

*Fondazione Eni Enrico Mattei*  
*Working Papers*  
Fondazione Eni Enrico Mattei

---

*Year 2009*

*Paper 297*

---

Team Formation in a Network

Markus Kinaterder  
Universidad de Navarra

This working paper site is hosted by The Berkeley Electronic Press (bepress).

<http://www.bepress.com/feem/paper297>

Copyright ©2009 by the author.

# Team Formation in a Network

## **Abstract**

Two project leaders (or entrepreneurs) in a network, which captures social relations, recruit players in a strategic, competitive and time-limited process. Each team has an optimal size depending on the project's quality. This is a random variable with a commonly known distribution. Only the corresponding project leader observes its realization. Any decision is only observed by the involved agents. The set of pure strategy Sequential Equilibria is characterized by giving an algorithm that selects one equilibrium at a time. An agent's expected payoff is related to his position in the network, though no centrality measure in the literature captures this relation. A social planner frequently would achieve a higher welfare.

# Team Formation in a Network

Markus Kinateder\*  
Universidad de Navarra†

5 February 2009

## Abstract

Two project leaders (or entrepreneurs) in a network, which captures social relations, recruit players in a strategic, competitive and time-limited process. Each team has an optimal size depending on the project's quality. This is a random variable with a commonly known distribution. Only the corresponding project leader observes its realization. Any decision is only observed by the involved agents. The set of pure strategy Sequential Equilibria is characterized by giving an algorithm that selects one equilibrium at a time. An agent's expected payoff is related to his position in the network, though no centrality measure in the literature captures this relation. A social planner frequently would achieve a higher welfare.

*JEL classification numbers: C72, C73, D85*

*Keywords: Network, Dynamic Competitive Group Formation, Imperfect Information*

---

\*I am very grateful for the support received from my supervisor Jordi Massó and I thank Toni Calvó-Armengol and Dunia López-Pintado for their advice and time. I benefited hugely from conversations with Sebastian Bervoets, Vicente Bielza de Ory, Michael Chwe, Pierre Courtois, Sjaak Hurkens, Rahmi Ilkic, Michael Kosfeld, Philippos Louis, Joan de Martí, Pedro Mendi, Luca Merlino, Juan D. Moreno-Ternero, Francesc Obiols, Dov Samet, Joel Sobel, Marco van de Leij, Xavier Vilà, Yves Zenou and from comments made by participants of this model's presentation at Universitat Autònoma de Barcelona (UAB), at Universidad de Navarra, at Technical University Dortmund and at conferences in Maastricht, Zaragoza and Pamplona. An earlier version of this paper forms part of my PhD thesis defended at UAB in September 2008. I thank the committee members for their generous advice. Financial support from the Spanish Ministry of Education and Science through grant SEJ2005-01481 is acknowledged.

†Departamento de Economía, Edificio de Biblioteca (Entrada Este), Universidad de Navarra, 31080 Pamplona, Spain; email: mkinateder@unav.es

# 1 Introduction

In this paper, the formation of start-up businesses is analyzed when entrepreneurs rely on their social or business contacts in the early process of founding a company. It is important to understand this process better, to identify inefficiencies and to remove them. This would benefit economic growth which entrepreneurs foster. They innovate and improve existing technologies. The economic literature on the impact of social networks in labor markets is surveyed by Ioannides and Loury (2004). To analyze an entrepreneur's recruitment in a network theoretically is novel.<sup>1</sup>

In reality, entrepreneurs need, apart from funding, skilled and reliable people, such as business contacts, friends and relatives. Brüderl and Preisendörfer (1998) show empirically that entrepreneurs which start a business in the region in which they grew up obtain more and better support. Access to local networks and business contacts significantly improve a start-up's success probability. This is due to better information, existing contacts with customers and suppliers, access to financing and an existent family and friends network which provides emotional support and unpaid work. Michelacci and Silva (2007) show empirically that locals have better access to funding and Blumberg and Pfann (2001) find a positive relationship between an individual's level of social capital and his decision to become self-employed.

Those which join the entrepreneur may recruit friends or former colleagues. The team's growth is restricted by the network which captures social relations, if it runs out of money (in this model there is a time limit) or if the business idea is poor. Moreover, other teams may compete for the same individuals.

A stylized and dynamic model is developed. Different stages of offers and replies are called time periods and a last period limits the game. Each agent is either a player or a project leader and the analysis focuses on two projects. Each project leader knows his project's quality, which influences the optimal number of contributors and each contributor's payoff, while all other agents only know the prior distribution of project qualities. The setup and each project's location are commonly known.<sup>2</sup>

A unique Sequential Equilibrium is selected by letting the agents move sequentially and breaking any agent's indifference. To characterize the equilibrium is non-trivial since it depends among other things on each player's position in the network relative to that of the projects. Inefficient unemployment may prevail in equilibrium, sometimes because the players try to extract information about the more distant project's quality by waiting.

---

<sup>1</sup>A large sociological analysis of entrepreneurship in networks is based on Aldrich and Zimmer (1986).

<sup>2</sup>Although, as is discussed below, all results obtain under weaker informational assumptions.

The players are categorized according to the number of offers they may receive. Additionally, the group of players with the highest expected payoff before the game is identified and said to be most central. Conceptually related centrality measures generally yield different results, such as Freeman’s (1977) *betweenness* centrality and Burt’s (1992) *structural holes*. A player has the highest betweenness centrality if he is on more paths between two agents than any other player and information brokers which bridge a structural hole between two groups of agents benefit most from exchange between them. Example 2 illustrates the stark difference between the three measures.

This paper is related with network formation games (see Jackson (2005) for a survey), in which the static equilibrium concept of pairwise stability is used. In such games the analysis is usually less complex than in this dynamic model in which each agent’s type and location matter. For example, in Goyal and Vega-Redondo’s (2007) network formation game each pair of directly or indirectly linked players creates a surplus. Intermediaries on a path between two players pocket part of the surplus since they occupy a structural hole. The stable network is a star, or a circle if the players are restricted to form a limited number of links. Buskens and van der Rijt (2006) simulate a model in which each player strives to take such an intermediate position and find that balanced complete bipartite networks<sup>3</sup> obtain. They conclude that no agent can occupy a structural hole if everyone aspires to do so. The coalition formation literature uses similar ideas (see Ray (2008) for a survey). Each subset of players generates a value which is distributed among its members. A coalition is stable if no member wants to leave it and form a new one.

Bala and Goyal’s (1998) model has similar features as the one in this paper. They study the relationship between network structure and social learning. Initially, players do not know which action yields the highest payoff. Over time they observe the outcome of their own and their neighbors’ choices. This yields a dynamic learning process. As in this paper, players obtain information gradually over time and the informational restrictions are captured by a fixed network. In Stein (2008), each player has information which is only valuable together with that obtained from his neighbors. Hence, a player’s relative position in the network matters. By communication, underdeveloped ideas spread throughout the network while valuable ones remain local. Conversely, in this paper, high quality projects potentially spread further than those of low quality.

The game is defined in the next section. Two examples are provided in section 3. In section 4, equilibrium existence and uniqueness are established and illustrated in various examples. The players are categorized based on the equilibrium. Moreover, a new cen-

---

<sup>3</sup>In such a network, there are two (polarized) sides each with the same number of agents; each agent on one side is connected to all agents on the other side but to none on his.

trality measure is derived and allocated to the network literature. Before concluding, the model's welfare implications as well as assumptions and possible extensions are discussed.

## 2 Preliminaries

### 2.1 The Game in a Network

The agents in finite set  $A = \{1, \dots, n\}$  dispose of a set of projects denoted by  $\mathcal{P} = \{1, 2\}$ . The two project leaders take the first positions in  $A$  and belong to set  $PL$ , while the agents which occupy positions 3 to  $n$  in  $A$  are players and belong to set  $I$ . Thus,  $A = PL \cup I$ . Let project leader 1 obtain project 1 and project leader 2 project 2. If  $\gamma \in PL$  is considered, abusing notation, the other project leader is referred to as  $-\gamma$ .

Each agent has a fixed position in a network. Formally, the agents in  $A$  are nodes of a network  $\eta$ , whose graph is defined as the pair  $(A, E)$ , where  $E \subseteq A \times A$  denotes the set of links between them. A link from agent  $i$  to agent  $j$  is denoted by  $(i, j)$ . Graph  $(A, E)$  is undirected, that is, for all  $i, j \in A$ ,  $(i, j) \in E$  if, and only if,  $(j, i) \in E$ .

Given network  $\eta$ , a path between two distinct agents  $i$  and  $j$  is defined as a sequence of distinct agents  $i_1, \dots, i_r$  with  $i_1 = i$ ,  $i_r = j$ , and  $(i_{l-1}, i_l) \in E$ , for all  $1 < l \leq r$ . Its length is  $r - 1$ . Let network  $\eta$  be connected, that is, each agent is connected to at least one other agent directly and to all others via paths of finite lengths. The length of the shortest path between two distinct agents  $i$  and  $j$  is called *distance* between  $i$  and  $j$  and is denoted by  $d_{ij}$ . The *largest distance* (along shortest paths) between agent  $i$  and any other agent in  $\eta$  is defined as  $d_i = \max_{j \in A} d_{ij}$ . Finally, denote agent  $i$ 's set of direct neighbors by  $i(1) = \{j \in A \mid d_{ij} = 1\}$ , and for any  $2 \leq m \leq d_i$ , define his set of *m-neighbors* as  $i(m) = \{j \in A \mid d_{ij} \leq m\}$ . This set includes all agents at distance  $m$  or less from  $i$ .

Each project  $\gamma \in \mathcal{P}$  may be of *high* or *low* quality, denoted by  $k_\gamma^H$  and  $k_\gamma^L$ , respectively. Both are positive real numbers and  $k_\gamma^H > k_\gamma^L$ .<sup>4</sup> Let  $K_\gamma = \{k_\gamma^L, k_\gamma^H\}$  and  $K = \times_{\gamma \in \mathcal{P}} K_\gamma$ . Each project's quality is randomly and independently drawn before the game begins. A project's realized quality level is only observed by its leader. For each  $\gamma \in \mathcal{P}$ , nature selects  $k_\gamma^H$  with probability  $p_\gamma \in (0, 1)$  and  $k_\gamma^L$  with complementary probability  $(1 - p_\gamma)$ . The probabilities may vary across projects. Denote the commonly known vector of probabilities with which each project's quality is high by  $p = (p_1, p_2)$ . Finally, denote by  $k \in K$  the vector of realized project qualities.

<sup>4</sup>For simplicity, the analysis focuses on the case of two quality levels per project. However, the extension of this setup to more than two quality levels per project is straightforward.

Following Harsanyi (1967-68), this game of incomplete information (about each project's quality, and as shown below, its payoff function) can be studied as one of imperfect information. By taking place in a network, the game becomes one of multiple stages called time periods. They are denoted by  $t = 1, \dots, T$ , with  $T \geq 1$  fixed exogenously. The fixed network  $\eta$  including each player's and project leader's position as well as the game's setup are commonly known. Finally, let  $(\eta, T, K, p)$  be the game's tuple of parameters.

## 2.2 Strategies and History of the Game

Before the game begins, project leader  $\gamma \in PL$  may reject his project. If he carries it out, at  $t = 1$ ,  $\gamma$  may offer any neighbor except of  $-\gamma$ , that is, any  $j \in \gamma(1) \setminus \{-\gamma\}$  to join. Formally,  $\gamma$ 's action at  $t = 1$  is defined as  $f_\gamma^1 \in \{offer, no\ offer\}^{\gamma(1) \setminus \{-\gamma\}}$ . Given  $f_\gamma^1$ , the set  $o_\gamma^1$  of players which are offered to join project  $\gamma$  at  $t = 1$  is determined. It is empty if  $\gamma$  issues no offers. Only a player who receives an offer observes it. He only knows from which project it is but not the project's quality or how many other players are asked.

Each player  $j \in o_\gamma^1$  may accept or reject the offer. His action is defined as  $g_j^1 \in \{Yes, No\}$  and is only observed by project leader  $\gamma$ . If player  $j$  receives offers from both projects at  $t = 1$ , his action is defined as  $g_j^1 \in \{(Yes, No), (No, Yes), (No, No)\}$ , that is, he can at most join one project. If he receives no offer at  $t = 1$ ,  $g_j^1 = \emptyset$ . After all players chose a (possibly "empty") action and the corresponding project leader(s) observed them, the second period starts. Each project leader may ask any unrecruited neighbor or a neighbor of a player who joined him at  $t = 1$ . This determines  $o_1^2$  and  $o_2^2$ . In general, denote the set of players offered to join project  $\gamma$  at  $t$  by  $o_\gamma^t$ . Any unrecruited player who receives an offer decides whether to accept it or not. He does not know the project's quality, how many other players are offered to join it or joined it already.

A player commits to a project forever (see section 5 for a discussion of this assumption). Hence, if  $Yes \in g_j^s$  for any  $s \geq 1$ , player  $j$  cannot choose *Yes* any more at any  $t$ , where  $s < t \leq T$ . He rejects any offer from the other project leader and justifies his rejection—also if he receives two offers simultaneously.<sup>5</sup> A project leader may reissue his offer to any player who rejected it before.<sup>6</sup> This yields a dynamic process of offers and replies.

Denote player  $i$ 's private history at the end of period  $t$  by  $h_i^t$ . It contains every offer  $i$  received with his reply at all  $1 \leq s \leq t$ . A period's history in which he received no offer is empty. For any project leader  $\gamma \in PL$  and any  $t$ ,  $h_\gamma^t$  contains all offers  $\gamma$  made and the

<sup>5</sup>A player's action space thus changes once he accepted an offer. This is not modelled formally, nor is the justification the other project leader receives from a player who is not available any more.

<sup>6</sup>Project leaders do not use offers to obtain information about the other project's quality. They simply intend to fill up their team on time and recruit well-connected players only for this purpose.

replies he received at all  $1 \leq s \leq t$ . Any player  $i$ 's and any project leader  $\gamma$ 's history at the beginning of the game are empty, that is,  $h_i^0 \equiv \emptyset$  and  $h_\gamma^0 \equiv \emptyset$ , respectively.

Let  $F_\gamma$  denote the strategy set of any project leader  $\gamma \in PL$ . It contains all sequences of actions, called strategies, which  $\gamma$  may choose. Each is defined as  $f_\gamma \equiv \{f_\gamma^t\}_{t=1}^T$ , where  $f_\gamma^t$  is conditional on  $h_\gamma^{t-1}$  at any  $t$ . Let  $F = \times_{\gamma \in PL} F_\gamma$  be the project leaders' strategy space with generic element  $f$  called (project leaders') strategy profile. Similarly, denote by  $G_i$  the strategy set of any player  $i \in I$ . Each element of this set is a sequence of actions, called strategy, which player  $i$  may choose. It is denoted by  $g_i \equiv \{g_i^t\}_{t=1}^T$ , where  $g_i^t$  is conditional on  $h_i^{t-1}$  and the offer(s)  $i$  receives at  $t$ . Let  $G = \times_{i \in I} G_i$  be the players' strategy space with generic element  $g$  called (players') strategy profile. To emphasize player  $i$ 's and project leader  $\gamma$ 's role,  $g$  and  $f$  are written as  $(g_i, g_{-i})$  and  $(f_\gamma, f_{-\gamma})$ , respectively.

## 2.3 Payoff Function and Equilibrium Concept

Denote by  $b_\gamma$  the set of contributors of project  $\gamma \in \mathcal{P}$  and by  $|b_\gamma|$  its cardinality. Project leader  $\gamma$ 's payoff function is denoted by  $\pi_\gamma(|b_\gamma|, k_\gamma^q)$ . It is assumed to be strictly concave in  $|b_\gamma|$  and strictly increasing in  $k_\gamma^q$  for  $q \in \{L, H\}$ . To calculate his optimal number of contributors  $|b_\gamma^*(q)|$ , he maximizes his payoff function with respect to  $|b_\gamma|$ .<sup>7</sup> The solution is the lowest nonnegative integer for which the payoff is maximal. If  $k_\gamma^q$  enters the payoff function in a multiplicative way, then since  $k_\gamma^H > k_\gamma^L$ ,  $|b_\gamma^*(H)| > |b_\gamma^*(L)|$  holds, unless both are zero. Finally, for a given positive number of contributors up to the optimal one, a high quality project yields its members a larger payoff than one of low quality.

Given  $f$  and  $g$ , project  $\gamma$ 's actual set of contributors is  $b_\gamma(f, g)$ . It contains  $\gamma$ , unless it is empty since even  $\gamma$  does not contribute himself. His payoff in this case is zero. Similarly, player  $i$ 's payoff is zero if  $i \notin \cup_{\gamma \in \mathcal{P}} b_\gamma(f, g)$ . Denote project leader  $\gamma$ 's realized payoff by  $\pi_\gamma(f, g) \equiv \pi_\gamma(|b_\gamma(f, g)|, k_\gamma^q)$ . The realized payoff of any  $j \in b_\gamma(f, g)$  is the same as  $\gamma$ 's.<sup>8</sup> Formally, let the realized payoff of any player  $i \in I$  be

$$\pi_i(f, g) = \begin{cases} \pi_1(f, g), & \text{if } i \in b_1(f, g); \\ \pi_2(f, g), & \text{if } i \in b_2(f, g); \\ 0, & \text{otherwise.} \end{cases}$$

If  $i \in b_\gamma(f, g)$ , player  $i$ 's payoff is sometimes written as  $\pi_i(|b_\gamma(f, g)|, k_\gamma^q)$ . Any agent's payoff function maps  $F \times G$  into  $\mathbb{R}$ . All agents observe their payoff only at the end of period  $T$ .

<sup>7</sup>Since  $\gamma$  contributes to his own project, he requires one player less than  $|b_\gamma^*(q)|$  for  $q \in \{L, H\}$ .

<sup>8</sup>To assume equal sharing of a project's payoff is certainly restrictive. However, to allow for different sharing rules, for bargaining or for wage posting is left for future research since it is not straightforward.

At any  $1 \leq t \leq T$ , all agents use expected payoffs. To calculate them they form beliefs about each project's quality. When the game begins, agent  $j$ 's belief  $\mu_j^0$  is identical to the common prior  $p$  except of the  $\gamma$ th entry of project leader  $\gamma$ 's belief  $\mu_\gamma^0$  which is 1 if  $\gamma$ 's quality is high or 0 if it is low. Before player  $i$  moves at  $1 \leq t \leq T$ , he updates his belief from  $\mu_i^{t-1}$  to  $\mu_i^t$  (in a way explained below). Both are elements of  $\{0, p_1, 1\} \times \{0, p_2, 1\}$ . At any  $t$ ,  $\mu_i^t$  is uniquely determined by  $\mu_i^{t-1}$ ,  $h_i^{t-1}$  and the offer(s)  $i$  receives at  $t$ . Project leaders update beliefs analogously. Finally, let  $\mu^t = \{\mu_1^t, \dots, \mu_n^t\}$ .

For any  $t$ , given  $f, g$  and  $\mu_\gamma^t$ , denote the payoff  $\gamma$  expects to receive at  $T$  by  $\tilde{\pi}_\gamma^t(f, g)$ . It is the weighted sum (by  $\mu_\gamma^t$ ) of his payoff at  $T$  in case project  $-\gamma$ 's quality is low or high, respectively. A player's expected payoff after joining a project is calculated analogously, that is, he weights his payoff at  $T$  for each of the four pairs of realized project qualities by his belief  $\mu_i^t$ . Any unrecruited player determines which project he may still join until  $T$  and his corresponding payoff, conditional on the realized project quality. He then weights the four payoff functions by his belief. Denote any player  $i$ 's expected payoff at any  $t$  by  $\tilde{\pi}_i^t(f, g)$  and by  $\tilde{\pi}_j^0(f, g)$  the expected payoff of any agent  $j \in A$  before the game begins.

Given  $(\eta, T, K, p)$ , define the Team Formation Game ( $TFG$ ) as the tuple  $(A, F, G, \pi)$ , where  $\pi = (\pi_1(f, g), \dots, \pi_n(f, g))$ . A Sequential Equilibrium ( $SE$ ) strategy profile requires each agent to have an optimal action at any  $1 \leq t \leq T$  given any history and his belief. In particular, the continuation strategy must be sequentially rational after any history for the remainder of the  $TFG$ . The system of beliefs consistent with the strategy profile is not defined formally since it is very simple: on a  $SE$  path, each agent at most updates his belief about each project's quality once (when it is revealed to him).<sup>9</sup>

**Definition 1.** Given  $(\eta, T, K, p)$  and any Team Formation Game  $(A, F, G, \pi)$ , a Sequential Equilibrium is a pair of strategy profiles  $(f, g)$  such that at any  $1 \leq t \leq T$ ,

i) for all  $\gamma \in PL$ ,  $\mu_\gamma^{t-1}$ ,  $h_\gamma^{t-1}$  and  $\hat{f}_\gamma \in F_\gamma$ , given  $k_\gamma^q$  for any  $q \in \{L, H\}$ ,

$$\tilde{\pi}_\gamma^t(f, g) \geq \tilde{\pi}_\gamma^t(\hat{f}_\gamma, f_{-\gamma}, g), \text{ and}$$

ii) for all  $i \in I$ ,  $\mu_i^{t-1}$ ,  $h_i^{t-1}$  and  $\hat{g}_i \in G_i$ ,

$$\tilde{\pi}_i^t(f, g) \geq \tilde{\pi}_i^t(f, \hat{g}_i, g_{-i}) \text{ provided that } \text{Yes} \notin g_i^s \text{ for all } s < t.$$

After accepting an offer player  $i$ 's strategy is trivial. As is shown below, a pure strategy  $SE$  exists in any  $TFG$ . It is found using *sequential rationality* (SR). At each  $1 \leq t \leq T$ , the agents move in increasing order of their indexes, that is, first project leaders 1 and

---

<sup>9</sup>On out of equilibrium paths, an agent may update his belief more often. This is feasible since each agent always puts a positive belief on any history. After any "strange" observation which an agent cannot attribute to one type of project leader, his (updated) belief about this project's quality is the prior  $p_\gamma$ .



Figure 1: Four Players on a Line

2, and then players 3 to  $n$ . A player which receives no offer, obviously, takes no decision. While the results are unchanged if project leaders move simultaneously, the sequentiality of the players' moves together with breaking any agent's indifference allows to select a unique  $SE$ , as is shown in section 4.

## 3 Two Examples

### 3.1 Example 1: A Simple Coordination Game

Let project leaders 1 and 2, and players 3 and 4 be organized on a line as depicted in Figure 1. Let  $T = 2$ ,  $k_\gamma^H = 1$ ,  $k_\gamma^L = \frac{2}{3}$ ,  $p_\gamma = \frac{1}{2}$  and  $\pi_\gamma(|b_\gamma|, k_\gamma^q) = 3k_\gamma^q|b_\gamma| - \frac{1}{2}|b_\gamma|^2$  for  $\gamma \in \mathcal{P}$  and  $q \in \{L, H\}$ ;<sup>10</sup> hence,  $|b_\gamma^*(H)| = 3$  and  $|b_\gamma^*(L)| = 2$ .<sup>11</sup>

A player's decision at  $T = 2$  is determined by SR. In case he is offered to join the more distant project, he updates his belief about its quality to 1. If he receives two offers (since he did not join his project leader neighbor at  $t = 1$  while the other player did) he thus joins the more distant (high quality) project, he accepts the single offer his project leader neighbor makes (since this yields him a higher payoff than his outside option of 0), or he cannot take any decision if he joined him already at  $t = 1$  (apart from rejecting a second offer he may receive).

At  $t = 1$ , in a  $SE$ , each project leader offers his neighbor to join.<sup>12</sup> Conditional on recruiting him, the project leader asks the other player at  $T = 2$  if, and only if, his project's quality is high. At  $t = 1$ , the players *coordinate*<sup>13</sup> on one project, that is, one of them waits and the other accepts his neighbor's offer. The player which waits hopes to join the more distant project in case its quality is high. Otherwise, he can still join his

<sup>10</sup>Each player contributes 3 times the project quality units to the payoff of every  $j \in b_\gamma$  and causes a cost, for example, due to the coordination effort, which increases exponentially in the number of contributors.

<sup>11</sup>Throughout this paper, both projects' payoff functions take this form, and thus are identical. However, the general result is valid for any pair of payoff functions which fulfill the assumptions made above.

<sup>12</sup>The leader of a low quality project is indifferent to ask his neighbor at  $t = 1$  (and  $T = 2$ ) or only at  $T = 2$ . However, by assumption, he asks his neighbor at  $t = 1$  (see Assumption 1 below).

<sup>13</sup>In this paper, the expression *coordination* is used to describe a situation in which a group of players intends to join the same project even if some of them are closer to the other project.

project leader neighbor at  $T = 2$ . A player who joins a project at  $t = 1$  hopes that its leader recruits the other player in case the project's quality is high.

The player which waits obtains a higher expected payoff. With probability  $\frac{1}{2}$  the more distant project's quality is high and the player's payoff is 4.5. With complementary probability of  $\frac{1}{2}$ , the player joins his project leader neighbor at  $T = 2$ . Since this project's quality is high or low with equal probability as well, the player obtains a payoff of 4 or 2, respectively, each with compound probability of  $\frac{1}{4}$ . Hence, a player's expected payoff from waiting is  $\frac{1}{2}4.5 + \frac{1}{2}(\frac{1}{2}2 + \frac{1}{2}4) = \frac{15}{4}$  and it is  $\frac{1}{2}2 + \frac{1}{2}4.5 = \frac{13}{4}$  if he accepts his offer at  $t = 1$  (and the other player waits). A player which waits benefits from the other project if its quality is high. If it is not, he has a second chance to join a high quality project. In case both players accept or reject their neighbors' offers at  $t = 1$ , both end up in different teams and obtain a lower expected payoff of  $\frac{1}{2}2 + \frac{1}{2}4 = \frac{12}{4}$ . In the unique *SE*, player 3 waits at  $t = 1$  since waiting yields a higher payoff and he can choose to wait or not because he moves first. Player 4 joins project 2 at  $t = 1$  by SR since he moves second. If project 2's quality is high, the teams  $\{1\}, \{2, 3, 4\}$  form. Otherwise, the teams  $\{1, 3\}, \{2, 4\}$  emerge: player 4 joins project 1 at  $t = 1$  and player 3 project 2 at  $T = 2$ .

Given the unique *SE*, the agents' beliefs are derived. Project leader  $\gamma$ 's belief about project  $-\gamma$ 's quality is identical to the prior always. At  $t = 1$ , he has no other information than the prior. At  $T = 2$ , before he moves, he only observed his neighbor's reply to his offer at  $t = 1$ , which reveals nothing about project  $-\gamma$ 's quality.

At  $t = 1$ , each player's belief about the projects' quality levels is identical to the prior (even if he receives no offer from his project leader neighbor). At  $T = 2$ , a player updates his belief about the more distant project's quality to 1 if, and only if, he receives an offer to join it. Otherwise, player 4's belief is identical to the prior. If player 3 is not asked to join project 2 at  $T = 2$  (since its leader did not ask player 4 at  $t = 1$  or player 3 at  $T = 2$ ), he updates his belief about its quality to 0.

### 3.2 Example 2: The Illusion to be most "Powerful"

Suppose that the agents in  $A = \{1, \dots, 6\}$  form the network depicted in Figure 2, that  $k_\gamma^H = 2$ ,  $k_\gamma^L = 1$ ,  $\pi_\gamma(|b_\gamma|, k_\gamma^q) = 3k_\gamma^q|b_\gamma| - \frac{1}{2}|b_\gamma|^2$  for  $\gamma \in \mathcal{P}$  and  $T = 3$ ; then,  $|b_\gamma^*(H)| = 6$  and  $|b_\gamma^*(L)| = 3$ . At first sight, player 4 has the best position. He is linked to three players and both project leaders may compete for him. He fills a structural hole and has the highest betweenness centrality (together with player 3) with respect to the two project leaders. Nevertheless, as is shown now, in the unique *SE* his expected payoff is lowest since players 3, 5 and 6 take away his power by coordinating on one project.

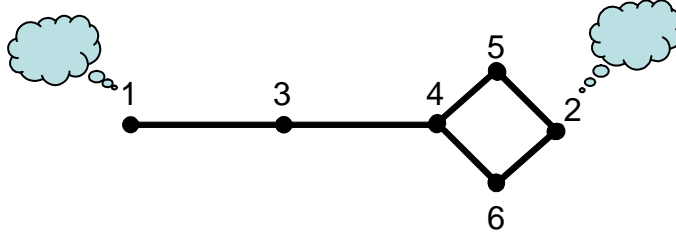


Figure 2: Network in Example 2

Given any quality level, project leader 1 needs to recruit at least two players. Hence, he offers player 3 and (if possible) 4 to join. Provided they join him on time, he asks players 5 and 6 if, and only if, his project's quality is high. Project leader 2 tries to recruit all players as well if, and only if, his project's quality is high. In case it is low, he only asks players 5 and 6. Their strategies are identical if it is assumed that no player delays his decision out of indifference (see Assumption 1 in section 4). Player 4 is not asked to join project 2 if its quality is low. He updates his belief about its quality to 1 if, and only if, he receives an offer to join it which he accepts.

For player 3 to join project 1 at  $t = 1$  strictly dominates to join it at  $t = 2$  since he gives up his chance to join project 2 anyway but prevents project leader 1 from reaching players 5 and 6. Hence, he either joins it at  $t = 1$  or decides at  $T = 3$ . In the first case, he hopes that project 1's quality is high and that players 5 and 6 are recruited; in the second, he hopes to join project 2 at  $T = 3$ , though he still may join project 1.

### 3.2.1 The Equal Likelihood Case

Thus far, the probability with which each project's quality is high was not used. In order to proceed, let  $p = (\frac{1}{2}, \frac{1}{2})$ . Due to the network's geography, an offer from the more distant project leader may reach players 3, 5 and 6 only at  $T = 3$ . Each of them, provided he is still available, would accept it (since he updates his belief about the project's quality to 1). In case it is not made, the player joins his project leader neighbor at  $T = 3$ . The three players receive a higher expected payoff if they coordinate on one project. If player 3 joins project 1 at  $t = 1$ , players 5 and 6 decide only at  $T = 3$ . Conversely, if player 3 waits until  $T = 3$ , players 5 and 6, as assumed before, both join project 2 at  $t = 1$ . Hence, player 4 receives at most a single offer at  $t = 2$  and accepts it in order to let the corresponding project leader access the player(s) on his other side.

Player 3's expected payoff in case he waits is 12.25 while player 4's is 8.75 and that of 5 and 6 is 11. Their expected payoff is 13.25 if they wait while that of 3 and 4 is 11.

At  $t = 1$ , in the unique  $SE$ , player 3 waits since he moves first, and by SR, players 5 and 6 join project 2. Player 3 joins project 2 at  $T = 3$  if offered and otherwise project 1. Player 4 is worst off in the  $SE$  since he remains unrecruited if project 2's quality is low. Although he fills a structural hole and has the highest betweenness centrality, in the unique  $SE$ , players 3, 5 and 6 "take away his power."

### 3.2.2 Changes in the Time Parameter and the Quality Probabilities

If  $T > 3$ , player 3 would wait until period  $T - 1$  for an offer to join project 2. If it is not made, he joins project 1. This prevents players 5 and 6 from receiving project 1's offer.<sup>14</sup> By SR and since they do not delay their decision, they join project 2 at  $t = 1$  (though they are indifferent to join it at any  $1 \leq s \leq T - 3$ ). Player 4 joins project 2, if offered, at  $t = 2$  or accepts project 1's offer at  $T$ . Player 3 joins project 2 at  $t = 3$  or project 1 at  $T - 1$ . Player 4's payoff then equals player 3's.<sup>15</sup>

Let  $T = 3$  again and consider changes in  $\hat{p} = p_1 = p_2$ , the probability with which each project's quality is high. Denote by  $g'$  and  $g''$  the players' strategy profiles in which they coordinate on project 1 and 2, respectively. Player 3 chooses to wait or not since he moves first, and players 5 and 6, by SR, join or wait, respectively. Player  $i$ 's payoff if he joins project  $\gamma$  is  $\pi_i(|b_\gamma(f, g)|, k_\gamma^q)$  for  $q \in \{L, H\}$  and  $g \in \{g', g''\}$ . The project leaders' strategy profile  $f$  is identical in both cases. Player 3 waits until  $T = 3$  if  $g''$  is played and joins project 1 at  $t = 1$  under  $g'$ . Since he moves first, in the unique  $SE$ , he is better off to wait (since the other side joins) than to join (since the other side waits) if, and only if,

$$\hat{p} \pi_3(|b_2(f, g'')|, k_2^H) + (1 - \hat{p}) \hat{p} \pi_3(|b_1(f, g'')|, k_1^H) + (1 - \hat{p}) (1 - \hat{p}) \pi_3(|b_1(f, g'')|, k_1^L) \geq \hat{p} \pi_3(|b_1(f, g')|, k_1^H) + (1 - \hat{p}) \pi_3(|b_1(f, g')|, k_1^L).$$

Obviously,  $\hat{p} \pi_i(|b_1(f, g')|, k_1^H) = \hat{p} \pi_i(|b_2(f, g'')|, k_2^H)$  for all  $i \in I$  and it cancels on both sides. Then,  $(1 - \hat{p})$  drops out of the inequality as well. This yields

$$\hat{p} \pi_3(|b_1(f, g'')|, k_1^H) + (1 - \hat{p}) \pi_3(|b_1(f, g'')|, k_1^L) \geq \pi_3(|b_1(f, g')|, k_1^L),$$

and thus,

$$\hat{p} \geq \frac{\pi_3(|b_1(f, g')|, k_1^L) - \pi_3(|b_1(f, g'')|, k_1^L)}{\pi_3(|b_1(f, g'')|, k_1^H) - \pi_3(|b_1(f, g'')|, k_1^L)}. \quad (1)$$

<sup>14</sup>If player 3 joined project 1 before  $T - 1$ , players 5 and 6 would wait to decide until project 1's quality is revealed to them. Obviously, this is not part of a  $SE$  given the players' fixed order of moves.

<sup>15</sup>If player 3 joins project 1 only at  $T$ , his expected payoff is larger than player 4's.

The numerator and denominator of (1) are nonnegative. This equation is derived in a general way since it holds analogously for players 5 and 6 (however, substituting the players' and projects' indexes,  $g'$  with  $g''$  and vice versa). Using (1), the range of  $\hat{p}$  can be calculated for which each side prefers to wait if the other side joins. For player 3 the threshold value of  $\hat{p}$  is  $\frac{4.5-4}{10-4} = \frac{1}{12}$  and it is  $\frac{4.5-4.5}{13-4.5} = 0$  for players 5 and 6. For  $\hat{p} \in (0, \frac{1}{12})$ , in the unique  $SE$ , player 3 prefers to join project 1 at  $t = 1$ , while for all larger values of  $\hat{p}$  he waits. Since  $\hat{p}$  is larger than zero, players 5 and 6 always prefer to wait rather than to join project 2 at  $t = 1$ ,<sup>16</sup> though they best-reply to player 3's choice since he moves first.<sup>17</sup> Moreover, both sides never wait or join simultaneously for any  $\hat{p} \in (0, 1)$ .

## 4 Equilibrium in any Team Formation Game

Although the fixed order of moves eliminates many equilibria, frequently, the agents out of indifference have various possible equilibrium strategies. In order to select a unique  $SE$ , two more assumptions are made. Assumption 1 ( $A1$ ) states that no agent "unnecessarily" delays his decision and was already introduced in Examples 1 and 2.

**Assumption 1.** *A player which at most receives one offer (possibly from some point in time on) immediately accepts it. A project leader recruits in a circular cascade, if possible.*

The behavior implied by  $A1$  arises also if there is a small waiting cost, each agent's payoff is discounted over time, or  $T$  is a random variable and with a small probability any period before  $T$  is the last one.  $A1$  implies that a project leader offers any player to join which previously was not accessible until his project is filled or  $T$  is reached and imposes players not to delay their decision out of indifference to join a project now or later.<sup>18</sup>

Given  $A1$ , suppose that at a distance up to  $T$  from each project there are less or just enough players to fill it if its quality is low and that no player is competed for by both projects, that is,  $1(T) \cap 2(T) = \emptyset$ . Then, each agent's  $SE$  strategy is unique as is shown in Proposition 1.

**Proposition 1.** *Given  $(\eta, T, K, p)$  and any  $TFG$ , suppose that  $A1$  holds. Then, there is a unique  $SE$   $(\dot{f}, \dot{g})$ , if  $|\gamma(T)| \leq |b_\gamma^*(L)| - 1$  for all  $\gamma \in PL$  and  $1(T) \cap 2(T) = \emptyset$ .*

*Proof.* Given any  $TFG$ , suppose that  $A1$  holds, and assume that  $|\gamma(T)| \leq |b_\gamma^*(L)| - 1$  for all  $\gamma \in PL$  and that  $1(T) \cap 2(T) = \emptyset$ . Then, each project leader  $\gamma$ 's optimal strategy  $\dot{f}_\gamma$  is

<sup>16</sup>For low values of  $\hat{p}$ , players 5 and 6 by joining project 2 at  $T = 3$  form a three-agent team while player 3 is left with project leader 1 only, in case he waits and project 2's quality is low.

<sup>17</sup>If players 5 and 6 moved first, they would wait and player 3 would join project 1 at  $t = 1$ .

<sup>18</sup>The agents' behavior also arises when the informational assumptions are relaxed (see section 5).

to recruit in a circular cascade, that is, at  $t = 1$  he asks his direct neighbors, conditional on receiving positive answers, at  $t = 2$ , he asks his second-neighbors and so on. Finally, at  $T$  he offers all players at distance  $T$  from him to join (conditional on having recruited all closer players before). Each player  $i \in I$  receives at most one offer since  $1(T) \cap 2(T) = \emptyset$ . His optimal strategy  $\hat{g}_i$  is to accept it immediately. A1 ensures uniqueness since project leaders and players do not delay their decision.  $\square$

If A1 does not hold, a unique  $SE$  obtains under all other conditions in Proposition 1 only if the agents form a tree, a circle or a line. Otherwise, more equilibria arise if a player, by delaying his decision to join a project, does not obstruct its leader's access to players at a larger distance. Similarly, a project leader could initially bypass a player if this does not prevent his access to players at a larger distance. If he recruits all bypassed players on time, this is a  $SE$  ruled out by A1.

If  $|b_\gamma^*(H)| - 1 \leq |\gamma(T)|$ , but all other conditions in Proposition 1 hold, project leader  $\gamma$ 's strategy need not be unique. At each distance there are either less, more or just enough players such that together with all players at all lower distances his project is filled. If at some distance there are just enough players to fill it, uniqueness obtains under A1 which prescribes project leaders to recruit closer players before more distant ones. However, should a project leader reach a distance at which there are more players to fill his project, then he is indifferent which to ask. Each would accept his offer since the other project is too far away (that is, since  $1(T) \cap 2(T) = \emptyset$ ). In this case, Assumption 2 (A2) applies.

**Assumption 2.** *A player which is indifferent between two projects joins (or aspires to join) the lower indexed project. In case of being indifferent, project leader 1 asks lower indexed players and project leader 2 higher indexed players.*

A2 is a selection criterion for an agent who has various payoff-equivalent options. Alternatively, the agents can be assumed to have lexicographic preferences over all teams that may form in case of being indifferent. An immediate corollary of Proposition 1 and the discussion thereafter is stated next.

**Corollary 1.** *Given  $(\eta, T, K, p)$  and any TFG, suppose that A1 and A2 hold. Then, there is a unique  $SE$   $(\tilde{f}, \tilde{g})$ , if  $1(T) \cap 2(T) = \emptyset$ .*

The assumption that projects do not compete for players, made in Proposition 1 and Corollary 1, need not hold, such as in Examples 1 and 2. A project leader's  $SE$  strategy, given A1 and A2 in case  $1(T) \cap 2(T) \neq \emptyset$  and  $o_1^t \cap o_2^t \neq \emptyset$  for some  $1 \leq t \leq T$ , is strictly dominant and the players' best-reply is unique as is shown in Proposition 2.

**Proposition 2.** *Given  $(\eta, T, K, p)$  and any  $TFG$ , suppose that  $A1$  and  $A2$  hold. Then, there is a unique  $SE$   $(f^*, g^*)$ .*

*Proof.* The proof is divided into two parts. Part I shows that each project leader  $\gamma$  has a strictly dominant strategy  $f_\gamma^* \in F_\gamma$ , provided that  $A1$  and  $A2$  hold. Part II shows that the players' best-reply  $g^* \in G$  is unique. Together this yields the unique  $SE$ .

**Part I.** Given  $(\eta, T, K, p)$  and any  $TFG$ , suppose that  $A1$  and  $A2$  hold. Then, project leader  $\gamma$ 's strictly dominant strategy given  $|b_\gamma^*(q)|$ , for  $q \in \{L, H\}$ , is as follows. By  $A1$ ,  $\gamma$  offers any player he can access to join unless  $T$  is reached, his project is filled or all accessible players join(ed) project  $-\gamma$ . By  $A2$ ,  $\gamma$  uniquely selects the players he asks if more than required are accessible and he is indifferent which to ask;  $\gamma$  is not indifferent if a player rejects his offer in order to wait for  $-\gamma$ 's offer. In this case,  $\gamma$  does not reissue his offer to this player if, and only if, the player may reject it again and  $\gamma$  can access enough other players to fill his project which (he expects will) join him. Finally,  $A1$  implies that a project leader never resumes to issue offers once he stopped. This yields  $f^* \in F$ .

**Part II.** Given  $f^* \in F$ , at any  $t$ , the set of unrecruited players is partitioned in categories 0, 1 and 2 as follows. Players in category 0 never receive an offer. Players in category 1 may at most receive one offer and players in category 2 may receive offers from both projects. At any  $s$ , where  $t < s \leq T$ , the groups' composition may change. A player drops from category 2 to category 1 if he loses the option to join one of the two projects, either because it is too far away to reach him on time or because he updates his belief about its quality to 0. Similarly, a player may drop to category 0. Players which accept an offer are not categorized any more.

Players in category 0 never take a decision. By  $A1$ , those in category 1 immediately accept the only offer they may receive. The players in category 2 closest to each of the two projects calculate their expected payoff of waiting and joining (given that the other side waits or joins, respectively). The players coordinate on one project if one side waits and the other joins. If both join, the players one distance further away from each project calculate their expected payoff and decide whether to join or to wait given the other group's behavior. Both groups may also wait for some time (until at least one project's quality is revealed to them). A player's indifference to wait or to join is broken by  $A2$  and the order of moves together with  $SR$  reduces any remaining equilibrium multiplicity.  $\square$

Given any  $TFG$ , the players can be categorized at any  $t$ , taking into account their position in  $\eta$  relative to that of the two projects and each agent's equilibrium strategy. This categorization is unique since there is a one to one correspondence with the  $SE$ .

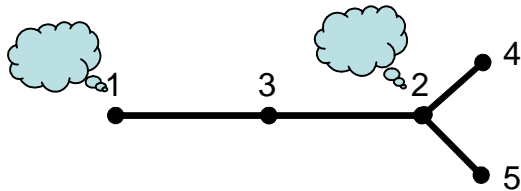


Figure 3: A Tree with Origin in Project Leader 2

**Corollary 2.** *Given  $(\eta, T, K, p)$  and any TFG, suppose that A1 and A2 hold. Then, at any  $1 \leq t \leq T$ , each player  $i \in I$  is uniquely allocated to one category.*

Obviously, Corollary 2 also applies to the categorization before the game begins. This is the basis to identify a new centrality measure in the next subsection. Since the players do not know the realized project qualities when the game begins, for every  $k \in K$ , the corresponding equilibrium path is derived. Each player which may never join a project is allocated to category 0 when the game begins. A player which may join both projects, possibly in different realizations of the project qualities, is allocated to category 2, denoted by  $\mathcal{C}_2^0$  and all other players belong to category 1, denoted by  $\mathcal{C}_1^0$ .

Since for  $k = (k_1^H, k_2^H)$  the projects' extension is largest, this case serves to identify a subset of the competed for players in  $\mathcal{C}_2^0$ . Each project leader  $\gamma \in PL$  counts the players at distances 1, 2 and so on from him until his high quality project is filled. Formally, given  $|b_\gamma^*(H)|$ , for each  $\gamma \in PL$  let  $m_\gamma$  be the distance such that  $|\gamma(m_\gamma)| \geq |b_\gamma^*(H)| - 1$ , but  $|\gamma(m_\gamma - 1)| < |b_\gamma^*(H)| - 1$ , disregarding any path that includes project leader  $-\gamma$ . Distance  $m_\gamma$  is equal to project leader  $\gamma$ 's largest distance  $d_\gamma$  if there are less players than  $|b_\gamma^*(H)|$  in the network and  $-\gamma$  does not obstruct  $\gamma$ 's access to any of them. In Figure 3 with  $|b_1^*(H)| \geq 2$ , for example,  $m_1 = 1$  since project leader 1 can only access player 3. If  $|\gamma(m_\gamma)| > |b_\gamma^*(H)| - 1$ , project leader  $\gamma$  selects the players at distance  $m_\gamma$  by A2. Since the recruitment is time constrained and  $T$  may be smaller than  $m_\gamma$ , let  $\hat{m}_\gamma = \min\{T, m_\gamma\}$ .

Slightly abusing notation, denote the set of players project leader  $\gamma$  would like to ask if project  $\gamma$ 's quality is high by  $\gamma(\hat{m}_\gamma(H))$ . Any player in this set expects to receive  $\gamma$ 's offer with positive probability before the game begins. Analogously, allocate all players which fill  $\gamma$  if its quality is low to  $\gamma(\hat{m}_\gamma(L))$ , again abusing notation. This set is a strict subset of  $\gamma(\hat{m}_\gamma(H))$ , unless  $T$  limits  $\gamma$ 's recruitment or he cannot access enough players. Players in the intersection of  $1(\hat{m}_1(H))$  and  $2(\hat{m}_2(H))$  expect to receive offers from both projects before the game begins and are allocated to a subset of  $\mathcal{C}_2^0$ , namely to

$$\mathcal{C}_{2,1}^0 = \{i \in I \mid i \in 1(\hat{m}_1(H)) \cap 2(\hat{m}_2(H))\}.$$

The  $SE$  is trivial if  $\mathcal{C}_{2,1}^0 = \emptyset$ . Then, Corollary 1 holds and  $\mathcal{C}_2^0 = \emptyset$ . Each player receives at most one offer and accepts it immediately. If each project's quality is high and  $\mathcal{C}_{2,1}^0 \neq \emptyset$ , then there are not enough players in sets  $1(\hat{m}_1(H))$  and  $2(\hat{m}_2(H))$  to fill both of them and at least one project leader needs to ask more distant players, provided he can access them on time. Therefore, category  $\mathcal{C}_2^0$  may contain other players than those in  $\mathcal{C}_{2,1}^0$ . The players in  $\mathcal{C}_{2,1}^0$  "closest" to each project are called barrier players since they (may have to) decide whether a project leader gains access to the players in  $\mathcal{C}_{2,1}^0$  and thus (may) constitute a barrier for him. For each  $\gamma \in \mathcal{P}$ , define the set of barrier players as

$$BP_\gamma = \{i \in \mathcal{C}_{2,1}^0 \mid d_{\gamma i} = 1 \text{ or } d_{\gamma i} = d_{\gamma j} + 1 \text{ for some } j \in i(1) \text{ s.t. } j \in I \setminus \mathcal{C}_{2,1}^0\}.$$

Unless  $BP_\gamma$  is a singleton,  $d_{\gamma i} \neq d_{\gamma i'}$  may hold for  $i, i' \in BP_\gamma$ . One project leader may reach his barrier player(s) faster than the other. All players in  $\eta$  belong to  $\mathcal{C}_2^0 \equiv \mathcal{C}_{2,1}^0$  if, such as in Examples 1 and 2,  $|I| \leq \min_{\gamma \in PL} |b_\gamma^*(H)|$ ,  $m_\gamma \leq T$ , and both project leaders can access all players. Then, each project leader's barrier players are his neighbors.

To provide an algorithm for the categorization at any  $1 \leq t \leq T$  is involved and does not yield additional insights. In general, the players need not coordinate on one project (immediately), and hence, anything may occur. This is illustrated in Example 3 below. Other examples of the categorization and the unique  $SE$  are provided in section 4.2.

## 4.1 A New Centrality Measure

In this subsection, the players with the highest (ex ante) expected payoff are identified.

**Definition 2.** *Given  $(\eta, T, K, p)$  and any TFG, suppose that A1 and A2 hold. Any player  $i \in I$  with the highest (ex ante) expected payoff is said to be most central.*

Frequently, several players are most central. Although other centrality measures also identify groups of players as most central, usually the group's size is fixed exogenously before its members are determined.<sup>19</sup> In this model, the group's size is endogenous: all players with the highest expected payoff given the unique  $SE$  ( $f^*, g^*$ ) are most central.

If the players coordinate on one project, their expected payoff can be calculated and compared, and thus the most central players identified. Suppose that the players coordinate on project 1 and project 2's barrier players wait until project 1's quality is revealed to them. (In case the players coordinate on project 2 just relabel the projects' indexes.)

---

<sup>19</sup>In chapter 5 of Wasserman and Faust (1994), it is shown how various measures which identify a single most central player extend to group centrality.

The expected payoff of any player  $j \in BP_2$  in this case is

$$p_1 \pi_j(b_1(f^*, g^*), k_1^H) + (1 - p_1) [p_2 \pi_j(b_2(f^*, g^*), k_2^H) + (1 - p_2) \pi_j(b_2(f^*, g^*), k_2^L)]. \quad (2)$$

If  $\pi_j(b_2(f^*, g^*), k_2^L) \neq 0$ , player  $j$  in  $BP_2$  joins project 2 even if both projects' quality is low and his expected payoff is larger than any player  $i$ 's who joins project 2 in any case (and only receives the second term  $p_2 \pi_i(b_2(f^*, g^*), k_2^H) + (1 - p_2) \pi_i(b_2(f^*, g^*), k_2^L)$  as expected payoff). Any player  $i \in 1(\hat{m}_1(L))$  receives an expected payoff of  $p_1 \pi_i(b_1(f^*, g^*), k_1^H) + (1 - p_1) \pi_i(b_1(f^*, g^*), k_1^L)$ . This is strictly smaller than the expected payoff of player  $j$  if

$$\pi_i(b_1(f^*, g^*), k_1^L) < p_2 \pi_j(b_2(f^*, g^*), k_2^H) + (1 - p_2) \pi_j(b_2(f^*, g^*), k_2^L).$$

All barrier players of project 2 for which this conditions holds have the highest expected payoff. If the inequality is reversed, any player  $i \in 1(\hat{m}_1(L))$  has the highest expected payoff and if it holds with equality, both groups of players are most central.

If  $\pi_j(b_2(f^*, g^*), k_2^L) = 0$  in (2), the players in  $BP_2$  are recruited only if at least one project's quality is high. Any player  $i \in \gamma(\hat{m}_\gamma(L))$ , for  $\gamma \in \mathcal{P}$ , has an expected payoff of  $p_\gamma \pi_i(b_\gamma(f^*, g^*), k_\gamma^H) + (1 - p_\gamma) \pi_i(b_\gamma(f^*, g^*), k_\gamma^L)$  and again, the players whose expected payoff is largest are identified and declared most central.

In general, the analysis is not as simple as suggested here since the most central players may belong to different categories at the beginning of the game. Moreover, the barrier players may not coordinate on a project or only after some time (see Example 3 below). This centrality measure's predictions are illustrated for various examples in section 4.2 and compared with those of existing concepts in the literature in section 4.3.

## 4.2 Examples of the Categorization

In various examples, the categorization of players before the game begins is illustrated and the most central players are identified. Therefore, the unique  $SE$  is obtained.

### 4.2.1 Example 3: The optimal strategy of barrier players

In Examples 1 and 2, the two groups of barrier players coordinate their behavior. Example 3 provides two instances in which this is not the case. First, both groups of barrier players do not wait for an offer from the more distant project, that is, coordination does not take place. Second, both groups of barrier players wait (at least for some time).

Consider first the network depicted in Figure 4. Let  $T = 3$ ,  $p_1 = p_2 = \frac{1}{3}$ ,  $k_\gamma^H = \frac{5}{3}$ ,  $k_\gamma^L = 1$  and  $\pi_\gamma(|b_\gamma|, k_\gamma^q) = 3k_\gamma^q|b_\gamma| - \frac{1}{2}|b_\gamma|^2$  for  $\gamma \in \mathcal{P}$ ; then,  $|b_\gamma^*(H)| = 5$  and  $|b_\gamma^*(L)| = 3$ .

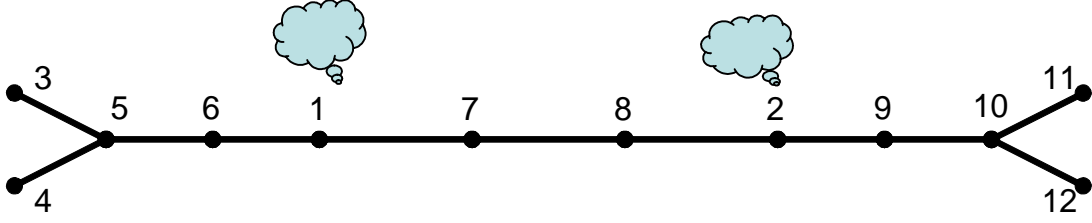


Figure 4: Both groups of Barrier Players do not wait

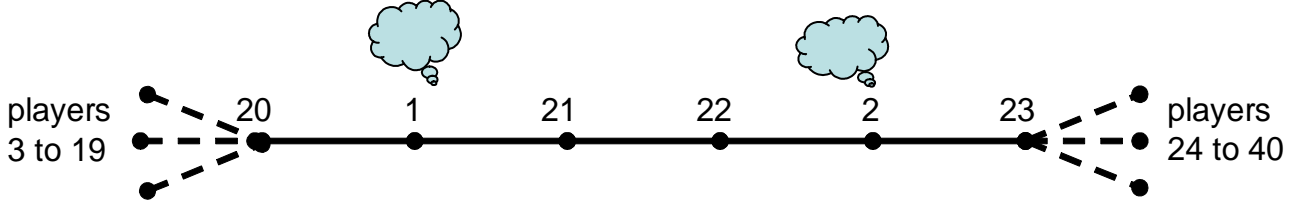


Figure 5: Both groups of Barrier Players wait initially

In the unique *SE*, players 3 to 6 and 9 to 12 can only receive one offer and accept it immediately; they belong to  $\mathcal{C}_1^0$ . Barrier players 7 and 8 belong to  $\mathcal{C}_2^0 \equiv \mathcal{C}_{2,1}^0$ . Only player 7's strategy is analyzed since his position in  $\eta$  is symmetric to 8's. If he joins project 1 at  $t = 1$ , his expected payoff is  $\frac{2}{3}4.5 + \frac{1}{3}12.5 = 7\frac{1}{6}$ . If he waits until  $t = 2$ , he is offered to join project 1 again if, and only if, its quality is high. (Otherwise, project leader 1 asks player 5.) If the players coordinate on project 2, player 7 is asked to join it at  $t = 2$  if, and only if, its quality is high. His expected payoff in this case is  $\frac{1}{3}12.5 + \frac{2}{3}(\frac{1}{3}12.5 + \frac{2}{3}0) = 6\frac{17}{18}$ . Hence, at  $t = 1$ , players 7 and 8 in the *SE* join projects 1 and 2, respectively, and coordination does not take place. Players 6 to 9 receive an expected payoff of  $7\frac{1}{6}$  and are most central.

Consider next the network depicted in Figure 5. Let  $T = 4$ ,  $p_1 = p_2 = \frac{9}{10}$ ,  $k_\gamma^H = 7$ ,  $k_\gamma^L = 1$  and  $\pi_\gamma(|b_\gamma|, k_\gamma^q) = 3k_\gamma^q|b_\gamma| - \frac{1}{2}|b_\gamma|^2$  for  $\gamma \in \mathcal{P}$ ; then,  $|b_\gamma^*(H)| = 21$  and  $|b_\gamma^*(L)| = 3$ . In the unique *SE*, only the strategy of barrier players 21 and 22 is interesting. Player 21 is asked to join project 1 at  $t = 1$ . This offer is repeated at  $t = 2$  if, and only if, project 1's quality is high. (If it is low its leader asks player 3 by *A2*.) The reasoning for player 22 is analogous. Both prefer to join the same high quality project. Since they wait at  $t = 1$ , they coordinate on project 1 (by *A2*) from  $t = 2$  on and player 21 joins it if offered. At  $t = 3$ , player 22 either accepts project leader 1's offer or joins project 2 if offered. Player 21 could then be included in project 2 at  $T = 4$ .

A barrier player's expected payoff from joining his project leader neighbor at  $t = 1$  is  $\frac{1}{10}4.5 + \frac{9}{10}220.5 = 198.9$ , provided the other barrier player joins it as well. In case both

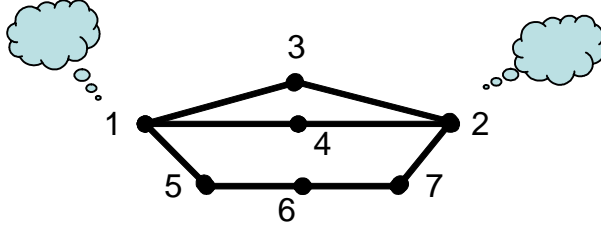


Figure 6: The "boiling soup pot"

of them wait until the projects' qualities are revealed to them, both join the same high quality project with probability 0.99 and receive a payoff of 0 with probability 0.01. Their expected payoff in this case is 218.295 which is strictly larger than 198.9. Since no other player has the option to join both projects, the two barrier players are most central.

#### 4.2.2 Example 4: The "boiling soup pot"

In the network depicted in Figure 6, assumption *A2* breaks the symmetry between players 3 and 4 and makes player 4 most central. Let  $T = 2$ ,  $p_1 = p_2$ ,  $k_\gamma^H = \frac{4}{3}$ ,  $k_\gamma^L = 1$  and  $\pi_\gamma(|b_\gamma|, k_\gamma^q) = 3k_\gamma^q|b_\gamma| - \frac{1}{2}|b_\gamma|^2$  for  $\gamma \in \mathcal{P}$ ; hence,  $|b_\gamma^*(H)| = 4$ ,  $|b_\gamma^*(L)| = 3$  and  $\hat{m}_\gamma = 1$  for  $\gamma \in PL$ . Players 3 and 4 who belong to  $\mathcal{C}_{2,1}^0$  are barrier players of both projects. Player 6 is in  $\mathcal{C}_2^0$  as well and players 5 and 7 belong to  $\mathcal{C}_1^0$ .

In the unique *SE*, at  $t = 1$ , project leader 1 asks players 3 (by *A2*) and 5 in any case, and additionally offers player 4 to join if, and only if, project 1's quality is high. Similarly, project leader 2 asks players 4 (by *A2*) and 7 in any case, and additionally 3 if, and only if, project 2's quality is high. Players 5 and 7 accept their offers. Player 3 would join project 2 and player 4 project 1 since to receive this offer reveals them the corresponding project's high quality. Otherwise, each of them joins the other project.

The teams  $\{1, 3, 5\}$  and  $\{2, 4, 7\}$  form if both projects' quality is low. If project 1's quality is high and project 2's is low, project 1 fills at  $t = 1$  with players 3, 4 and 5, while project leader 2 recruits player 7 at  $t = 1$  and player 6 at  $T = 2$ . The teams  $\{1, 3, 4, 5\}$  and  $\{2, 6, 7\}$  form. If project 2's quality is high and project 1's low, project 2 fills at  $t = 1$  with players 3, 4 and 7, while project leader 1 recruits player 5 at  $t = 1$  and player 6 at  $T = 2$ . The teams  $\{1, 5, 6\}$  and  $\{2, 3, 4, 7\}$  form. Finally, if both projects' quality is high, the teams at  $t = 1$  are  $\{1, 4, 5\}$  and  $\{2, 3, 7\}$ . Both leaders offer player 6 to join at  $T = 2$ . He only receives two offers in this case. Hence, he updates his belief about both projects' quality to 1 and joins project 1, by *A2*.

Given any pair  $p_1 = p_2$ , only player 4 is included in each high quality team of size

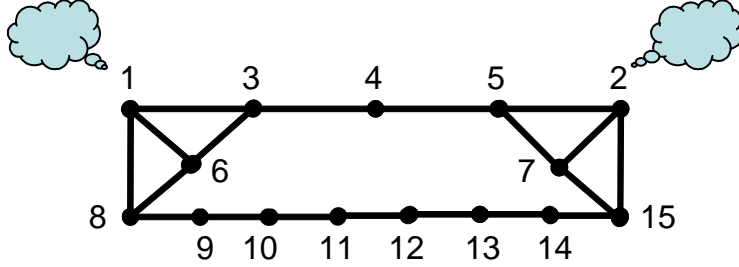


Figure 7: The  $SE$  depends on  $p$

four which forms. He joins a low quality team of size three when both projects' quality is low. Hence, no other player's expected payoff is higher than his and he is most central. Player 3 whose position seems to be symmetric to player 4's receives a lower expected payoff than 4 since player 6, by A2, joins project 1 in case both projects' quality is high.

#### 4.2.3 Example 5: The $SE$ depends on $p$

To find the unique  $SE$  can be involved, as is illustrated for the network depicted in Figure 7. Let  $T = 5$ ,  $p_1 = p_2 = \dot{p}$ ,  $k_\gamma^H = 3\frac{1}{3}$ ,  $k_\gamma^L = 2\frac{1}{3}$  and  $\pi_\gamma(|b_\gamma|, k_\gamma^q) = 3k_\gamma^q|b_\gamma| - \frac{1}{2}|b_\gamma|^2$  for  $\gamma \in \mathcal{P}$ ; then,  $|b_\gamma^*(H)| = 10$ ,  $|b_\gamma^*(L)| = 7$  and  $\hat{m}_\gamma = 4$ . The leader of high quality project 1 aims to recruit players 3 to 11 and that of high quality project 2 players 3 to 7 and 12 to 15. Hence,  $\mathcal{C}_{2,1}^0 = \{3, 4, 5, 6, 7\}$ ,  $BP_1 = \{3, 6\}$  and  $BP_2 = \{5, 7\}$ . Player 12 also belongs to  $\mathcal{C}_2^0$ . Obviously, he is within reach of project 2 and joins it at  $t = 4$  in case its quality is high. However, if project 2's quality is low and project 1's high, project leader 1 recruits player 12 at  $T = 5$ . All other players are in  $\mathcal{C}_1^0 = \{8, 9, 10, 11, 13, 14, 15\}$ . The  $SE$  depends on the value of  $\dot{p}$  and three cases may occur.

**Case 1:** At  $t = 1$ , player 3 moves first and waits while, by A1 and SR, players 5, 7 and 15 join project 2 and player 8 project 1. Let  $team_\gamma^t$  be the set of  $\gamma$ 's contributors at the end of period  $t$ . Then,  $team_1^1 = \{1, 8\}$  and  $team_2^1 = \{2, 5, 7, 15\}$ . At the end of period 2,  $team_1^2 = \{1, 8, 9\}$  and  $team_2^2 = \{2, 4, 5, 7, 14, 15\}$ . From period 3 on, the teams depend on  $k \in K$ . How they evolve is depicted in Table 1. The barrier players coordinate on project 2 and at  $t = 3$  player 3 either join it (since then its quality is high) or project 1. At  $T = 5$ , the teams are identical to those formed at  $t = 4$ , except of project 1 which includes player 12 if, and only if,  $k_1^H > k_2^L$ . Without A1, player 3 might wait until  $T = 5$  to decide.

	$k_1^H$ $k_2^H$	$k_1^H$ $k_2^L$	$k_1^L$ $k_2^H$	$k_1^L$ $k_2^L$
$team_1^3$	{1, 8, 9, 10}	{1, 3, 8, 9, 10}	same as	same as
$team_2^3$	{2, 3, 4, 5, 7, 13, 14, 15}	{2, 4, 5, 7, 13, 14, 15}	$k_1^H$ $k_2^H$	$k_1^H$ $k_2^L$
$team_1^4$	{1, 8, 9, 10, 11}	{1, 3, 6, 8, 9, 10, 11}	same as	same as
$team_2^4$	{2, 3, 4, 5, 6, 7, 12, 13, 14, 15}	{2, 4, 5, 7, 13, 14, 15}	$k_1^H$ $k_2^H$	$k_1^H$ $k_2^L$

Table 1: How the teams evolve in Case 1 of Example 5

Players 3 and 6 are most central. Players 4, 5 and 7 join a low quality project 2 in case  $k_1^H k_2^L$ , while 3 and 6 join high quality project 1 with one more contributor. Otherwise, their payoffs coincide. All other players always join the same team, except of player 12 who, however, remains unrecruited in case  $k_1^L k_2^L$ .

It is left to show that players 4 and 5 do not deviate. Suppose first that player 5 waits for an offer from project 1 (instead of joining project 2). Then, player 3 updates his belief about project 2's quality to 0 and joins project 1 at  $t = 3$ . The teams at this point in time are  $team_1^3 = \{1, 3, 8, 9, 10\}$  and  $team_2^3 = \{2, 7, 13, 14, 15\}$ . At  $t = 4$ , project leader 1 asks players 6 and 11 if project 1's quality is low and additionally player 4 (just in case he is still available) if it is high. All players accept his offer. Project leader 2 recruits player 12 who updates his belief about its quality to 1 (since he anticipates to be on the *SE* path). At  $T = 5$ , player 5 receives project leader 2's offer for sure and project leader 1's if, and only if, its quality is high. Hence, he joins project 1 if offered and otherwise project 2.

Player 5's deviation is profitable if his expected payoff is larger than or equal to the one in the *SE*. (By *A2*, he aspires to join project 1 if the two payoffs are equal and deviates from the *SE*.) Player 5's payoff in both cases is depicted in Table 2. His expected payoff in the *SE* is  $\dot{p}\pi_5(10, k_2^H) + (1-\dot{p})\pi_5(7, k_2^L)$ . If he deviates it is  $\dot{p}\pi_5(9, k_1^H) + (1-\dot{p})(\dot{p}\pi_5(7, k_2^H) + (1-\dot{p})\pi_5(7, k_2^L))$ . Substituting for the payoff player 5 obtains in each case, the first term is larger than the second if  $\dot{p} > \frac{41}{42}$ . Hence, player 5 does not deviate for  $\dot{p} \in (\frac{41}{42}, 1)$ .

Player 5's payoff	in the <i>SE</i>	after deviating
$k_1^H$ $k_2^H$	$\pi_5(10, k_2^H) = 50$	$\pi_5(9, k_1^H) = 49.5$
$k_1^H$ $k_2^L$	$\pi_5(7, k_2^L) = 24.5$	$\pi_5(9, k_1^H) = 49.5$
$k_1^L$ $k_2^H$	$\pi_5(10, k_2^H) = 50$	$\pi_5(7, k_2^H) = 45.5$
$k_1^L$ $k_2^L$	$\pi_5(7, k_2^L) = 24.5$	$\pi_5(7, k_2^L) = 24.5$

Table 2: Player 5's Payoff

Suppose that player 5 does not deviate, but that player 4 waits for an offer from project leader 1 (instead of joining project 2). Player 3 again updates his belief about

project 2's quality to 0 and joins project 1 at  $t = 3$ . Then,  $team_1^3 = \{1, 3, 8, 9, 10\}$  and  $team_2^3 = \{2, 5, 7, 13, 14, 15\}$ . At  $t = 4$ , project leader 1 recruits players 6 and 11 for sure, and player 4 if, and only if, project 1's quality is high. Project leader 2 recruits player 12 who updates his belief about its quality to 1 (since he anticipates to be on the *SE* path). At  $T = 5$ , player 4 receives project leader 2's offer if its quality is high and accepts it if he did not yet join project 1. Otherwise he remains unrecruited.

His expected payoff in the *SE* is  $\dot{p}\pi_4(10, k_2^H) + (1 - \dot{p})\pi_4(7, k_2^L)$ . If he deviates it is  $\dot{p}\pi_4(8, k_1^H) + (1 - \dot{p})\dot{p}\pi_4(8, k_2^H)$ . Substituting for the payoff player 4 obtains in each case, the first term is larger than the second if  $48\dot{p}^2 - 70.5\dot{p} + 24.5 > 0$ . This inequality is fulfilled for  $\dot{p} \in (0, 0.564) \cup (0.904, 1)$ . For this range of  $\dot{p}$  player 4 joins project 2 at  $t = 2$ . (By A2, player 4 aspires to join project 1 if this inequality is weak and deviates from the *SE*.)

If the inequalities derived for players 4 and 5 both hold, that is, for  $\dot{p} \in (\frac{41}{42}, 1)$ , players 4 and 5 do not deviate from the *SE* in Case 1. Otherwise, at least one of them would deviate and a different *SE* has to be found. If the inequality for player 5 is violated but the one for player 4 not, that is, for  $\dot{p} \in (0, 0.564) \cup (0.904, \frac{41}{42}]$ , Case 2 applies. Finally, if the inequality for player 4 is violated, that is, for  $\dot{p} \in [0.564, 0.904]$ , Case 3 applies (independently of player 5's inequality).

**Case 2:** If player 5 deviates, player 3 would wait until  $t = 4$  and player 6 until  $T = 5$  before joining project 1. Both join project 2 before if offered. This induces player 5 to join project 2 at  $t = 1$  since project 1's offer never reaches him. The game evolves as initially described in Case 1, though players 3 and 6 join project 1 only at  $t = 4$  and  $T = 5$ , respectively, if project 2's quality is low. This is a *SE* for  $\dot{p} \in (0, 0.564) \cup (0.904, \frac{41}{42}]$ .

**Case 3:** In case player 4 would deviate from the *SE* derived in Cases 1 or 2, player 3 waits instead until  $T = 5$  to join project 1 and player 4 joins project 2 at  $t = 2$  since he never receives an offer from project 1. The game evolves as described in Case 1, though players 3 and 6 join project 1 only at  $T = 5$ , if project 2's quality is low. This is the unique *SE* for  $\dot{p} \in [0.564, 0.904]$ .

Finally, it is shown that player 3 never deviates in a *SE* (given any case) as long as  $\dot{p} = p_1 = p_2$ . If he waits at  $t = 1$ , his expected payoff is  $\dot{p}\pi_3(10, k_2^H) + (1 - \dot{p})[\dot{p}\pi_3(8, k_1^H) + (1 - \dot{p})\pi_3(7, k_1^L)]$  while it is  $\dot{p}\pi_3(10, k_1^H) + (1 - \dot{p})\pi_3(7, k_1^L)$  if he joins project 1 at  $t = 1$ . The first term is larger than or equal to the second if

$$\begin{aligned} \dot{p} \pi_3(10, k_2^H) + (1 - \dot{p}) \dot{p} \pi_3(8, k_1^H) + (1 - \dot{p}) (1 - \dot{p}) \pi_3(7, k_1^L) &\geq \\ \dot{p} \pi_3(10, k_1^H) + (1 - \dot{p}) \pi_3(7, k_1^L). \end{aligned}$$

This simplifies to  $\dot{p} \pi_3(8, k_1^H) + (1 - \dot{p}) \pi_3(7, k_1^L) \geq \pi_3(7, k_1^L)$  and further to  $\pi_3(8, k_1^H) \geq$

$\pi_3(7, k_1^L)$ . Thus, player 3 waits at  $t = 1$  given any  $\dot{p} \in (0, 1)$ . Players 3 and 6 are most central in Cases 2 and 3 as well since the same teams emerge as in Case 1.

### 4.3 Centrality Measure and Related Literature

Conceptually, the centrality measure proposed here is related with Freeman's (1977) betweenness centrality and Burt's (1992) structural holes. However, the three measures usually select different players as most central, such as in Example 2.

A player which fills a structural hole obtains the highest payoff since he brokers information or controls its flow between two groups. He receives an informational rent in an environment in which information is complementary and cross fertilization of ideas benefits both groups. Burt's concept is based on Freeman's betweenness centrality which measures the proportion of paths between two distinct players which contain a third one. For example, in Figure 2, players 3 and 4 have the highest betweenness centrality if the two project leaders are taken as reference since any path between them contains both players. Nevertheless, player 4's expected payoff is lowest. If player 4 were connected to both project leaders, his betweenness centrality would be highest. Then, players 3 and 4 obtain the highest expected payoff as shown in the next section.

Whereas betweenness centrality is defined purely with respect to the network, the concept of structural holes requires an interpretation of the network. There are two groups of players which can access each other only via a player who occupies a structural hole. Each group is seen as a separate entity. Conversely, the centrality measure developed in this paper depends on the *TFG*, and in particular, on the project leaders' position in the network. It takes into account the agents' strategic considerations, that is, the project leaders' competition for contributors and the players' intention to join together a high quality project. Obviously, the assumptions of commitment, the projects' equal payoff division,  $A1$ ,  $A2$  and the agents' fixed order of moves are crucial to select the most central players (see section 5 for a discussion of these assumptions).

The agents' strategic behavior might prevent a player which is most central according to Freeman or Burt, such as player 4 in Example 2, from taking advantage of his position. In Example 1, player 3 is most central, while Freeman and Burt select players 3 and 4.<sup>20</sup> According to Freeman's betweenness centrality, players 7 and 8 are most central in Figure 4 of Example 3 while players 6 to 9 are most central in the *TFG*. In Figure 5 of Example 3, both measures select players 21 and 22 as most central. In Figure 6, all players have the same betweenness centrality while player 4 is most central in the *TFG*. In Figure 7, players

---

<sup>20</sup>If player 4 moves before 3 in this *TFG*, he obtains the highest expected payoff and is most central.

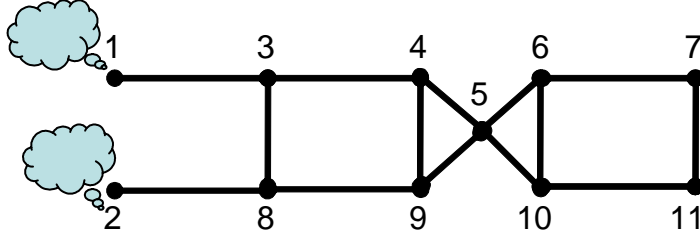


Figure 8: Restricted Access

3 to 5 and 8 to 15 have the same betweenness centrality and player 4 might arguably fill a structural hole. However, in the *TFG* of Example 5 players 3 and 6 are most central.

#### 4.4 Welfare Implications

In a *SE* inefficient unemployment may arise. Due to the network structure a project leader who wants to recruit contributors may not have access to free players while there are unrecruited players which would like to join him. This occurs, for example, if only one project leader can access category 2, such as in a star whose hub is the only barrier player while two of the spokes are project leaders. Suppose that in Figure 3 various players are added to the tree that originates in project leader 2. Then, inefficient unemployment obtains in the *SE*, if project leader 1 needs more than one player and project leader 2 leaves some players in the tree unrecruited. Example 6 is another instance of this.

**Example 6:** *Restricted Access*

*In the network depicted in Figure 8, the project leaders are on one side of the network and the players on the other. Let  $T = 3$ ,  $p_1 = p_2$ ,  $k_\gamma^H = 2$ ,  $k_\gamma^L = \frac{4}{3}$  and  $\pi_\gamma(|b_\gamma|, k_\gamma^q) = 3k_\gamma^q|b_\gamma| - \frac{1}{2}|b_\gamma|^2$  for  $\gamma \in \mathcal{P}$ ; then,  $|b_\gamma^*(H)| = 6$ ,  $|b_\gamma^*(L)| = 4$ ,  $\hat{m}_\gamma = 3$  for  $\gamma \in PL$  and  $\mathcal{C}_2^0 = \mathcal{C}_{2.1}^0 = \{3, 4, 5, 8, 9\}$  with  $BP_1 = \{3\}$  and  $BP_2 = \{8\}$ .*

*In the unique *SE*, at  $t = 1$ , player 3 accepts project leader 1's offer and player 8 rejects project leader 2's. At  $t = 2$ , project 1 includes players 4 and 8 in any case, and at  $T = 3$ , players 5 and 9 if, and only if, project 1's quality is high. Project 2's only contributor is its leader. Players 6, 7, 10 and 11 belong to category 0 throughout the game and  $\mathcal{C}_1^0 = \emptyset$ . Players 3, 4 and 8 are most central since they join project 1 independently of its quality.*

Although these are stylized examples, in reality, such situations frequently occur as is implied by the empirical results in Brüderl and Preisendörfer (1998) and Blumberg and Pfann (2001). A remedy to fix this inefficiency in the model is to link a project leader with unrecruited players. However, links may be costly and the theoretical analysis of a network

formation game that precedes the team formation game would yield useful insights. In reality, the network structure is usually not commonly known. Nevertheless, it might not be difficult and costly to provide platforms for existing and potential entrepreneurs to form additional links, such as fairs where local banks, customers, suppliers and representatives of the local administration offer their advice and availability to form links.

From a social point of view it is desirable to include more players in a project than its leader aspires to do (due to the concavity of the payoff function and the assumption of equal sharing among project participants). In a *SE*, even this number may not be reached. Moreover, the players frequently coordinate on one project. One group of barrier players waits in order to induce the other one to join its project (immediately) and thereby to reveal the project's quality. This is inefficient if once one team is filled the other may pick up the remaining players, but its barrier players prevent it from accessing available players. This inefficiency is inherent in the *SE*. Both groups of barrier players would like to obtain information about the more distant project's quality before taking a decision.

In general, more dense networks, that is, networks in which there are more links and a lower diameter, fare better in terms of welfare, as defined by the sum of all agents' payoffs, since players are more easily accessible and more players can be reached before  $T$ . However, the project leaders' position matters and a newly created link may allow one of them to "steal" players from the other which might decrease welfare. A change in the network structure may increase welfare as shown in section 5.

In case more project leaders are added to the network, any player's expected payoff is nondecreasing. Although, the competition for players becomes more severe and this harms project leaders. In reality, there are few examples, such as Silicon Valley, in which such a process is self-reinforcing. Many skilled individuals are competed for by many projects but this attracts more skilled individuals and entrepreneurs. Projects may go bust and release players again. As a consequence, the network is dynamic and its structure is constantly changing. In most other places all over the world, the network is not that dynamic and competition between entrepreneurs is different as illustrated by Brüderl and Preisendörfer (1998). The model developed in this paper applies to these situations in which the network is roughly stable over a long period of time.

## 5 Discussion

In this section, various assumptions and extensions of the model are discussed.

**Assumptions to obtain a unique *SE*.** The assumptions to select a unique *SE*

are restrictive. However, the entire set of  $SE$  can be identified. First, all results are given when the agents move in increasing order of their indexes. For any other order of moves, the corresponding  $SE$  can be found—Proposition 2 is valid for any order of moves. Moreover, Assumptions 1 and 2 can be adapted to break an indifference in any possible way. For each set of assumptions, the corresponding unique  $SE$  is derivable. Once the entire set of  $SE$  for a given  $TFG$  is identified, one can be selected according to certain criteria, such as welfare maximization. Then, comparative statics would be meaningful.

Finally, consider a change of the assumptions in the preceding examples. A different order of moves does not affect the  $SE$  in Figure 4 of Example 3 since coordination does not take place. In case coordination takes place, the total welfare in the  $TFG$  is frequently unchanged when the assumptions are altered while each player’s payoff changes. In Example 1, player 4 would wait at  $t = 1$  if he moved before 3. In Example 2, players 5 and 6 would wait at  $t = 1$  as long as both of them move before player 3. In Example 4, a different  $SE$  arises for each set of assumptions, though the total welfare is identical in each case. In Figure 5 of Example 3, the order of moves does not affect the  $SE$  while a change of  $A2$  would make players 21 and 22 coordinate on project 2. In Example 5, a change in the assumptions has a bigger impact on the  $SE$  which also depends on  $\hat{p}$ .

**Informational Assumptions.** It is possible to relax the informational requirements for project leaders. Initially, each may be restricted to know his direct neighbors. After recruiting one of them, he gets to know the recruited neighbor’s neighbor(s) and so on. In this way the project leader’s behavior implied by  $A1$  arises endogenously in a  $SE$ .

The players’ information about the network could be restricted as well. If the players are also categorized for low quality projects, then a player only needs to know in which category he is given the realized project qualities. To know his exact position in the network is not necessary. This implies that it is not essential in reality to be linked with the most focal agent(s), such as a project leader or barrier player.

If all agents observe all offers and replies and not only those involving them, then the players update their beliefs earlier, and the  $SE$  is trivial.

**The Categorization in more General Cases.** The players’ categorization extends to more than two quality levels per project and to more than two projects. More categories and intersections between them exist and a player’s belief updating is more complicated, though qualitatively the result is similar. If there are more than two projects, for any number of quality levels, the number of categories is equal to the number of projects since some players initially might expect to join each project with a positive probability. Players drop to lower categories over time or join a project and all players which expect

to receive a certain number of offers might do so from different projects. Hence, for each category there is a subcategory with different project combinations. In general, a unique  $SE$  can be selected if analogous assumptions to  $A1$  and  $A2$  hold.

Finally, suppose that each agent is equally likely to be selected as project leader. First, one is drawn from the set of agents and then the other among the remaining agents (in order to avoid that one agent obtains two projects). The ensuing game could be solved analogously to the  $TFG$ . Additionally, each player would take into account the probability with which each other agent is a project leader and form corresponding beliefs (which enter his expected payoff function). The coordination feature would be lost unless  $T$  is quite large and a player does not lose the option to join one project by waiting for a second offer. A project leader's optimal strategy would be unchanged.

**Changes in the Network Structure.** If links are added or destroyed in network  $\eta$ , the centrality of players may change and a different  $SE$  may arise. In order to illustrate this, consider Example 2 with  $T = 3$  and  $p_1 = p_2$ , but suppose that in the network depicted in Figure 2 player 4 is connected to both project leaders. Then, all players are barrier players but only player 4 is one of both projects. In the unique  $SE$ , at  $t = 1$ , players 5 and 6 join project 2. Its leader asks them independently of its quality. If project 2's quality is high, player 4 joins it as well at  $t = 1$  and player 3 at  $t = 2$ . If it is low, players 3 and 4 join project 1 at  $T = 3$  since both update their beliefs about project 2's quality to 0. Players 3 and 4 obtain a higher expected payoff than 5 and 6 since they have two chances to join a high quality project.

**Commitment.** If a player need not commit, he accepts the first offer he receives but then reneges on his promise if he gets a better offer. Since all players prefer to join the same project, once the two projects have recruited neighbors in category 2, one group of players would renege on its promises.

In Example 2, given  $T = 3$  and no commitment, player 3 joins project 1 and players 5 and 6 project 2 at  $t = 1$ . If offered, player 4 joins project 2 at  $t = 2$  since this reveals him its high quality. Player 3 then joins it at  $T = 3$ . If player 4 is not asked by project 2 at  $t = 2$  (since its quality is low), he joins project 1 instead. If, and only if, project 1's quality is high, players 5 and 6 then renege on their initial promise to project leader 2 and join project 1 at  $T = 3$ . All players obtain the same payoff. Either all of them join the same high quality project or two low quality projects each with three agents form.

Each project leader would then recruit players out of reach for the other since they stay with him. Players in category 2 are only recruited if no others are accessible. If a project leader thereby reveals his project's high quality, his chance to keep them increases.

If project leaders need not commit either, they would initially pretend to have a high quality project. At  $T$ , they then discard players in case the project's quality is low. Hence, the players only get to know a project's quality once they cannot react to it any more.

In reality, project leaders and players frequently commit. Usually, contracts are signed for a certain period and can only be cancelled at early enough prior notice. If one party breaks a contract, fines may be imposed on it. Moreover, in many industries after leaving a company, important employees are prevented from changing to a competitor, supplier or customer for some time.

## 6 Final Remarks

This paper studies team formation in a network taking into account social ties of economic agents. Two projects compete for players which have imperfect information about their quality. A unique pure strategy  $SE$  always exists if certain conditions hold. The players are categorized according to their position in the network relative to that of the two projects and according to whether they may receive two, one or no offers. The categorization is related with a player's  $SE$  strategy and his expected and realized payoff. Moreover, a new centrality measure is derived, which is unrelated to existent concepts in the network literature. Usually, a group of players is most central.

The model's solution implies that economic agents only need to locate themselves in the right category, but that their exact position does not matter. Unemployment may prevail in a  $SE$ , although project leaders still want to recruit. In a  $SE$ , the players may delay their decision to join a closer project in order to solicit information about the more distant's quality. This behavior is individually rational, though it may be socially inefficient if it prevents a team from reaching its optimal size.

This model can be extended in several ways. The players' skill level may be heterogeneous. A project leader may have to recruit several low skilled players just to access a high skilled one. The agents may choose an effort level, which is a continuous variable, or each can allocate an amount of time or money to various projects. Bargaining among project participants could take place or a project leader could post a wage. Moreover, a market for the teams' products might exist. The team which produces earlier captures a higher market share, while the other is more successful if its product's quality is higher and it offers a larger quantity. Though this depends on whether the products are substitutes or complements. Finally, a network formation game that precedes the  $TFG$  can be analyzed.

## References

- Aldrich, H. and C. Zimmer (1986), "Entrepreneurship through Social Networks," *The Art and Science of Entrepreneurship*, edited by D. Sexton and R. Smilor, 3-23, Ballinger Publishing Company.
- Bala, V. and S. Goyal (1998), "Learning from Neighbours," *Review of Economic Studies* 65, 595-621.
- Blumberg, B. and G. Pfann (2001), "Social Capital and the Uncertainty Reduction of Self-Employment," *IZA Discussion Paper* No. 303.
- Brüderl, J. and P. Preisendörfer (1998), "Network Support and the Success of Newly Founded Businesses," *Small Business Economics* 10, 213-225.
- Burt, R. (1992), *Structural Holes: The Social Structure of Competition*, Harvard University Press.
- Buskens, V. and A. van der Rijt (2008), "Dynamics of Networks if Everyone Strives for Structural Holes," *American Journal of Sociology*, 114, 371-407.
- Freeman, L. (1977), "A Set of Measures of Centrality Based on Betweenness," *Sociometry* 40, 35-41.
- Goyal, S. and F. Vega-Redondo (2007), "Structural Holes in Social Networks," *Journal of Economic Theory* 137, 460-492.
- Harsanyi, J. (1967-68), "Games with Incomplete Information Played by Bayesian Players," *Management Science* 14, 159-182, 320-334, 486-502.
- Ioannides, Y. and L. Loury (2004), "Job Information Networks, Neighborhood Effects, and Inequality," *Journal of Economic Literature* 42, 1056-1093.
- Jackson, M. (2005), "A Survey of Models of Network Formation: Stability and Efficiency," *Group Formation in Economics: Networks, Clubs, and Coalitions*, edited by G. Demange and M. Wooders, 11-57, Cambridge University Press.
- Michelacci, C. and O. Silva (2007), "Why so Many Local Entrepreneurs?," *Review of Economics and Statistics* 89, 615-633.
- Ray, D. (2008), *A Game-Theoretic Perspective on Coalition Formation*, Oxford University

Press.

Stein, J. (2008), "Conversations Among Competitors," *American Economic Review*, 98, 2150-2162.

Wasserman, S. and K. Faust (1994), *Social Network Analysis*, Cambridge University Press.