and induces the conglomerate to announce the true level of scope economies to the regulator. By so doing, a conglomerate with scope economies θ selects the policy $(q(\hat{\theta}), T(\hat{\theta}))$ by announcing $\hat{\theta} = \theta$.[14]

As a matter of fact, unregulated firms in sector U do not observe communication between the conglomerate and the regulator (i.e. the announcement $\hat{\theta}$). However, the implemented regulatory policy $q(\hat{\theta}), T(\hat{\theta})$ is clearly public information (for example, each consumer observes the regulated price on her own bill), so that, knowing the regulatory policy $\mathcal{C}$, rival firms obtain information on θ by simply observing the (implemented) regulated price $\widehat{p}$ or, equivalently, the quantity $\widehat{q}$ (in the sequel we will indicate updating with respect to $\widehat{q}$). This is an important *informational externality* of regulation which allows competitors to update their beliefs about the level of the scope economies and then accordingly set their strategic variables in the unregulated market. It is important to realize also that this informational externality in turn affects the regulated firm's incentives to report $\hat{\theta}$, as we discuss next.

Given the (truthful) announcement of economies of scope, the competitive market game may or may not be one of complete information. More precisely, if the optimal regulatory contract

[13] If the regulator were only concerned by welfare in market R, regulated quantities would be smaller but still larger than q^S due to economies of scope.

[14] Acting before competition takes place in market U, the regulator cannot infer any information on θ by observing firms' activities in that market. Furthermore, from the view point of the regulator, the unregulated output is a moral hazard variable. Hence, the appropriate reference for the application of direct mechanisms is here the Generalized Revelation Principle of Myerson (1982).

contemplates *discriminatory regulation* (i.e. a "screening" contract) with different quantities and prices for different announcements $\widehat{\theta}$, the updating process is then perfect so that $v(\widehat{q}) \equiv \Pr(\overline{\theta}|\widehat{q}) = 1$ when $\widehat{q} = q(\overline{\theta})$ and $v(\widehat{q}) = 0$ when $\widehat{q} = q(\underline{\theta})$. On the other hand, if the regulator sets *uniform regulation* (i.e. a "pooling" contract) in which the regulated quantity $\widehat{q}$ does not depend on the firm's type, then unregulated competitors are not able to perform any updating so that $v(\widehat{q}) = v$.[15]

Given $\widehat{q}$, we can then illustrate the Bayesian (continuation) equilibrium in the unregulated market in which the strategic variables (either prices or quantities) satisfy the following set of necessary conditions,

$$\begin{aligned} &\tfrac{\partial}{\partial x_i} E_\theta\left[\pi_i(x_i, X_{-i})|\widehat{q}\right] = 0, \text{ for } i = 2, ..., n \\ &\tfrac{\partial}{\partial x_1} \Pi^I(\widehat{q}, x_1, X_{-1}; \theta) = 0, \text{ for } \theta \in \Theta \end{aligned}$$

where $E_\theta\left[\pi_i(x_i, X_{-i})|\widehat{q}\right]$ is the rivals' expected profit (with expectation over θ), conditional on the information provided by $\widehat{q}$. We denote with $y_1\left(\widehat{q}, \theta, v(\widehat{q})\right)$ the equilibrium output in the competitive market for a conglomerate with (true) scope economies θ, producing a regulated output $\widehat{q}$ and when the rival firms' updated beliefs are $v(\widehat{q})$. Similarly, let $y\left(\widehat{q}, v(\widehat{q})\right)$ be the rivals' output which clearly does not depend on the *true* level of scope economies but only on observed quality $\widehat{q}$ and associated $\widehat{\theta}$. Consistently with our notation, we will denote with $y_1\left(\widehat{q}, \theta, 1\right), y_1\left(\widehat{q}, \theta, 0\right)$ and $y\left(\widehat{q}, 1\right), y\left(\widehat{q}, 0\right)$ outputs of a type θ conglomerate and of its rivals when they believe that the true level of scope economies are either $\overline{\theta}$ or $\underline{\theta}$.

Since the regulator is uninformed on the level of scope economies, she must design a regulatory contract that induces truthful revelation by any type θ.[16] Consider a conglomerate with scope economies θ which declares $\widehat{\theta}$ and gets the contract $(\widehat{q}, \widehat{T}) \in \mathcal{C}$. This firm obtains a profit,

$$\begin{aligned} \Pi^I(\widehat{\theta}, \theta) \equiv\ & \widehat{q}\ p(\widehat{q}) + y_1\left(\widehat{q}, \theta, v(\widehat{q})\right)\ p_1^U\left[y_1\left(\widehat{q}, \theta, v(\widehat{q})\right), Y_{-1}\left(\widehat{q}, v(\widehat{q})\right)\right] + \\ & - C\left[\widehat{q}, y_1\left(\widehat{q}, \theta, v(\widehat{q})\right); \theta\right] - \widehat{T}. \end{aligned}$$

On the other hand, by truthfully announcing its scope economies this firm obtains a profit $\Pi^I(\theta) \equiv \Pi^I(\widehat{\theta}, \theta)$ with $\widehat{\theta} = \theta$. Hence, any type of firm θ will truthfully announce the level of scope economies

[15] Few comments are in order on rivals' updating. First, we implicitly assume that the multi-utility cannot credibly communicate θ to the rivals. Second, entry in sector R is uninformative since regulation induces entry by any type θ, as discussed below. Finally, we do not consider the possibility that the regulator could "fine tune" information disclosed to the unregulated market. This would require a stochastic regulatory contract that we will discuss in Section 6.

[16] Although the announcement $\widehat{\theta}$ indirectly affects (also) rivals' beliefs $v(\widehat{q})$, the effects of $\widehat{\theta}$ uniquely take place through the regulator's instruments $(q(\widehat{\theta}), T(\widehat{\theta}))$. It then follows that the Revelation Principle is valid independently of rivals' updating.

if

$$\Pi^I(\theta) \geq \Pi^I\left(\widehat{\theta};\theta\right) \quad \forall \widehat{\theta} \in \Theta.$$

This incentive compatibility constraint for type θ can be conveniently rewritten as follows. Let $\Pi_U(\widehat{q},\theta,v(\widehat{q}))$ be the profit earned in the unregulated market U by the conglomerate with (true) scope economies θ, producing $\widehat{q}$ in sector R and inducing the rivals to believe that scope economies are $\widehat{\theta}$, i.e.

$$\begin{aligned} \Pi_U(\widehat{q},\theta,v(\widehat{q})) \equiv\ & y_1(\widehat{q},\theta,v(\widehat{q}))\ p_1^U\left[y_1(\widehat{q},\theta,v(\widehat{q})),Y_{-1}(\widehat{q},v(\widehat{q}))\right]+ \\ & -\left\{C\left[\widehat{q},y_1(\widehat{q},\theta,v(\widehat{q}));\theta\right]-C\left[\widehat{q},0;\theta\right]\right\} \end{aligned} \tag{12}$$

where the costs attributed to unregulated production is simply the incremental cost of y_1.[17] Then, truthful revelation is guaranteed by the following equivalent condition

$$\Pi^I(\theta) \geq \Pi^I(\widehat{\theta}) + \Pi_U(\widehat{q},\theta,v(\widehat{q})) - \Pi_U(\widehat{q},\widehat{\theta},v(\widehat{q})), \text{ with } \widehat{\theta} \neq \theta.$$

In particular, the conglomerate with high scope economies $\overline{\theta}$ prefers not to mimic the one with small scope economies $\underline{\theta}$ and vice-versa if

$$\begin{aligned} &\overline{\Pi}^I \geq \underline{\Pi}^I + \Delta_\theta \Pi_U\left(\underline{q},v(\underline{q})\right) && IC(\overline{\theta}) \\ &\underline{\Pi}^I + \Delta_\theta \Pi_U\left(\overline{q},v(\overline{q})\right) \geq \overline{\Pi}^I && IC(\underline{\theta}) \end{aligned} \tag{13}$$

where

$$\Delta_\theta \Pi_U(q,v(q)) \equiv \Pi_U\left(q,\overline{\theta},v(q)\right) - \Pi_U\left(q,\underline{\theta},v(q)\right)$$

identifies the extra gain that type $\overline{\theta}$ obtains with respect to $\underline{\theta}$ when they both produce the same regulated output q and induce beliefs $v(q)$ on the rivals.

The scope economies announcement $\widehat{\theta}$ has here several interesting effects. First of all, as in standard models of regulation with asymmetric information, more efficient firms have incentives to understate their level of scope economies and to mimic less efficient firms in order to obtain more lenient and favorable regulation. This *cost-efficiency effect* of the announcement is captured in the term $\Delta_\theta \Pi_U\left(\underline{q},v(\underline{q})\right)$ by the difference $-[C\left(\underline{q},y_1;\overline{\theta}\right) - C\left(\underline{q},y_1;\underline{\theta}\right)] \geq 0$. Indeed, if the efficient firm with type $\overline{\theta}$ mimics type $\underline{\theta}$, it produces the same regulated quantity $\underline{q}$ with a cost saving corresponding the previous cost difference.

On the other hand, the presence of an unregulated market generates two additional effects of the announcement. A *direct strategic effect* emerges since rival firms observe the regulated output

[17] Being $C[\widehat{q},0;\widehat{\theta}] = C[\widehat{q},0;\theta]$, the cost in Π_U can be indeed written in terms of incremental costs as in (12).

and they know that, because of property (2), if $\widehat{q}$ is large their cost disadvantage (with respect to the conglomerate) is also large, for any given level of θ. We also have a *beliefs-driven strategic effect* which is the consequence of asymmetric information in market U and would not exist if rivals knew θ. Indeed, observing $\widehat{q}$ the rivals may be induced to believe that scope economies are either large or low depending on $\widehat{\theta}$, whatever the true level of scope economies is. Hence, the incentive for the conglomerate to declare its type also depends on the reaction of its rivals which is driven by these two strategic effects (direct and belief-driven), as we will further illustrate.

Anticipating all these effects, the regulator then sets the optimal regulatory policy $\mathcal{C}^*$ maximizing the expected social welfare subject to the incentive compatibility constraints $IC(\theta)$ as in (13) and the participation constraints

$$\Pi^I(\theta) \geq \pi^S \qquad \forall \theta \in \Theta \quad IR(\theta).$$

This leads to the following result.

Proposition 1 (Optimal regulation) *Let $\tilde{q}(\theta)$ be defined for any $\theta \in \Theta$ by*

$$p(\tilde{q}(\theta)) = SMC(\tilde{q}(\theta), \theta) + \mathcal{I}(\theta)(1-\alpha)\frac{v}{1-v}\frac{\partial \Delta_\theta \Pi_U(\tilde{q}(\theta), 0)}{\partial q}, \tag{14}$$

where the indicator function is $\mathcal{I}(\theta) = 1$ if $\theta = \underline{\theta}$ and 0 otherwise.

*(i) Optimal regulated quantity $q^*(\theta)$ is **discriminatory** with $q^*(\theta) = \tilde{q}(\theta)$ if*

$$\Delta_\theta \Pi_U\left(\tilde{q}(\overline{\theta}), 1\right) \geq \Delta_\theta \Pi_U\left(\tilde{q}(\underline{\theta}), 0\right), \tag{15}$$

*otherwise it is **uniform** with $q^*(\theta) = \tilde{q}$ where $\tilde{q}$ is independent of θ and solves*

$$p(\tilde{q}) = E_\theta\left[SMC(\tilde{q}, \theta)\right] + (1-\alpha)\frac{v}{1-v}\frac{\partial \Delta_\theta \Pi_U(\tilde{q}, v)}{\partial q}. \tag{16}$$

(ii) The profit of a conglomerate with scope economies θ is $\Pi^I(\theta) = \pi^S + (1-\mathcal{I}(\theta))\Delta_\theta \Pi_U\left(\underline{q}^*, v\left(\underline{q}^*\right)\right)$ with $\underline{q}^*$ as in point (i).

To interpret the results in the Proposition assume for the moment that constraint (15) is always satisfied so that optimal quantities are discriminatory. Then, as in standard models of regulation with asymmetric information, the regulator must guarantee the conglomerate with large scope economies (i.e. type $\overline{\theta}$) an additional rent $\Delta_\theta \Pi_U(\underline{q}, v(\underline{q}))$ which corresponds to the higher profit type $\overline{\theta}$ can obtain with respect to $\underline{\theta}$ when asked to produce the same quantity $\underline{q}$ and rivals believe that scope economies are low (i.e. constraint $IC(\overline{\theta})$ binds at the optimum). The (socially costly) rent of type $\overline{\theta}$ is an increasing function of the quantity designed for low scope economies (i.e. $\partial \Delta_\theta \Pi_U(\underline{q}, 0)/\partial \underline{q} \geq 0$), so that the optimal $\underline{q}$ is distorted downwards relative to full information, and we (generically) have $q^*(\overline{\theta}) > q^*(\underline{\theta})$ (which justifies the rivals' beliefs described above). If the conglomerate's incentives to announce the level of scope economies were solely driven by the cost efficiency effect, then this monotonicity on regulated output would also guarantee that the inefficient conglomerate (i.e. type $\underline{\theta}$) had no incentives to mimic type $\overline{\theta}$ since, otherwise, it would have to produce a large output $q^*(\overline{\theta})$ that is too costly given its low efficiency.

However, we know that incentives to announce the level of scope economies are also affected by the two strategic effects which may either facilitate or hinder the regulatory process. Now, the role of the two strategic effects is best understood rewriting the incentive compatibility constraint for type $\underline{\theta}$ as follows,

$$\Pi_U\left(\overline{q}, \overline{\theta}, 1\right) - \Pi_U\left(\underline{q}, \overline{\theta}, 0\right) \geq \Pi_U\left(\overline{q}, \underline{\theta}, 1\right) - \Pi_U\left(\underline{q}, \underline{\theta}, 0\right). \tag{17}$$

which corresponds to (15) in the Proposition.[18] The left hand side is the change of profits in market U for a type $\overline{\theta}$ conglomerate when regulated quantity is $\overline{q}$ and rivals consequently believe scope economies are $\overline{\theta}$, as compared to profits with quantity $\underline{q}$ and rivals believing $\underline{\theta}$. Similarly, the right hand side is the same profit difference for a type $\underline{\theta}$ conglomerate.[19]

Notice that if only the cost-efficiency effect were at play, (17) would simply reduce to $C\left(\overline{q}, y_1; \overline{\theta}\right) - C\left(\overline{q}, y_1; \underline{\theta}\right) \geq C(\underline{q}, y_1; \overline{\theta}) - C(\underline{q}, y_1; \underline{\theta})$ which would be as usual satisfied by monotonicity on output $\overline{q} \geq \underline{q}$ due to properties (2)-(3) of the cost function.

With *quantity competition* in the unregulated market (i.e. strategic substitutability), a firm which appears to be more efficient induces its rivals' to behave less aggressively, thus reducing their outputs and increasing its profits. If $\overline{q} \geq \underline{q}$, then because of the direct strategic effect shifting regulated production from $\underline{q}$ to $\overline{q}$ induces a contraction of the rivals' outputs (for any given θ and associated beliefs). Similarly, for the beliefs-strategic effect, declaring large scope economies induces the rivals to revise their beliefs and again contract their outputs. Both these changes account for an *increase* of profits Π_U so that both sides of (17) become larger for the two strategic effects on the unregulated market. Furthermore, since higher scope economies amplify any change on profits Π_U, the two effects make the left hand side larger than the right hand side, thus making the overall

[18] Substituting $\overline{\Pi}^I = \underline{\Pi}^I + \Delta_\theta \Pi_U(\underline{q}, v(\underline{q}))$ (since $IC(\overline{\theta})$ binds at the optimum as shown in the Appendix) and $\underline{\Pi}^I = \pi^S$ (since $IR(\underline{\theta})$ also binds), constraint $IC(\underline{\theta})$ becomes $\underline{\Pi}^I = \pi^S \geq \pi^S + \Delta_\theta \Pi_U(\underline{q}, v(\underline{q})) - \Delta_\theta \Pi_U\left(\overline{q}, v(\overline{q})\right)$ which is equivalent to (17) and (15).

[19] If only the cost-efficiency effect were at play, (17) would reduce to $C\left(\overline{q}, y_1; \overline{\theta}\right) - C\left(\overline{q}, y_1; \underline{\theta}\right) \geq C(\underline{q}, y_1; \overline{\theta}) - C(\underline{q}, y_1; \underline{\theta})$ which is satisfied by $\overline{q} \geq \underline{q}$ for properties (2)-(3) of the cost function.

mimicking (potential) gain for type $\underline{\theta}$ even less attractive. In other terms, incentive compatibility for the inefficient firm is less demanding than without the two strategic effects and the regulated output may be incentive compatible even if standard monotonicity $\overline{q} \geq \underline{q}$ is violated, thus making the regulatory process "simpler".

If instead, the unregulated market is characterized by *price competition* (i.e. strategic complementarity), both the two strategic effects have adverse consequences on regulation. In fact, the conglomerate induces an accommodating response from its rivals if it now shifts from production $\overline{q}$ to $\underline{q}$ so that the two strategic effects reduce both sides in (17). Since these changes are intensified by higher scope economies, the left hand side decreases more than the right hand side and the two effects make more difficult to satisfy incentive compatibility for type $\underline{\theta}$. A further consequence is that, contrary to standard models of regulation with asymmetric information, constraint $IC(\underline{\theta})$ may be violated even if regulated output is monotone and, if this is the case, the regulator may be obliged to give up discriminating with respect to scope-economies, thus resorting to uniform regulation as indicated in the pricing condition (16).

The two strategic effects are also relevant for the firm's rent $\Delta_\theta \Pi_U(\underline{q}, v(\underline{q}))$ illustrated in Proposition 1. It is now clear that the informational rent $\Delta_\theta \Pi_U$ of the efficient conglomerate is larger if mimicking an inefficient firm induces the rivals to believe that it is really inefficient, i.e. when $v(\underline{q}) = 0$. On the contrary, if the rivals were informed on the level of scope economies, so that their beliefs would be $v(\underline{q}) = 1$ in any case, then they would reduce their price by much lesser extent than when their beliefs are $v(\underline{q}) = 0$ so that, ultimately, the rent would be smaller, i.e. $\Delta_\theta \Pi_U(\underline{q}, 1) \leq \Delta_\theta \Pi_U(\underline{q}, 0)$. It now should also be clear that the direct strategic effect similarly increases the firm's rent with price competition and that, conversely, the two strategic effects reduce the rent when firms compete on quantities.

These results are summarized in the following Proposition.

Proposition 2 (Regulation with quantity and price competition) .

(i) With ***quantity-competition*** *in the unregulated market, optimal regulation is discriminatory with* $\underline{q}^* = \tilde{q}(\underline{\theta}) \leq \underline{q}^I_{FI}$, $\overline{q}^* = \tilde{q}(\overline{\theta}) = \overline{q}^I_{FI}$. *The conglomerate is hurt by the lack of information on scope economies of the rival firms and by the direct strategic effect in the unregulated market. Both effects make the regulator's task simpler.*

(ii) With ***price-competition****, optimal regulation may be either discriminatory with* $q^*(\theta) = \tilde{q}(\theta)$ *or uniform with* $q^*(\theta) = \tilde{q}$ *for any* θ *and* $\underline{q}^I_{FI} \gtrless \tilde{q} \leq \overline{q}^I_{FI}$. *In any case, the conglomerate gains by the lack of information of the rivals and by the direct strategic effect in the unregulated market. Both effects make the regulator's task more complex.*

The discussion above highlights how different forms of market competition have different consequences on the ability of the regulator to design an efficient regulatory contract. With Cournot competition, eliciting information from the regulated firm is easier, so that the regulatory contract benefits from the existence of a competitive market, where the conglomerate can freely operate. The opposite holds under price competition, where revealing a firm's efficiency may stimulate the rivals' reaction.

These different effects and the role played by the unregulated market in the regulatory process will prove important also for the analysis in Section 5 in which we will discuss the (social) desirability of joint or separate productions. Before turning to this analysis it is instructive to present a simple explicit model which allows to further investigate the interplay between the regulated and unregulated activities of the conglomerate.

4.1 An explicit model

Let costs be described by (4) so that the level of scope economies is simply assessed by the term $-\theta q y_1$ in the cost function and consider the following demands,

$$\begin{array}{ll} \text{Market } R: & p_R = \gamma_R - q,\ \gamma_R \geq c \\ \text{Market } U: & \left\{ \begin{array}{ll} y_i = \gamma_U - b p_i^U + \sum\limits_{j \notin i} s p_j^U & \text{with price-competition} \\ p_i^U = \gamma_U - b y_i - \sum_{j \neq i} s y_j & \text{with quantity competition} \end{array} \right. \end{array} \tag{18}$$

where $\gamma_U \geq c$, $b \geq (n-1)s \geq 0$ and $s \geq 0$ is a substitutability parameter.[20]

When the conglomerate cannot integrate productions, regulation in market R takes place under full information, with optimal price $p(q^S) = c$ and quantity $q^S = \gamma_R - c$ and the conglomerate is left with profit π^S which depends on the type of competition in the unregulated markets. The regulatory contract in this case is $\mathcal{C}^S \equiv (\gamma_R - c, -\pi^S)$.

Quantity-competition. Imagine the conglomerate chooses a regulated output $\widehat{q}$ but is characterized by a true level of scope economies θ. It is then useful to describe the estimation error on conglomerate' scope-economies incurred by the rival firms when they observe $\widehat{q}$, i.e.

$$\Delta(\widehat{q}, \theta) \equiv v(\widehat{q})\,\overline{\theta}\overline{q} + (1 - v(\widehat{q}))\underline{\theta}\underline{q} - \theta\widehat{q}.$$

[20] As usual, in case of price-competition, this system of demand for market U can be derived from utility $V_U[y_1, y] = \mu(y_1 + (n-1)y) - \frac{1}{2}\beta\left(y_1^2 + (n-1)y^2\right) - \gamma(n-1)y_1 y$ where $\gamma_U = \frac{\mu}{\beta+(n-1)\sigma}$, $b = \frac{\beta+(n-2)\sigma}{(\beta+(n-1)\sigma)(\beta-\sigma)}$, $s = \frac{\sigma}{(\beta+(n-1)\sigma)(\beta-\sigma)}$.

With discriminatory regulation we clearly have,

$$\Delta\left(\overline{q},\overline{\theta}\right)=\Delta\left(\underline{q},\underline{\theta}\right)=0,\quad \Delta\left(\overline{q},\underline{\theta}\right)=\overline{q}\left(\overline{\theta}-\underline{\theta}\right)>0,\quad \Delta\left(\underline{q},\overline{\theta}\right)=-\underline{q}\left(\overline{\theta}-\underline{\theta}\right)<0.$$

If the announcement $\widehat{\theta}$ corresponds to the true level of scope economies $\widehat{\theta}=\theta$, then observing $\widehat{q}$ the rivals will make no error. On the contrary, when $\widehat{\theta}\neq\theta$ the error may induce the rivals to over- or under-estimate scope economies. Finally, with uniform regulation $\tilde{q}$ we clearly have $\Delta\left(\tilde{q},\theta\right)=\tilde{q}\left[v\overline{\theta}+(1-v)\underline{\theta}-\theta\right].$

Outputs in the unregulated market can then be written as follows,

$$\begin{aligned} y_1\left(q,\theta,v\left(q\right)\right) &= y_1^{FI}\left(q,\theta\right)+\tfrac{(n-1)s^2}{2b(2b-s)[2b+s(n-1)]}\Delta\left(q,\theta\right) \\ y\left(q,v\left(q\right)\right) &= y^{FI}\left(q,\theta\right)-\tfrac{s}{(2b-s)[2b+s(n-1)]}\Delta\left(q,\theta\right) \end{aligned} \tag{19}$$

where $y_1^{FI}\left(q,\theta\right)$ and $y^{FI}\left(q,\theta\right)$ are the outputs that would prevail were the rivals informed on θ (more details are in the Appendix). The previous expressions show that when the rivals overestimate the expected scope economies so that $\Delta\left(q,\theta\right)\geq 0$, the conglomerate expands its production and the rivals contract theirs, and the opposite holds with underestimation $\Delta\left(q,\theta\right)\leq 0$. In the former case the conglomerate gains and the rivals lose and the opposite in the latter case.

In the Appendix we show that constraint $IC(\underline{\theta})$ is here equivalent to

$$\left(\overline{q}-\underline{q}\right)4b(2b-s)\left(\gamma_U-c\right)+\overline{q}^2k-\underline{q}^2a\geq 0$$

with $k\geq a\geq 0$. This shows that monotonicity $\overline{q}\geq\underline{q}$ is *sufficient but not necessary* for incentive compatibility of type $\underline{\theta}$, as previously discussed. Furthermore and along the same lines, in the Appendix we also show that the conglomerate's rent $\Delta_\theta\Pi_U(\underline{q},0)$ is reduced by the error $\Delta(\underline{q},\overline{\theta})<0$: the imperfect information of the rivals indeed negatively affects the conglomerate and favours the regulator.

Price-competition. Equilibrium prices in market U are

$$\begin{aligned} p_1^U &= p_1^{FI}\left(q,\theta\right)-\tfrac{(n-1)s^2}{2(2b+s)(2b-(n-1)s)}\Delta\left(q,\theta\right) \\ p^U &= p^{FI}\left(q,\theta\right)-\tfrac{bs}{(2b+s)(2b-(n-1)s)}\Delta\left(q,\theta\right) \end{aligned} \tag{20}$$

where $p_1^{FI}(q,\theta)$ and $p^{FI}(q,\theta)$ are the prices prevailing were the rivals informed on θ. Now, contrary to quantity competition, if rivals expect larger economies of scope than real ones (i.e. $\Delta(q,\theta)\geq 0$), then they reduce their price and, by complementarity, also the conglomerate reduces its price. It can be shown that conglomerate's profit Π_U in market U is then decreasing in $\Delta(q, \theta)$.

As previously explained, with price-competing firms conglomerate's incentives to understate scope economies are aligned in the two markets so that the regulator will find it more difficult to obtain information revelation. This reflects into the constraint $IC(\underline{\theta})$ which is here equivalent to

$$\left(\overline{q} - \underline{q}\right) \left[2A + B\left(\overline{q} + \underline{q}\right)\left(\overline{\theta} + \underline{\theta}\right)\right] \geq \left(\overline{q}^2 + \underline{q}^2\right)(n-1)s^2\left(\overline{\theta} - \underline{\theta}\right)$$

with $A > 0$ and $B > 0$ so that monotonicity $\overline{q} \geq \underline{q}$ is necessary but not sufficient for incentive compatibility. Furthermore, we also show in the Appendix that, the firm's rent $\Delta_\theta \Pi_U(\underline{q}, 0)$ is now increased by the error $\Delta(\underline{q}, \overline{\theta}) < 0$.

We can now exploit this simple explicit model also to investigate the effects of some specific properties of the competitive market on regulation and firm's rent.

Proposition 3 (An explicit model) *Let cost and demand in the two markets be as in (4) and (18).*

Quantity-competition. *A less concentrated or smaller unregulated market (i.e. larger n or smaller γ_U) both imply a smaller informational rent for the conglomerate and a smaller asymmetric-information distortion on regulation.*

Price-competition. *The effects of concentration and size in the unregulated market are ambiguous: A more competitive and smaller unregulated market may increase both the informational rent and the regulatory distortion.*

With quantity-competing firms, any characteristics of market U, such as its dimension and the number of competitors, that lead to an increase of y_1 unambiguously affect conglomerate's incentives over cost announcement. Indeed, the larger is y_1, the larger the scope economies term $\theta q y_1$ reducing costs, as well as the cost saving of firm with $\theta = \overline{\theta}$ as compared with type $\theta = \underline{\theta}$. Hence, when y_1 is large, for example due to a large unregulated market or limited competition, the regulator has to leave a larger profit $\Delta_\theta \Pi_U$ to type $\overline{\theta}$ and this also increases the distortion arising when scope economies are low. This shows that if a regulated firm wants to expand its activities in one out of several unregulated markets, when competition is on quantities it should enter into larger and less competitive markets, as expected.

Things are different with price-competing firms. In fact, we know in this case the two strategic effects increase the firm's rents and the regulatory distortions. Since the characteristics of the unregulated market (i.e. n and γ_U) now have a complex impact on these strategic effects (recall that optimal regulation may be also uniform), it turns out that, unexpectedly, a more competitive and larger unregulated market may increase the informational rent and negatively affect consumers in the regulated market, as documented in the proof. In this case, the conglomerate would then prefer to expand into more competitive and relatively smaller unregulated markets.

5 The desirability of horizontal integration

We now investigate whether allowing the conglomerate to integrate production of regulated and unregulated outputs is desirable at all. Equivalently, we study the desirability of allowing a regulated firm to expand its activities into an unregulated and competitive market.

On the one hand, integrating production brings (large or small) scope economies, but on the other hand scope economies are privately known by the conglomerate who takes advantage of this private information with respect to the regulator and also the rival firms. In particular, as shown in Proposition 1, asymmetric information associated with production integration allows the firm to earn informational rents that are a social cost, and induce inefficiencies in the regulatory process since the regulated price systematically entails a loss of allocative efficiency when economies of scope are small. Furthermore, when the regulator cannot differentiate its policy on the basis of the firm's efficiency (i.e. when uniform regulation is in place), then rival firms in the unregulated market face the additional problem that they operate under asymmetric information and this may negatively impact on their profits and on consumer surplus in that market, as we will further discuss.

To analyze the desirability of integration and its pros and cons, it is first useful to consider a couple of simplified informational environments that are instrumental to the analysis.

Consider first the benchmark where *both the regulator and the rivals know the level of scope economies* which has been developed in Section 3. When the conglomerate can integrate its activities, also in this setup several effects emerge on welfare as compared with the case of separation. First, the conglomerate is more efficient in its activities in the unregulated market where total industry costs are then lower, as well as equilibrium prices. Second, the overall profits earned by the firms in the two industries are reduced. In fact, total profits earned by the firms in the two markets are $\pi^S + \sum_{i \neq 1} \pi_i^I (q, \theta)$ with integration and $\sum_{i=1}^n \pi_i^S$ with separation and we know that $\pi_i^I (q, \theta) \leq \pi_i^S$ since with integration the rivals face a tougher competitor. Hence, with this informational setup allowing integrated conglomerate production is clearly desirable.

Consider now a second instrumental environment in which the *regulator is informed but rivals are not informed* on θ. In this case regulation would convey all the information on θ to the rivals since (generically) we would have regulated outputs $\overline{q}^I_{FI} \neq \underline{q}^I_{FI}$ and prices $\overline{p}^I_{FI} \neq \underline{p}^I_{FI}$. Hence, again joint production would be preferable to separation.[21]

This discussion seems to point out that, if anything, the problem with joint production should eventually relate to the worsening of the *regulator's information.* A partial answer is provided in the following Lemma.

Lemma 1 (Integration and partial information) *(i) If the regulator is fully informed on scope-economies, letting the conglomerate to integrate its productions is socially desirable, independently of the information of the rivals in the unregulated market;*

(ii) If the rivals are fully informed, then integration is socially desirable independently of the regulator's information.

In addition to point (i) discussed above, point (ii) in the Lemma illustrates a different informational environment in which the regulator does not know the level of scope economies but the rivals are fully informed on θ. The intuition for the desirability of joint production also in this case is as follows. Imagine that with integration the regulator simply offered the conglomerate exactly the same regulatory contract $\mathcal{C}^S$ that she would offer in case of separation. Clearly, the consumer surplus in the regulated sector would be unaffected relative to the case of separation because the firm produces exactly the same quantity q^S. The conglomerate would obtain a larger profit due to scope economies which increase social welfare proportionally to the weight α (the transfer is also clearly unchanged). Finally, being the rivals fully informed, the unregulated market simply becomes more efficient because one of the active firms (the conglomerate) now has lower costs. Hence, the effect of joint production is clearly positive.

However, as emphasized in the introduction, rival firms in unregulated markets often lament their impossibility to ascertain the actual magnitude of scope economies of conglomerate firms, as much as regulators do. When this is the case, none of the previous arguments is sufficient to reach a conclusion on the desirability of integration and a more detailed analysis is called for, also requiring to distinguish the nature of competition in the unregulated market.

Consider first quantity competition in the unregulated market. Indeed, one cannot rely on the contract designed for separation $\mathcal{C}^S$ because with quantity competition leaving the rivals with no information may hurt the unregulated market. When the rivals do not know the value of θ and do not receive any information from the regulatory process (as it is the case when contract $\mathcal{C}^S$ is

[21] The desirability of integration would hold in this case even if optimal regulation conveyed only partial information on θ to the unregulated market. In fact, welfare associated with full disclosure is always attainable and larger than that with separation, as discussed in the text.

the policy in place), they act as if the conglomerate had an "average" level of scope economies. In particular, when the real value of θ is $\overline{\theta}$, rivals underestimate scope economies and produce more than they would otherwise do. On the contrary, when they overestimate the level of scope economies, they reduce production and it may well happen that the contraction of total production of the $n-1$ rivals exceeds the expansion of conglomerate's output.[22] Now, contract $\mathcal{C}^S$ may then induce the following ranking for total output in the unregulated market $Y\left(q^S,\overline{\theta},v\right) \geq Y^S \geq Y\left(q^S,\underline{\theta},v\right)$ and, since gross consumer surplus is a concave function of total output, the net effect of integration on (expected) consumer surplus in the unregulated market may be negative.

Consider now price competition. From the analysis of Propositions 1 and 2 we know that, when the rivals are uninformed, information revelation in the regulatory process is problematic; this may well lead to large distortions on regulated outputs. The conglomerate has additional incentives to lie to the regulator, hiding its efficiency. Because of strategic complementarity, if the rivals perceive that the conglomerate is more efficient they will react more aggressively. As a consequence, inducing the regulated firm to reveal its type is more difficult, and regulation becomes less efficient. Hence a trade-off emerges again: the greater technical efficiency which comes from integration entails a larger distortion in the regulated price and a larger informational rent for the conglomerate.

Notwithstanding these negative effects of integrated productions both with price and quantity competition, the following result holds.

Proposition 4 (Desirability of Integration) *Irrespective of the type of competition in the unregulated market, letting the regulated firm integrate and run joint production for regulated and unregulated markets is socially desirable, even if both the rival firms and the regulator do not know the value of scope economies.*

In order to get an intuition for this important result, consider again the contract $\mathcal{C}^S$ which the regulator optimally designs for the case of separation. As already discussed, applying this policy when the conglomerate instead integrates production raises a problem: with this contract, uninformed rivals would end up with no information on the magnitude of scope economies. However, with price competition (in general with strategic complementarity) the possibility that the regulated firm has lower costs makes *rivals more aggressive* even if they do not know exactly the magnitude of scope economies, thus inducing a larger welfare in the unregulated market. Hence, although regulation $\mathcal{C}^S$ is suboptimal and leaves the rivals uninformed, with price competition it still allows

[22] In the explicit model of Section 4.1 this is the case when $Y^S - Y(q^S,\underline{\theta},v) = (n-1)v\overline{q}\overline{\theta} - \underline{q}\underline{\theta}\,[2+v(n-1)]$ is positive, e.g. when $\underline{\theta}$ is sufficiently low. Furthermore, although here not explicitly considered, this uncertainty over θ may even induce some rivals to exit the market.

to reach a larger welfare than with separate productions, thus making joint productions even more desirable when an optimal regulatory contract is in place.

This reasoning is no longer true and cannot be employed in the case of quantity competition, as illustrated above. Alternatively, to understand why integration is ultimately desirable even if competition is in quantities, one needs to combine our previous results on optimal regulation in Section 4 and in Lemma 1. The important reference point to consider now, is optimal regulation when the regulator is uninformed whilst the rivals are fully informed on θ. Let us indicate the associated optimal policy with $\mathcal{C}'$. Imagine now to employ regulation $\mathcal{C}'$ when rivals do not know θ. Notice first that $\mathcal{C}'$ (although sub-optimal in this case) remains incentive compatible. To see this, recall our discussion in Section 4 showing that, when the rivals are uniformed and firms compete on quantities, the regulator will find it easier to elicit information by the conglomerate. In fact, the firm with large scope economies obtains a smaller profit and, at the same time, the inefficient firm finds it less convenient to mimic high scope economies when the rivals are uninformed. All this implies that even if regulation $\mathcal{C}'$ is designed for the case in which rivals are informed, with quantity competition in the unregulated market it remains incentive compatible also when applied to the case in which rivals are uninformed.

Hence, although this policy $\mathcal{C}'$ is potentially suboptimal in the latter case, it induces truthful revelation and, what is more, it allows to reach a social welfare that is not smaller than that arising when the rivals do know the level of conglomerate' scope economies. It follows that, *a fortiori*, integration is desirable even if firms compete in quantities and neither the regulator nor the rivals know θ.

Despite the asymmetric information on the level of scope economies that integration brings about, and irrespective of strategic complementarity or substitutability in the unregulated market, integration is preferable to separation: the reaction of competition in the unregulated market can be ultimately turned to the benefits of consumers in the two markets and overall welfare.[23]

6 Discussion and concluding remarks

We have analyzed optimal regulation of a conglomerate firm that serves both a regulated and an unregulated market. When the conglomerate is allowed integrate its production, economies of scope

[23] As stated in Section 2, it is impractical to first let the firm integrate and then split it apart. Furthermore, although one might conceive a contract, where the decision to integrate is taken by the Government, conditional on the observed level of scope economies, even in that (probably implausible) situation our result that separation is dominated by allowing integration would anyway hold with similar arguments. The general principle of unbundling, often considered by regulatory authorities, is thus a dominated policy.

reduce costs, but the magnitude of these economies is not perfectly known to the regulator and to competitors in the unregulated market, so regulation is distorted by asymmetric information and competition in the unregulated market may also be affected adversely. The regulator must therefore take into account how the unregulated market reacts to decisions in the regulated one, because this in turn affects the conglomerate's incentives in its regulated activity. A notable effect of regulation is an informational externality: regulatory policy action conveys valuable information to the rival firms and its effects (on both markets) depend on the nature of competition in the unregulated market. Accordingly, we discussed optimal regulation and its distortions due to asymmetric information when competition in the unregulated market bears, alternatively, on quantities or on prices. We have shown that with quantity competition this externality simplifies the task of the regulator, whereas price competition complicates it.

We then addressed the issue of desirability of joint production in the conglomerate's activities, where a potential trade-off emerges. On one hand, allowing the conglomerate to integrate productions reduces its costs and, if this is at least partially passed on in the form of lower prices, then consumers may benefit (possibly in both markets). On the other hand, the conglomerate's private information makes the regulator's task more difficult, engendering distortions in regulatory policy and may also make the unregulated market less competitive. Notwithstanding this trade-off, we show that if uncertainty bears solely on the magnitude of scope economies and diseconomies are ruled out, then integrated production is socially desirable; and if allowed to do so, the conglomerate will exploit this opportunity.

Other potential benefits of integrated production relate to the demand side. For example, customers would clearly find it advantageous having only one provider for both services (joint billing, lower transaction costs summarized in the expression "one stop shop"). Our model can be actually reinterpreted as one where consumers get higher utility from single bill: the cross-market effect may go through the utility function rather than the economies of scope that we have considered.

We have not explicitly considered here diseconomies of scope (which in our model would correspond to $\underline{\theta} < 0$). If such diseconomies were possible, then clearly the desirability of integration of the conglomerate's activities (for example motivated by managers' desire to build their own "empires") may not hold and the regulator should add to the drawbacks of integration also the risk of a less efficient conglomerate.

Interestingly, our analysis could also be extended over a long run horizon with free entry. Indeed, in this case the zero-profit condition (for rival firms) would make our arguments even simpler.

Without going into analytic details, one may consider that with zero profits in the unregulated market what counts is really consumer surplus, so that allowing integration has a more straightforward impact on price and hence on welfare. In case production in the unregulated market entails a fixed cost, notice moreover that integration would also reduce the duplication effect noted by Vickers (1995).

At least two relevant extensions of the current framework can be conceived. So far, we have considered a situation, where the public authority deciding on integration and the one which sets the regulatory policy share the same objective function. In the EU, while some structural decisions in sectors such as energy or transport are taken at European level, specific regulatory policies are decided by national regulatory authorities. In this case, it may well be that the regulated price dose not fully consider the surplus generated in the competitive sector.

This case has some similarities with our model, but it also entails a few differences. A national sectoral regulator would in any case anticipate that the firm's incentives are affected by its activities in the unregulated market, so that the analysis of incentive compatibility, participation decisions and regulation (which we have carried out in Section 4) would be left qualitatively unaffected. However, a delegation problem would emerge, in that the sectoral regulator would have an objective function, which is not fully in line with the one of the European "principal" who is in charge of structural decisions. We leave this line of research to future work.

A second possible extension could further exploit the informative role played by regulatory policy. In this paper, this informational externality from regulation towards the unregulated market has been framed as a straight dichotomy: either the policy informs the rivals fully or it provides no information at all. Although this simple policy framework is robust (eliminating the possibility that the regulator and the conglomerate collude on the information externality to the unregulated market), it might be suboptimal, if the regulator could "fine tune" information to the unregulated market. As the regulated price is naturally observable, a more sophisticated disclosure policy would then require stochastic regulatory contracts that reveal information only partially.[24] However, it is important to notice that our results on the desirability of an integrated conglomerate would not be affected by this extension. Indeed, a more sophisticated regulatory policy that optimally controls for the information flow would actually make integrated production by the conglomerate even more beneficial. It may be interesting to study the properties of optimal regulation associated with an optimal disclosure policy, for example along the lines illustrated in Calzolari and Pavan

[24]Interestingly, the optimality of stochastic regulatory contracts may emerge in a context in which, absent the informative role of the regulatory policy, the optimal contract would be deterministic, as in standard models of regulation with asymmetric information.

(2006). With this respect, our results suggest that when competition in the unregulated market bears on quantities, disclosure should be optimal, while with price competition a "no disclosure" policy appears to be preferable. This is an interesting challenge that we plan to explore further in future research.

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7 Appendix

Proof of Proposition 1. *Step 1.* The regulatory program is

$$(\mathcal{P}^I)\quad \begin{cases} \underset{\{(q(\theta),\Pi^I(\theta))\}_{\theta\in\Theta}}{Max} \quad E_\theta\left\{W^I\left[q,\left(y_1\left(q,\theta,v(q)\right),Y_{-1}\left(q,v(q)\right)\right),\theta\right]\right\} \\ \text{s.t.} \\ \Pi^I\left(\theta\right)\geq\Pi^I\left(\widehat{\theta};\theta\right) \quad \forall\left(\widehat{\theta},\theta\right)\in\Theta\times\Theta \quad IC(\theta) \\ \Pi^I\left(\theta\right)\geq\pi^S \qquad \forall\theta\in\Theta \qquad IR(\theta) \end{cases}$$

where the objective is defined as in (9) with the difference that outputs in market U also depend on beliefs $v(q)$.[25]

The set of constraints $IC(\theta)$ and $IR(\theta)$ for any $\theta\in\Theta$ can be rewritten as follows,

$$\begin{aligned} &\overline{\Pi}^I\geq\underline{\Pi}^I+\Delta_\theta\Pi_U\left(\underline{q},v(\underline{q})\right), && IC(\overline{\theta})\\ &\underline{\Pi}^I+\Delta_\theta\Pi_U\left(\overline{q},v(\overline{q})\right)\geq\overline{\Pi}^I, && IC(\underline{\theta})\\ &\overline{\Pi}^I\geq\pi^S, && IR(\overline{\theta})\\ &\underline{\Pi}^I\geq\pi^S. && IR(\underline{\theta}) \end{aligned}$$

For given quantity q and associated beliefs of the rival firms $v(q)$, a more efficient firm obtains in market U a larger profit so that $\Delta_\theta\Pi_U\left(q,v(q)\right)>0$ for any $q>0$. Hence, constraints $IC(\overline{\theta})$ and $IR(\underline{\theta})$ imply that $IR(\overline{\theta})$ is slack and can be disregarded. This in turn means that constraint $IR(\underline{\theta})$ must be binding at the optimum. In fact, at least one of the two participation constraints has to be binding at the optimum, because, otherwise, the regulator could reduce both profits $\underline{\Pi}^I$, $\overline{\Pi}^I$ by an equal amount, thus keeping incentive compatibility unaffected and increasing the objective function. Furthermore, constraint $IC(\overline{\theta})$ must also be binding at the optimum. In fact, reducing $\overline{\Pi}^I$ the regulator is able to increase the objective function without negatively affecting $IC(\underline{\theta})$. Hence, she optimally reduces $\overline{\Pi}^I$ as much as possible up to the point in which constraint $IC(\overline{\theta})$ binds.

As for constraint $IC(\underline{\theta})$, this can be written as

$$\Delta_\theta\Pi_U\left(\overline{q},v(\overline{q})\right)\geq\Delta_\theta\Pi_U\left(\underline{q},v(\underline{q})\right). \tag{21}$$

Note that if $\overline{q}=\underline{q}$, then $v(\overline{q})=v(\underline{q})$ so that $\Delta_\theta\Pi_U\left(\overline{q},v\right)=\Delta_\theta\Pi_U\left(\underline{q},v\right)$ and constraint $IC(\underline{\theta})$ is trivially satisfied. The case with $\overline{q}\neq\underline{q}$ will be treated in the next steps.

[25] With the usual change of variables, maximization in program $(\mathcal{P}^I)$ is equivalently taken over the contract $\left\{\left(q\left(\theta\right),\Pi^I\left(\theta\right)\right)\right\}_{\theta\in\Theta}$ instead of $\left\{\left(q\left(\theta\right),T\left(\theta\right)\right)\right\}_{\theta\in\Theta}$. In both cases we will indicate the contract with $\mathcal{C}$.

Step 2. Using step 1, we can now further rewrite program $(\mathcal{P}^I)$ in the following equivalent way

$$(\mathcal{P}')\left\{\begin{array}{ll} \underset{(\overline{q},\underline{q})}{Max} E_\theta \Big\{ V_R(q) + V_U[Y(q,\theta)] - C[q, y_1(q,\theta);\theta] - \sum_{i\neq 1} C[y_i(q,\theta)] + & \\ \qquad -(1-\alpha)\sum_{i\neq 1}\pi_i^I(q,\theta)\Big\} - (1-\alpha)v\Delta_\theta\Pi_U\left(\underline{q}, v(\underline{q})\right) & \\ s.t. \quad \Delta_\theta\Pi_U\left(\overline{q}, v(\overline{q})\right) \geq \Delta_\theta\Pi_U\left(\underline{q}, v(\underline{q})\right) & IC(\underline{\theta}) \end{array}\right.$$

Hence, let $\tilde{q}(\theta)$ for $\theta \in \Theta$ be solution of the following two first order conditions

$$\begin{array}{l} \frac{\partial SMC(\overline{q},\overline{\theta})}{\partial q} = 0 \\ \frac{\partial SMC(\underline{q},\underline{\theta})}{\partial q} - (1-\alpha)\frac{v}{1-v}\frac{\partial \Delta_\theta \Pi_U(\underline{q},0)}{\partial q} = 0 \end{array}$$

where $v(q) = 0$ for $q = \tilde{q}(\underline{\theta})$, $v(q) = 1$ for $q = \tilde{q}(\overline{\theta})$ and generically we have $\tilde{q}(\overline{\theta}) \neq \tilde{q}(\underline{\theta})$.

If $\Delta_\theta\Pi_U\left(\tilde{q}(\overline{\theta}), 1\right) \geq \Delta_\theta\Pi_U\left(\tilde{q}(\underline{\theta}), 0\right)$, then the optimal regulated quantities $q^*(\theta)$ are $q^*(\theta) = \tilde{q}(\theta)$ for any θ, because these quantities maximize the objective in $(\mathcal{P}')$ and satisfy the unique constraint $IC(\underline{\theta})$.

If instead $\Delta_\theta\Pi_U\left(\tilde{q}(\overline{\theta}), 1\right) < \Delta_\theta\Pi_U\left(\tilde{q}(\underline{\theta}), 0\right)$, quantities $\tilde{q}(\overline{\theta}), \tilde{q}(\underline{\theta})$ violate $IC(\underline{\theta})$ so that the optimal solution requires that $IC(\underline{\theta})$ binds. Thus, consider a pair of quantities $\overline{q}, \underline{q}$ such that $\Delta_\theta\Pi_U\left(\overline{q}, v(\overline{q})\right) = \Delta_\theta\Pi_U\left(\underline{q}, v(\underline{q})\right)$. This implies $\overline{q} = \underline{q}$. In fact, suppose on the contrary that $\overline{q} \neq \underline{q}$ so that $v(\overline{q}) = 1$, $v(\underline{q}) = 0$ and $\Delta_\theta\Pi_U\left(\overline{q}, 1\right) = \Delta_\theta\Pi_U\left(\underline{q}, 0\right)$ or equivalently

$$\Pi_U\left(\overline{q}, \overline{\theta}, 1\right) - \Pi_U\left(\overline{q}, \underline{\theta}, 1\right) = \Pi_U\left(\underline{q}, \overline{\theta}, 0\right) - \Pi_U\left(\underline{q}, \underline{\theta}, 0\right).$$

This last equality is clearly generically impossible unless $\overline{q} = \underline{q}$, thus leading to a contradiction. Hence, when $IC(\underline{\theta})$ binds optimal regulation requires pooling so that quantities do not depend on θ. In this case, whatever its type θ, the conglomerate firm is required to produce a quantity $\tilde{q}$ independent of θ. This quantity can the be obtained by solving the following program,

$$\begin{array}{c} \underset{q}{Max} E_\theta \Big\{ V_R(q) + V_U[Y(q,\theta)] - C[q, y_1(q,\theta);\theta] - \sum_{i\neq 1} C[y_i(q,\theta)] + \\ -(1-\alpha)\sum_{i\neq 1}\pi_i^I(q,\theta)\Big\} - (1-\alpha)v\Delta_\theta\Pi_U(q,v) \end{array}$$

where constraint $IC(\underline{\theta})$ is omitted because, for what stated at the end of step 1, it is satisfied when $\overline{q} = \underline{q} = \tilde{q}$.

Step 3. Given the optimal quantities $q^*(\theta)$ obtained in step 2 we then have that the profit of the conglomerate with low scope economies is $\Pi^I\left(\underline{\theta}\right) = \pi^S$ from $IR(\underline{\theta})$ binding and for the efficient one is $\Pi^I\left(\overline{\theta}\right) = \pi^S + \Delta_\theta\Pi_U\left(\underline{q}^*, v\left(\underline{q}^*\right)\right)$ from $IC(\overline{\theta})$ binding. ■

Proof of Proposition 2. *Step 1.* We first derive the ranking on quantities $\tilde{q}(\underline{\theta}), \tilde{q}(\overline{\theta})$ defined in the text of Proposition 1.

We show that the distortion $\partial\Delta_\theta\Pi_U(q,v)/\partial q$ in the pricing conditions (14) and (16) is positive independently of the type of strategic interaction in the unregulated market. To see this, for the generic strategic variable x_1 of the conglomerate in market U consider the associated the first order condition,

$$\frac{\partial}{\partial x_1}\Pi^I(\widehat{q}, x_1, X_{-1}; \theta) = 0.$$

This condition depends on θ through the marginal cost $\frac{\partial C(q,y_1;\theta)}{\partial y_1}$. Now, the properties of the cost function (1)-(3) state that (i) this marginal cost is reduced by a larger q, due to scope economies, and (ii) this reduction is stronger the higher is θ (i.e. with large scope economies). Hence, for the implicit function theorem, it follows that the equilibrium profit $\Pi_U(q,\theta,v)$ is increasing in q, in θ and that the profit increase caused by a larger q is larger the higher is θ. Hence, keeping constant the rivals' beliefs for (i.e. for a given v) we have

$$\frac{\partial\Pi_U\left(q,\overline{\theta},v\right)}{\partial q} \geq \frac{\partial\Pi_U\left(q,\underline{\theta},v\right)}{\partial q} \geq 0, \tag{22}$$

and then

$$\frac{\partial\Delta_\theta\Pi_U(q,v)}{\partial q} \geq 0. \tag{23}$$

With the sign of (23) we then obtain the ranking on optimal regulated output. In particular, if with full information scope economies induce a larger regulated output $\underline{q}^I_{FI} \leq \overline{q}^I_{FI}$, then (23) implies $\tilde{q}(\underline{\theta}) \leq \tilde{q}(\overline{\theta})$ with strict inequality if $\overline{\theta} > \underline{\theta}$.

Step 2. Notwithstanding the monotonicity proved in the previous step, Proposition 1 illustrates that, quantities $\tilde{q}(\underline{\theta}), \tilde{q}(\overline{\theta})$ may fail to be incentive compatible. Here we analyze when this is the case and we check whether these outputs satisfy constraint $IC(\underline{\theta})$. As illustrated in (21), the incentive compatibility constraint for type $\underline{\theta}$ is equivalent to

$$\left[\Pi_U\left(\overline{q},\underline{\theta},1\right) - \Pi_U\left(\underline{q},\underline{\theta},0\right)\right] - \left[\Pi_U\left(\overline{q},\overline{\theta},1\right) - \Pi_U\left(\underline{q},\overline{\theta},0\right)\right] \leq 0. \tag{24}$$

We now decompose this inequality into the three effects of cost announcement. To consider the simple *cost-efficiency effect* of announcement, let us fictitiously assume that outputs y_1 and y do not depend on θ and q, in which case (24) would be

$$\begin{aligned}&\left[-C\left(\overline{q},y_1;\underline{\theta}\right)+C\left(\overline{q},0;\underline{\theta}\right)+C\left(\underline{q},y_1;\underline{\theta}\right)-C\left(\underline{q},0;\underline{\theta}\right)\right]+\\&-\left[-C\left(\overline{q},y_1;\overline{\theta}\right)+C\left(\overline{q},0;\overline{\theta}\right)+C\left(\underline{q},y_1;\overline{\theta}\right)-C\left(\underline{q},0;\overline{\theta}\right)\right]\leq 0\end{aligned}$$

or equivalently

$$\int_{\underline{q}}^{\overline{q}}\int_0^{y_1}\frac{\partial^2 C\left(h,u;\overline{\theta}\right)}{\partial y_1\partial q}dudh-\int_{\underline{q}}^{\overline{q}}\int_0^{y_1}\frac{\partial^2 C\left(h,u;\underline{\theta}\right)}{\partial y_1\partial q}dudh\leq 0.$$

It is then immediate that properties of the cost function (1)-(3) imply that the previous inequality is satisfied by standard monotonicity, i.e. for $\underline{q}\leq\overline{q}$.. Hence, the cost-efficiency effect alone would imply that outputs $\left(\tilde{q}(\underline{\theta}),\tilde{q}(\overline{\theta})\right)$ are implementable.

We now add the *direct strategic effect* reintroducing the dependence of y_1 and y on θ and q, but *keeping the rivals' beliefs unchanged*. To this end let assume that rivals are fully informed so that even if regulated output is $q(\hat{\theta})$ and real scope economies are associated to type θ, rivals' beliefs are still such that $\Pr(\theta|q(\hat{\theta}))=1$. Constraint (24) would then be

$$\begin{aligned}&\left[\Pi_U\left(\overline{q},\underline{\theta},0\right)-\Pi_U\left(\underline{q},\underline{\theta},0\right)\right]-\left[\Pi_U\left(\overline{q},\overline{\theta},1\right)-\Pi_U\left(\underline{q},\overline{\theta},1\right)\right]=\\&\textstyle\int_{\underline{q}}^{\overline{q}}\frac{\partial\Pi_U(h,\underline{\theta},0)}{\partial q}dh-\int_{\underline{q}}^{\overline{q}}\frac{\partial\Pi_U\left(h,\overline{\theta},1\right)}{\partial q}dh\leq 0\end{aligned}\tag{25}$$

where the notable difference with (24) is that rivals' beliefs on θ are always correct: independently of q then $v(q)=1$ if type is $\overline{\theta}$ and $v(q)=0$ if $\underline{\theta}$. Now, from (1)-(3) we know that the marginal cost of y_1 is decreasing in q for any θ, i.e. $\frac{\partial^2 C(q,y_1;\theta)}{\partial q\partial y_1}\leq 0$ and this marginal cost reduction associated with a larger q is larger the higher is θ.. Hence, independently of the type of competition we have,

$$0\leq\frac{\partial\Pi_U\left(q,\underline{\theta},0\right)}{\partial q}\leq\frac{\partial\Pi_U\left(q,\overline{\theta},1\right)}{\partial q}.$$

These inequalities imply that for both the cost-efficiency and the direct strategic effects, constraint (25) is verified by the simple monotonicity condition for outputs $\underline{q}\leq\overline{q}$.

We are now left to study the *belief-related strategic* effect. Adding and subtracting $\Pi_U\left(\overline{q},\underline{\theta},0\right)$ and $\Pi_U\left(\overline{q},\overline{\theta},0\right)$ from (24), constraint $IC(\underline{\theta})$ becomes

$$\begin{aligned}&\left[\Pi_U\left(\overline{q},\underline{\theta},1\right)-\Pi_U\left(\overline{q},\underline{\theta},0\right)-\left[\Pi_U\left(\underline{q},\underline{\theta},0\right)-\Pi_U\left(\overline{q},\underline{\theta},0\right)\right]\right]-\\&\left[\Pi_U\left(\overline{q},\overline{\theta},1\right)-\Pi_U\left(\overline{q},\overline{\theta},0\right)-\left[\Pi_U\left(\underline{q},\overline{\theta},0\right)-\Pi_U\left(\overline{q},\overline{\theta},0\right)\right]\right]\leq 0,\end{aligned}$$

or, equivalently

$$\begin{aligned}
&\Pi_U\left(\overline{q},\underline{\theta},1\right)-\Pi_U\left(\overline{q},\underline{\theta},0\right)-\left[\Pi_U\left(\overline{q},\overline{\theta},1\right)-\Pi_U\left(\overline{q},\overline{\theta},0\right)\right]+\\
&-\left[\int_{\underline{q}}^{\overline{q}}\left(\frac{\partial\Pi_U\left(h,\overline{\theta},0\right)}{\partial q}-\frac{\partial\Pi_U\left(h,\underline{\theta},0\right)}{\partial q}\right)dh\right]\leq 0.
\end{aligned} \tag{26}$$

The second line is negative whenever $\underline{q} \leq \overline{q}$ for the same reasons illustrated above on the direct strategic effect. On the contrary, the sign of the first line depends on the type of competition in market U. The function $\Pi_U(q,\theta,1) - \Pi_U(q,\theta,0)$ uniquely refers to the effect of a change of rivals' beliefs for any q and θ, that is for given marginal costs of y_1. With quantity competition, or more generally with strategic substitutability, we clearly have $\Pi_U(q,\theta,1) \geq \Pi_U(q,\theta,0)$, whilst $\Pi_U(q,\theta,1) \leq \Pi_U(q,\theta,0)$ with price competition or strategic complementarity. Furthermore, the absolute value $|\Pi_U(q,\theta,1) - \Pi_U(q,\theta,0)|$ is increasing in θ because a smaller marginal cost of y_1 (induced by a larger θ) amplifies the change induced by different beliefs, so that we have

$$\begin{aligned}
&\Pi_U\left(\overline{q},\underline{\theta},1\right)-\Pi_U\left(\overline{q},\underline{\theta},0\right)-\left[\Pi_U\left(\overline{q},\overline{\theta},1\right)-\Pi_U\left(\overline{q},\overline{\theta},0\right)\right]\leq 0 \quad \text{with substitutability,}\\
&\Pi_U\left(\overline{q},\underline{\theta},1\right)-\Pi_U\left(\overline{q},\underline{\theta},0\right)-\left[\Pi_U\left(\overline{q},\overline{\theta},1\right)-\Pi_U\left(\overline{q},\overline{\theta},0\right)\right]\geq 0 \quad \text{with complementarity.}
\end{aligned} \tag{27}$$

From the signs in (27) and $IC(\underline{\theta})$ written as (26) we then obtain the following.

First, with strategic complementarity in market U the sign in (27) implies that monotonicity $\underline{q} \leq \overline{q}$ be not sufficient to satisfy $IC(\underline{\theta})$. When the (absolute value of the) first line in (26) is larger than the second line in the case $\underline{q} = \tilde{q}(\underline{\theta}), \overline{q} = \tilde{q}(\overline{\theta})$, then quantities $\tilde{q}(\theta)$ are not incentive compatible in which case optimal regulation is uniform and defined by (16). Quantity $\tilde{q}$ is obtained from a pricing condition averaging with respect to type $\overline{\theta}$ and $\underline{\theta}$ so that $\tilde{q} \leq \overline{q}^I_{FI}$ but $\tilde{q} \gtreqless \underline{q}^I_{FI}$ because two countervailing effects are at play. On one side, the distortionary term $\frac{\partial\Delta_\theta\Pi_U(\tilde{q},v)}{\partial q}$ in (16) reduces $\tilde{q}$.. On the other side, the averaging with respect tot $\overline{\theta}$ and $\underline{\theta}$ increases $\tilde{q}$ as compared with $\underline{q}^I_{FI}$.. Furthermore, for the same reasons, we also have that $\tilde{q}(\underline{\theta}) \leq \tilde{q} \leq \tilde{q}(\overline{\theta})$ so that $\Delta_\theta\Pi_U(\tilde{q},v) \geq \Delta_\theta\Pi_U(\tilde{q}(\underline{\theta}),0)$ which implies that the conglomerate gains and the regulator's task is complicated by the rivals being uninformed on the level of scope economies. Finally, if the first line in (26) is larger than the second line with $\underline{q} = \tilde{q}(\underline{\theta}), \overline{q} = \tilde{q}(\overline{\theta})$, then optimal regulation is discriminatory, $\underline{q}^* = \tilde{q}(\underline{\theta}) \leq \underline{q}^I_{FI}$, $\overline{q}^* = \tilde{q}(\overline{\theta}) = \overline{q}^I_{FI}$. That the conglomerate gains from the rivals being uniformed can be seen in this case with discriminatory regulation by considering the firm's rent $\Pi^I\left(\overline{\theta}\right) = \pi^S + \Delta_\theta\Pi_U(\underline{q}^*,0)$ where $\Delta_\theta\Pi_U(\underline{q}^*,0) = \Pi_U(\underline{q},\overline{\theta},0) - \Pi_U(\underline{q},\underline{\theta},0)$. For strategic complementarity we have that $\Pi_U(\underline{q},\overline{\theta},0) \geq \Pi_U(\underline{q},\overline{\theta},1)$ and also in this case the conglomerate benefits being the rivals uninformed.

With strategic substitutability, the monotonicity $\underline{q} \leq \overline{q}$ implies that the incentive compatibility constraint (24) is satisfied from which it follows that optimal regulated quantities are $\tilde{q}(\underline{\theta}), \tilde{q}(\overline{\theta})$ defined by (14). This in turn gives the the comparison with quantities in the case of full information. Since the first line in (26) is negative the regulator's task is easened by the rivals being uninformed. Furthermore, since for strategic complementarity $\Pi_U(\underline{q}, \overline{\theta}, 0) \leq \Pi_U(\underline{q}, \overline{\theta}, 1)$, the conglomerate gains a smaller rent $\Pi^I\left(\overline{\theta}\right)$ being the rivals uninformed. Finally, monotonicity $\underline{q} \leq \overline{q}$ is here sufficient but not necessary for incentive compatibility. ■

Proof of Proposition 3. .

Quantity-competition.

Outputs in market U can be written as in (19) and the conglomerate's profits Π_U as follows,

$$\Pi_U\left(\widehat{q}, \theta, v\left(\widehat{q}\right)\right) = \frac{\left[2b\left[(2b-s)(\gamma_U - c) + (2b+(n-2)s)\,\widehat{q}\theta\right] + (n-1)s^2\Delta\left(\widehat{q}, \theta\right)\right]^2}{4b(2b-s)^2\left(2b+s(n-1)\right)^2}. \tag{28}$$

We know from Proposition 2 that optimal regulation is discriminatory so that the optimal regulated quantities are $\overline{q}^* \neq \underline{q}^*$. Substituting $y_1\left(\widehat{q}, \theta, v\left(\widehat{q}\right)\right)$ and $y\left(\widehat{q}, v\left(\widehat{q}\right)\right)$ we have

$$\begin{aligned}
\Delta_\theta \Pi_U\left(\overline{q}, 1\right) &= \overline{q}\left(\overline{\theta} - \underline{\theta}\right)\frac{4b(2b-s)(\gamma_U - c) + \overline{q}k}{4b(2b-s)\left(2b+s(n-1)\right)} \\
\Delta_\theta \Pi_U\left(\underline{q}, 0\right) &= \underline{q}\left(\overline{\theta} - \underline{\theta}\right)\frac{4b(2b-s)(\gamma_U - c) + \underline{q}a}{4b(2b-s)\left(2b+s(n-1)\right)}
\end{aligned}$$

where a and k are constant with respect to output and defined as

$$\begin{aligned}
a &\equiv \overline{\theta}(2b-s)\left(2b+s(n-1)\right) + \underline{\theta}\left(2b(2b+s(n-2)) + (n-1)s^2\right) \\
k &\equiv \overline{\theta}\left(2b(2b+s(n-2)) + (n-1)s^2\right) + \underline{\theta}(2b-s)\left(2b+s(n-1)\right)
\end{aligned}$$

with $a \geq 0$, $k \geq 0$ and $a - k = -2(n-1)s^2\left(\overline{\theta} - \underline{\theta}\right) \leq 0$.

>From these expressions for $\Delta_\theta \Pi_U\left(\overline{q}, 1\right)$ and $\Delta_\theta \Pi_U\left(\underline{q}, 0\right)$ we obtain

$$\frac{\partial \Delta_\theta \Pi_U\left(\underline{q}, 0\right)}{\partial \underline{q}} = \left(\overline{\theta} - \underline{\theta}\right)\frac{2b(2b-s)(\gamma_U - c) + \underline{q}a}{2b(2b-s)\left(2b+s(n-1)\right)} \geq 0$$

so that, as long as $\overline{\theta} > \underline{\theta}$, the distortion $(1-\alpha)\frac{v}{1-v}\frac{\partial \Delta_\theta \Pi_U\left(\underline{q}, 0\right)}{\partial \underline{q}}$ in the pricing condition (14) for $\theta = \underline{\theta}$ illustrated in Proposition 1 is strictly positive. Hence, generically we have $\overline{q} \neq \underline{q}$ and $\overline{q} > \underline{q}$ (which also confirm that optimal regulation is discriminatory).

Now, constraint $IC(\underline{\theta})$ can be here equivalently written as

$$\left(\overline{q}-\underline{q}\right)4b(2b-s)\left(\gamma_U-c\right)+\overline{q}^2k-\underline{q}^2a\geq 0$$

which is satisfied when $\overline{q}\geq\underline{q}$ because $k\geq a\geq 0$. Note that, as explained in the text, $IC(\underline{\theta})$ could be satisfied even if $\overline{q}<\underline{q}$ (monotonicity is not necessary for incentive compatibility).

To verify the effect of the error about scope-economies incurred by the rival firms on the conglomerate rent recall that $\Delta_\theta\Pi_U\left(\underline{q},0\right)=\Pi_U\left(\underline{q},\overline{\theta},0\right)-\Pi_U\left(\underline{q},\underline{\theta},0\right)$. Clearly, given that $\Pi_U\left(\underline{q},\underline{\theta},0\right)$ is unaffected by the error Δ and $\Pi_U\left(\underline{q},\overline{\theta},0\right)$ is increasing in $\Delta\left(\underline{q},\overline{\theta}\right)$ we also have that the rent $\Delta_\theta\Pi_U\left(\underline{q},0\right)$ increases in the error $\Delta\left(\underline{q},\overline{\theta}\right)$.

Consider now comparative statics on the main parameters n, γ_U:

$$\begin{aligned}\frac{\partial\Delta_\theta\Pi_U\left(\underline{q},0\right)}{\partial\gamma_U}&=\frac{\left(\overline{\theta}-\underline{\theta}\right)\underline{q}}{2b+s(n-1)}\geq 0\\ \frac{\partial\Delta_\theta\Pi_U\left(\underline{q},0\right)}{\partial n}&=\frac{\left(\overline{\theta}-\underline{\theta}\right)\underline{q}\left[-2b(\gamma_U-c)+s(\gamma_U-c+\underline{q}\underline{\theta})\right]}{(2b-s)\left[2b+s(n-1)\right]^2}\leq 0\end{aligned}$$

where the second inequality follows from the fact that $y\left(\underline{q},0\right)=\frac{2b(\gamma_U-c)-s(\gamma_U-c+\underline{q}\underline{\theta})}{(2b-s)[2b+s(n-1)]}$ so that $y\left(\underline{q},0\right)\geq 0$ if and only the numerator in $\frac{\partial\Delta_\theta\Pi_U\left(\underline{q},0\right)}{\partial n}$ is negative. We also have

$$\begin{aligned}\frac{\partial\Delta_\theta\Pi_U\left(\underline{q},0\right)}{\partial\underline{q}\partial\gamma_U}&=\frac{\overline{\theta}-\underline{\theta}}{2b+s(n-1)}\geq 0\\ \frac{\partial\Delta_\theta\Pi_U\left(\underline{q},0\right)}{\partial\underline{q}\partial n}&=\frac{\left(\overline{\theta}-\underline{\theta}\right)\left[-2b(\gamma_U-c)+s(\gamma_U-c+2\underline{q}\underline{\theta})\right]}{(2b-s)\left[2b+s(n-1)\right]^2}\leq 0\end{aligned}$$

where again the second inequality follows from $y\left(\underline{q},0\right)\geq 0$.

Price-competition.

With equilibrium prices as in (20) we can write conglomerate's profits Π_U in the market U as follows,

$$\Pi_U\left(\widehat{q},\theta,v\left(\widehat{q}\right)\right)=\frac{b\left[A+\widehat{q}\theta B-s^2(n-1)\Delta\left(\widehat{q},\theta\right)\right]^2}{4\left(2b+s\right)^2\left(2b-s(n-1)\right)^2}$$

where

$$\begin{aligned}A&\equiv 2(2b+s)(\gamma_U-c\left(b-(n-1)s\right))>0,\\ B&\equiv 2(2b^2-(n-1)s^2-(n-2)sb)>0.\end{aligned}$$

and the sign of B is implied by $b-(n-1)s>0$. In line with intuition, if the actual level of scope economies θ increases, conglomerate's profit increases and if rivals over-estimate scope economies (i.e. $\Delta\left(\widehat{q},\theta\right)\geq 0$), conglomerate's profit decreases. Hence, the informational rent $\Delta_{\theta}\Pi_{U}\left(\underline{q},0\right)$ of type $\overline{\theta}$ here can be written as,

$$\Delta_{\theta}\Pi_{U}\left(\underline{q},0\right)=\frac{b\left[A+\underline{q}\overline{\theta}B-s^{2}(n-1)\Delta\left(\underline{q},\overline{\theta}\right)\right]^{2}-b\left[A+\underline{q}\underline{\theta}B\right]^{2}}{4\left(2b+s\right)^{2}\left(2b-s(n-1)\right)^{2}}$$

which is decreasing in $\Delta\left(\underline{q},\overline{\theta}\right)$.

The asymmetric information distortion in the pricing condition (14) is

$$\frac{\partial\Delta_{\theta}\Pi_{U}\left(\underline{q},0\right)}{\partial q}=\frac{\left(B+1\right)\left(\overline{\theta}-\underline{\theta}\right)\left(A+\underline{q}\left(\left(\overline{\theta}-\underline{\theta}\right)+B\left(\overline{\theta}+\underline{\theta}\right)\right)\right)}{4\left(2b+s\right)^{2}\left(2b-s(n-1)\right)^{2}}\geq 0,$$

so that the solutions $\tilde{q}(\overline{\theta}),\tilde{q}(\underline{\theta})$ in Proposition 1 are generically monotone, i.e. $\tilde{q}(\overline{\theta})>\tilde{q}(\underline{\theta})$.

With some calculations we also have

$$\begin{aligned}
\Delta_{\theta}\Pi_{U}\left(\overline{q},1\right) &= \frac{b\overline{q}\left(\overline{\theta}-\underline{\theta}\right)\left[2A+\overline{q}\left(-(n-1)s^{2}\left(\overline{\theta}-\underline{\theta}\right)+B\left(\overline{\theta}+\underline{\theta}\right)\right)\right]}{4\left(2b+s\right)\left(2b-(n-1)s\right)}, \\
\Delta_{\theta}\Pi_{U}\left(\underline{q},0\right) &= \frac{b\underline{q}\left(\overline{\theta}-\underline{\theta}\right)\left[2A+\underline{q}\left((n-1)s^{2}\left(\overline{\theta}-\underline{\theta}\right)+B\left(\overline{\theta}+\underline{\theta}\right)\right)\right]}{4\left(2b+s\right)\left(2b-(n-1)s\right)},
\end{aligned}$$

Constraint $IC(\underline{\theta})$, $\Delta_{\theta}\Pi_{U}\left(\overline{q},1\right)\geq\Delta_{\theta}\Pi_{U}\left(\underline{q},0\right)$, can then be written as

$$2A\left(\overline{q}-\underline{q}\right)+B\left(\overline{q}^{2}-\underline{q}^{2}\right)\left(\overline{\theta}+\underline{\theta}\right)\geq\left(\overline{q}^{2}+\underline{q}^{2}\right)(n-1)s^{2}\left(\overline{\theta}-\underline{\theta}\right)$$

which clearly shows that output monotonicity is not sufficient for incentive compatibility.

Consider now the comparative statics on the main parameters n, γ_{U}. If optimal regulation is discriminatory then

$$\begin{aligned}
\frac{\partial\Delta_{\theta}\Pi_{U}\left(\underline{q},0\right)}{\partial\gamma_{U}} &= -\frac{b\underline{q}\left(4b^{2}-2b(n-2)s-3(n-1)s^{2}\right)}{\left(2b+s\right)^{2}\left(2b-s(n-1)\right)^{2}}\leq 0 \\
\frac{\partial\Delta_{\theta}\Pi_{U}\left(\underline{q},0\right)}{\partial\underline{q}\partial\gamma_{U}} &= -\frac{b\left(4b^{2}-2b(n-2)s-3(n-1)s^{2}\right)}{\left(2b+s\right)^{2}\left(2b-s(n-1)\right)^{2}}\leq 0
\end{aligned}$$

where the sign of $4b^{2}-2b(n-2)s-3(n-1)s^{2}\geq 0$ in the numerators can be derived by the expression for $\frac{\partial\Delta_{\theta}\Pi_{U}\left(\underline{q},0\right)}{\partial q}\geq 0$ substituting A and B. We also have that the expressions for $\frac{\partial\Delta_{\theta}\Pi_{U}\left(\underline{q},0\right)}{\partial n}$ and $\frac{\partial\Delta_{\theta}\Pi_{U}\left(\underline{q},0\right)}{\partial\underline{q}\partial n}$ can be positive or negative which is also the case for all the comparative statics when optimal regulation is instead uniform. ∎

Proof of Lemma 1. Point (i) in the Lemma is immediate from what stated in the text. Consider now point (ii).

Let $\mathcal{C}^S$ be the optimal regulatory contract with separation, let $\mathcal{I}'$ be the particular information set in which the rivals but not the regulator are informed about θ, and let $\mathcal{C}'$ be the optimal regulatory contract associated with the information set $\mathcal{I}'$. Finally, let $E_\theta\left[W^I(\mathcal{C},\theta)\,|\mathcal{I}\right]$ be the expected social welfare associated with the optimal regulatory contract $\mathcal{C}$ and the associated information set $\mathcal{I}$, whilst W^S has been defined as the optimal welfare with separation and its optimal contract $\mathcal{C}^S$.

Clearly, being independent of θ, the regulatory contract $\mathcal{C}^S$ is individually rational and incentive compatible when applied to integration with information set $\mathcal{I}'$, i.e. $\mathcal{C}^S$ satisfies constraints $IC(\theta)$ and $IR(\theta)$ for any $\theta \in \Theta$. This allows to evaluate the expected welfare with integration when the regulator offers the regulatory contract $\mathcal{C}^S$ and rivals are fully informed with $EW^I(\mathcal{C}^S,\mathcal{I}')$ and proceed with the following comparison:

$$
\begin{aligned}
&EW^I(\mathcal{C}^S,\mathcal{I}') - W^S = \\
&\quad E_\theta\left[V_U\left(Y(q^S,\theta,v(\theta))\right) - Y(q^S,\theta,v(\theta))p^U\left(Y(q^S,\theta,v(\theta))\right)\right] - \left[V_U\left(Y^S\right) - Y^S p^U\left(Y^S\right)\right] + \\
&\quad + \alpha\left[\sum_{i\neq 1}^{n}\pi^I\left(q^S,\theta\right) + \Pi_U\left(q^S,\theta\right) - \sum_{i=1}^{n}\pi_i^S\right]
\end{aligned}
$$

where we have indicated with $v(\theta)=1$ if $\theta=\overline{\theta}$ and $v(\theta)=0$ if $\theta=\underline{\theta}$ the rivals' degenerate beliefs.

Both the second and the third lines are positive because the unique difference in $EW^I(\mathcal{C}^S,\mathcal{I}')$ and W^S is that in the former the conglomerate benefits of scope economies and is thus more efficient in market U. Thus we have $EW^I(\mathcal{C}^S,\mathcal{I}') \geq W^S$. Now, notice that regulation $\mathcal{C}^S$ is suboptimal with information $\mathcal{I}'$ so that clearly $EW^I(\mathcal{C}',\mathcal{I}') \geq EW^I(\mathcal{C}^S,\mathcal{I}')$ which proves the result (ii) in the Lemma, i.e. $EW^I(\mathcal{C}',\mathcal{I}') \geq W^S$. ∎

Proof of Proposition 4. We separate the study of quantity and price competition in market U, respectively strategic substitutability and complementarity.

Strategic complementarity (price-competition) in market U. Let $\mathcal{I}^*$ be the information set in which neither the regulator nor the rivals know θ, as in the model setup, and $\mathcal{C}^*$ be the associated optimal regulatory contract illustrated in Proposition 1.

With information set $\mathcal{I}^*$, contract $\mathcal{C}^S$ satisfies all constraints $IC(\theta)$ and $IR(\theta)$ for any $\theta \in \Theta$ because $\mathcal{C}^S$ does not depend on θ and it is thus implementable. This allows to evaluate welfare with integration and information set $\mathcal{I}^*$ when the regulator offers the contract $\mathcal{C}^S$, i.e. $EW^I(\mathcal{C}^S,\mathcal{I}^*)$. We can now compare this welfare with the that associated with separation W^S. We now have

$$EW^I(\mathcal{C}^S,\mathcal{I}^*) - W^S = \quad E_\theta\left[V_U\left(Y(q^S,\theta,v)\right) - Y(q^S,\theta,v)p^U\left(Y(q^S,\theta,v)\right)\right] - \left[V_U\left(Y^S\right) - Y^S p^U\left(Y^S\right)\right] + \\ +\alpha\left[\sum_{i\neq 1}^{n}\pi^I\left(q^S,\theta\right) + \Pi_U\left(q^S,\theta\right) - \sum_{i=1}^{n}\pi_i^S\right]$$

The difference between this expression for $EW^I(\mathcal{C}^S,\mathcal{I}')-W^S$ with the expression for $EW^I(\mathcal{C}^S,\mathcal{I}')-W^S$ illustrated in the proof of Lemma 1 is that here rivals' beliefs correspond to their priors $\Pr(\theta = \overline{\theta}) = v$ and $\Pr(\theta = \underline{\theta}) = 1 - v$ and do not depend on $q(\theta)$. In fact, in the information set $\mathcal{I}^*$ they are not informed, contrary to $\mathcal{I}'$, and regulatory process associated with $\mathcal{C}^S$ is totally uninformative.

However, with price competition facing an integrated conglomerate induces the rivals' to reduce their prices and this increases both consumers' surplus and total profits in the market U. Hence, both the first and the second line in $EW^I(\mathcal{C}^S,\mathcal{I}^*) - W^S$ are positive so that we have

$$EW^I(\mathcal{C}^S,\mathcal{I}^*) \geq W^S.$$

Now note again that regulation $\mathcal{C}^S$ is sub optimal with information set $\mathcal{I}^*$ so that with the associated optimal regulation we have $EW^I(\mathcal{C}^*,\mathcal{I}^*) \geq EW^I(\mathcal{C}^S,\mathcal{I}^*)$ which finally implies the result,

$$EW^I(\mathcal{C}^*,\mathcal{I}^*) \geq EW^I(\mathcal{C}^S,\mathcal{I}^*) \geq W^S.$$

Strategic substitutability (quantity-competition) in market U. We first prove that optimal regulation $\mathcal{C}'$ for information set $\mathcal{I}'$ is discriminatory and in particular we generically have $\overline{q}' > \underline{q}'$. Optimal regulation with information set $\mathcal{I}'$ can be obtained following the proofs of Propositions 1 and 2, keeping in mind that the unique difference consists in the rivals being fully informed. Exactly as in the proof of Proposition 2, with (23) we show that, generically, $\frac{\partial\Delta_\theta\Pi_U(q,v)}{\partial q} > 0$, which immediately implies that the optimal regulation $\mathcal{C}'$ with information set $\mathcal{I}'$ is generically monotone $\overline{q}' > \underline{q}'$.

Now we show that contract $\mathcal{C}'$ is incentive compatible and individual rational also with information $\mathcal{I}^*$, i.e. it satisfies all constraints $IC(\theta)$ and $IR(\theta)$ for any $\theta \in \Theta$. This is again proved in step 2 of the proof of Proposition 2. In fact, the only difference between information sets $\mathcal{I}^*$ and $\mathcal{I}'$ is that in the former rivals are informed but they are informed in the latter. We know from the proof of Proposition 2 that with strategic substitutability the belief strategic effect due to the rivals' lack of information relaxes the compatibility constraint $IC(\underline{\theta})$ so that any pair of monotone outputs $\overline{q} \geq \underline{q}$ is incentive compatible (see (27) and related analysis).

This allows to evaluate the welfare $EW^I(\mathcal{C}', \mathcal{I}^*)$ that would prevail with information set $\mathcal{I}^*$ if the regulator allowed the conglomerate to integrate its activities and offered the contract $\mathcal{C}'$. For what stated above, contract $\mathcal{C}'$ is discriminatory so that it discloses perfect information on scope economies. Hence, the rivals' choices are the same in the two different information sets $\mathcal{I}^*$ and $\mathcal{I}'$ so that $EW^I(\mathcal{C}', \mathcal{I}^*)$ differs from $EW^I(\mathcal{C}', \mathcal{I}')$ uniquely as for the conglomerate's rent:

$$\begin{aligned} EW^I(\mathcal{C}', \mathcal{I}^*) - EW^I(\mathcal{C}', \mathcal{I}') &= -(1-\alpha)v \left\{ \Pi_U\left(\underline{q}', \overline{\theta}, 0\right) - \Pi_U\left(\underline{q}', \underline{\theta}, 0\right) - \left[\Pi_U\left(\underline{q}', \overline{\theta}, 1\right) - \Pi_U\left(\underline{q}', \underline{\theta}, 0\right)\right] \right\} = \\ &= -(1-\alpha)v \left[\Pi_U\left(\underline{q}', \overline{\theta}, 0\right) - \Pi_U\left(\underline{q}', \overline{\theta}, 1\right)\right], \end{aligned}$$

where for strategic substitutability we have $\Pi_U\left(\underline{q}', \overline{\theta}, 1\right) \geq \Pi_U\left(\underline{q}', \overline{\theta}, 0\right)$. It then follows

$$EW^I(\mathcal{C}', \mathcal{I}^*) \geq EW^I(\mathcal{C}', \mathcal{I}').$$

Now, recall that Lemma 1 shows

$$EW^I(\mathcal{C}', \mathcal{I}') \geq W^S$$

and we know that contract $\mathcal{C}'$ potentially suboptimal with information $\mathcal{I}^*$ so that

$$EW^I(\mathcal{C}^*, \mathcal{I}^*) \geq EW^I(\mathcal{C}', \mathcal{I}^*)$$

Hence, we finally obtain the following sequence of inequalities

$$EW^I(\mathcal{C}^*, \mathcal{I}^*) \geq EW^I(\mathcal{C}', \mathcal{I}^*) \geq EW^I(\mathcal{C}', \mathcal{I}') \geq W^S.$$

■