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# Total Factor Productivity Growth when Factors of Production Generate Environmental Externalities

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## Abstract

Total factor productivity growth (TFPG) has been traditionally associated with technological change. We show that when a factor of production, such as energy, generates an environmental externality in the form of CO2 emissions which is not internalized because of lack of environmental policy, then TFPG estimates could be biased. This is because the contribution of environment as a factor of production is not accounted for in the growth accounting framework. Empirical estimates confirm this hypothesis and suggest that part of what is regarded as technology's contribution to growth could be attributed to the use of environment in output production.

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Total factor productivity growth (TFPG) has been traditionally associated with technological change. We show that when a factor of production, such as energy, generates an environmental externality in the form of $CO_2$ emissions which is not internalized because of lack of environmental policy, then TFPG estimates could be biased. This is because the contribution of environment as a factor of production is not accounted for in the growth accounting framework. Empirical estimates confirm this hypothesis and suggest that part of what is regarded as technology's contribution to growth could be attributed to the use of environment in output production.

**Keywords**: Total Factor Productivity, Sources of Growth, Environmental Externalities, Energy, Environmental Policy

**JEL Classification:** O47, Q20, Q43

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# 1 Introduction

The sources of economic growth is an issue which has received much attention in economic science. One of the most popular and successful ways of summarizing the contribution of factors of production and technology to output growth is the growth accounting framework introduced by Solow (Solow 1957). Growth accounting allows for a breakdown of output growth into its sources which are the factors of production and technological progress, and makes possible the estimation of the contribution of each source to output growth. Growth accounting leads to the well known concept of the Solow residual, which measures total factor productivity growth (TFPG). TFPG is the part of output growth not attributed to the growth of factors of production such a capital or labour, but to technical change.[3] A strong positive TFPG has been regarded as a desirable characteristic of the growth process since, given the growth of conventional factors of production, it further promotes output growth. However, there are still conceptual disputes about the subject. For example, Easterly and Levine (2001) suggest "that economists need to provide much more shape and substance to the amorphous term TFP". In this paper we try to provide some additional "shape" by seeking to study the concept of TFPG when inputs which generate negative environmental externalities are used in production.

In the traditional growth accounting framework TFPG is what remains from output growth after the contribution of the factors used to produce output is subtracted. This residual has been traditionally attributed to

[3]During the last decades many different approaches have been used to measure TFPG. They include primal approaches using factor quantities, dual approaches using factor prices instead of factor quantities, and approaches which basically involve disaggregation and refinement of inputs in the production function. For presentation of these approaches and extended references, see for example Barro (1999), Barro and Sala-i-Martin (2004). Recent TFPG estimates are reported in Baier et al. (2006).

accumulation of knowledge and advances in engineering. In this framework the contribution of each factor is measured by the factor's share in total output multiplied by the factor's rate of growth. To obtain this share the factor's cost, as it is determined in a market economy, is used.

However, what if a factor is used in the production process but its cost is not accounted for in a market economy? That is, what if an unpaid factor is contributing to growth? This question is far from hypothetical since it has been understood in the recent decades that environment has been used as a factor of production. Environment is used in general for depositing by-products of the production process, the most striking example being the emission of greenhouse gasses which have been closely associated with severe negative externalities such as global warming and climate change (e.g. The Stern Report, 2006). When the cost of the environmental externality is not internalized in a market economy due to lack of an appropriate environmental policy, the use of the environment is equivalent to the use of an unpaid factor in the production of output. If however environment is an unpaid factor which contributes to growth, then at least part of what we think is TFPG is in fact the unaccounted contribution of environment to output growth. Thus, positive TFPG estimates that might suggest a "healthy" growth process could, at least partly, embody the unaccounted contribution of the environment. When this contribution is accounted for, TFPG might not be as strongly positive and the growth process might not be as "healthy" as we think, since the unpaid factor is excessively and inefficiently used. For example, a negative TFPG after the contribution of the environment is accounted for, could be interpreted as indicating that the "value" of the factors we use in production exceeds the "value" of what we produce by these factors. Negative TFPG estimates have been explained by institutional changes and conflicts (Baier et al. 2006). Our paper suggests another reason, the presence of an externality which results in an unaccounted, by the growth accounting framework, use of a factor of production. Analyzing the contribution of unpaid factors in the growth process could also have potentially significant policy implications. Given the fact that many economies have been characterized by high growth rates, one might want to examine whether and to what extend unpaid factors are contributing to this growth, and analyze what kind of policy is required in order to internalize the cost of these factors and thus use them efficiently.[4]

In order to capture the contribution of environment as an unpaid factor in growth accounting, environment's use in output production should be modelled. One way of doing this is by directly introducing emissions in the production function in the way originally proposed by Brock (1973).[5] When emissions are treated as a factor of production then a growth accounting exercise shows that the "traditional" TFPG measurements[6] will be in general biased since they do not account for the emissions, and thus environment's contribution to output growth.[7]

Another way of modelling environment's contribution to output growth, which might be more appropriate in the context of a market economy is to introduce as an input in the aggregate production function a factor of production which is "paid" in conventional terms, but which at the same time generates and "unpaid" or "uninternalized" environmental externality.

[4]TFP growth can be also influenced by positive externalities. Madsen (2008) shows that this applies to the case of the international patent stock which along with knowledge spillovers through the channel of imports, has contributed significantly to TFP growth. In this paper we concentrate on the impact of negative externalities.

[5]See also Tahvonen and Kuuluvainen (1993), or Xepapadeas (2005).

[6]We use the term 'traditional' TFPG for TFPG measures where unpaid factors or externalities are not taken into account.

[7]For theoretical analysis see Dasgupta and Maler (2000), Xepapadeas (2005). For an empirical application see Tzouvelekas et al. (2007).

In the context of externalities associated with climate change it is energy which is the input clearly satisfying this requirement.[8] Although energy is paid as a factor of production in a market economy, there is also an unpaid part of energy which is associated to an uninternalized environmental externality. This is the greenhouse gasses (GHGs) and in particular carbon dioxide ($CO_2$) emissions (or GHGs equivalent $CO_2$ emissions) generated by energy use. These emissions can be regarded as an unpaid environmental externality since no carbon tax policy has in general been applied until the relatively recent Kyoto protocol, which applies to a subset only of GHGs generating countries.

Thus the purpose of the present paper is to develop a conceptual framework and to provide estimates of the impact on TFPG measurements, of the uninternalized (or unpaid) part of energy, which is the environmental externality generated by emissions of GHGs. In this context we derive first an externality-adjusted TFPG measure using an optimal growth model with energy as a factor of production and emission accumulation which generates disutility, and then obtain empirical estimates of the externality-adjusted TFPG by applying our methodology to a panel of OECD countries. Our results suggests that TFPG measurements are significantly affected when the external cost of emissions associated with energy use is, up to a certain extend, internalized.

We measure TFPG by regarding energy as a factor of production which is not fully paid, in the sense that market prices for energy do not cover both private costs and external costs associated with energy use. However, there is a problem associated with TFPG measurements if energy related externalities are not internalized. If TFPG is estimated using data of factors'

[8] This means that the aggregate production function would be of the so called $KLE$ form (Griffin 1981, Griffin and Gregory 1976).

shares in output for a period when no environmental policy, such a taxation, tradeable permits, or command and control measures, has been applied to emissions associated with energy use, then these traditional TFPG estimates will be biased. This is because the share of the emissions part of energy in output is zero due to lack of environmental policy, which if existed, would have charged this part with its external cost.[9] We correct for this bias and arrive at an externality-adjusted TFPG by appropriately adjusting, for the external cost of $CO_2$ emissions, traditional TFPG measures obtained by estimating an aggregate production function for a panel 23 OECD countries, with energy used is an input in the production function.

The adjustment is carried out by subtracting the contribution of the unpaid (or uninternalized) part of energy costs, which is the environmental costs of $CO_2$ emissions, from output growth. To value this contribution we use current estimates of the marginal damages from $CO_2$ emissions. Our results suggest that when the emission's part of energy valued by the $CO_2$ emissions marginal damages, is accounted for in the growth accounting measurements, then the externality-adjusted Solow residual, or the externality-adjusted TFPG is reduced relative to the traditional TFPG estimates and might take even negative values. A negative Solow residual would imply that when *all* factors used in the production process are paid for their contribution to total output growth, then the contribution of technological progress to output growth is outweighed by the use of factors of production which generate uninternalized externalities.

[9]The bias emerges because the social marginal products deviate from private marginal products, due to the existence of uninternalized externalities.

## 2 The Solow Residual with Externality Generating Inputs

We start with a standard neoclassical production function:

$$Y = F(K, H, AL, BE) \tag{1}$$

where $K$ is physical capital, $H$ is human capital, $AL$ is effective labour with $L$ being labor in physical units and $A$ reflecting labor augmenting technical change, $BE$ is effective input of energy with $E$ being energy in physical units and $B$ reflecting energy augmenting technical change, . Differentiating (1) with respect to time, and denoting by $\epsilon_j, j = K, H, L, E$ the elasticity of output with respect to inputs, the basic growth accounting equation is obtained as:

$$\frac{\dot{Y}}{Y} = \epsilon_K\left(\frac{\dot{K}}{K}\right) + \epsilon_H\left(\frac{\dot{H}}{H}\right) + \epsilon_L\left(\frac{\dot{A}}{A}\right) + \epsilon_L\left(\frac{\dot{L}}{L}\right) + \epsilon_E\left(\frac{\dot{B}}{B}\right) + \epsilon_E\left(\frac{\dot{E}}{E}\right) \tag{2}$$

We assume that energy is related to emissions by the following function:

$$E(t) = \phi(Z(t)) \tag{3}$$

where $Z(t)$ is emissions created by the use of energy $E$ at time $t$. We assume that $\phi_Z > 0$, $\phi_{ZZ} \geq 0$ and that the inverse function exist, so we can alternatively express emissions as a function of energy use:

$$Z(t) = \phi^{-1}(E(t)) = \psi(E(t)) \tag{4}$$

differentiating (3) with respect to time and dividing by $E$ we obtain:

$$\frac{\dot{E}}{E} = \epsilon_{E_Z}\left(\frac{\dot{Z}}{Z}\right) \tag{5}$$

where $\epsilon_{E_Z}$ is the elasticity of energy with respect to emissions from (3). Then (2) becomes:

$$\frac{\dot{Y}}{Y} = \epsilon_K\left(\frac{\dot{K}}{K}\right) + \epsilon_H\left(\frac{\dot{H}}{H}\right) + \epsilon_L\left(\frac{\dot{A}}{A}\right) + \epsilon_L\left(\frac{\dot{L}}{L}\right) + \epsilon_E\left(\frac{\dot{B}}{B}\right) + \epsilon_E\epsilon_{Ez}\left(\frac{\dot{Z}}{Z}\right) \tag{6}$$

Therefore the growth accounting equation can be expressed either in terms of energy by (2) or in terms of emissions by (6). To transform (6) into a growth accounting equation in factor shares we use profit maximization in a competitive market set up. Profits for the representative competitive firm are defined as:

$$\pi = F(K, H, AL, BE) - R_K K - R_H H - wL - p_E E - \tau\psi(E) \tag{7}$$

where $p_E$ is the competitive price for energy and $\tau$ is an exogenous emission tax or an exogenous price for tradable emission permits. First-order conditions for profit maximization imply that input shares are defined as:

$$s_K = \frac{R_K K}{Y}, s_H = \frac{R_H H}{Y}, s_L = \frac{wL}{Y},\ s_E = \frac{p_E E + \tau\psi'(E)E}{Y} \tag{8}$$

It should be noted that the share of energy, $s_E$, consists of two parts. The part paid for energy in energy markets, $\frac{p_E E}{Y}$, and the share corresponding to the cost of emissions generated by energy $\frac{\tau\psi'(E)E}{Y}$. If $\tau$ reflects the external cost of emissions then this share is the share of externality in total output. If

$\tau = 0$ then the externality is not internalized and the use of the environment as factor of production is unpaid. Thus the externality-adjusted TFPG or the externality-adjusted Solow residual can be defined in terms of energy as:

$$\gamma_E = s_L \left(\frac{\dot{A}}{A}\right) + s_E \left(\frac{\dot{B}}{B}\right) = \frac{\dot{Y}}{Y} - s_K \left(\frac{\dot{K}}{K}\right) - s_H \left(\frac{\dot{H}}{H}\right) - s_L \left(\frac{\dot{L}}{L}\right) - s_E \left(\frac{\dot{E}}{E}\right) \tag{9}$$

or in terms of emissions as:

$$\gamma_Z = s_L \left(\frac{\dot{A}}{A}\right) + s_E \left(\frac{\dot{B}}{B}\right) = \frac{\dot{Y}}{Y} - s_K \left(\frac{\dot{K}}{K}\right) - s_H \left(\frac{\dot{H}}{H}\right) - s_L \left(\frac{\dot{L}}{L}\right) - s_E \left(\epsilon_{E_Z} \left(\frac{\dot{Z}}{Z}\right)\right)$$

where $s_E$ is defined by (8). Under constant returns to scale (9) becomes:

$$\gamma_E = \frac{\dot{y}}{y} - s_K \frac{\dot{k}}{k} - s_H \frac{\dot{h}}{h} - s_E \frac{\dot{e}}{e} \tag{10}$$

It can be seen from (10) that the contribution of the environment in TFPG is reflected in the term $s_E \frac{\dot{e}}{e}$. This indicates that there is one more source which generates output growth. This is environment used as an input in production, in addition to capital and labour. Thus, in order to obtain a "net" estimate of TFPG the environment's contribution should be properly accounted. Relationships (9) and (10) can be considered as externality-adjusted growth accounting equations and $\gamma$ is the "externality-adjusted Solow residual". In order to provide a meaningful definition of the TFPG for empirical estimation, when environment is an input, we need to define the share of energy in output both in terms of the social optimum, and a forward looking competitive equilibrium.

## 2.1 Interpreting the Shares of Inputs in Externality-Adjusted TFPG measurements:

### 2.1.1 The Social Optimum

To define, at the social optimum, the share of energy in output, when energy use releases emissions which are an environmental externality, we analyze the problem of a social planner. The social planner maximizes a standard Ramsey-Koopmans felicity functional defined over consumption and environmental damages, and determines an optimal tax $\tau$ which would internalize the externalities that the emission's part of energy creates during the production process. Let the evolution of the emission stock $S$ be described by the first order differential equation:

$$\dot{S}(t) = Z(t) - mS(t) \ , \ S(0) = S_0, m > 0 \tag{11}$$

$$\dot{S}(t) = \psi(E) - mS(t) \ \ , \ \ \psi(E) = Z = \phi^{-1}(E) \tag{12}$$

where $m$ reflects the environment's self cleaning capacity[10]. The stock of emissions generate damages according to a strictly increasing and convex damage function $D(S)\,,\ D' > 0, D'' \geq 0.$

Assume that utility for the "average person" is defined by a function $U(c(t), S(t))$ where $c(t)$ is consumption per capita, $c(t) = C(t)/N(t)\,,$ with $N(t)$ being population. We assume that $U_c(c,S) > 0$, $U_S(c,S) < 0$ $U_{cS}(c,S) \leq 0$, that $U$ is concave in $c$ for fixed $S$, and finally that $U$ is homogeneous in $(c,S)$. Then social utility at time $t$ is defined as $N(t)\,U(c(t), S(t)) = N_0 e^{nt} U(c(t), S(t))$ where $n$ is the exogenous population (and labour force)

[10] We use a very simple pollution accumulation process which has been often used to model global warming. The inclusion of environmental feedbacks and nonlinearities which represent more realistic situations is an area of further research, but we expect that it will not change the basic results.

growth rate and $N_0$ can be normalized to one. The objective of the social planner is to choose consumption and energy paths to maximize:

$$\max_{\{c(t),E(t)\}} \int_0^\infty e^{-(\rho-n)t} U(c,S)\,dt \tag{13}$$

where, $\rho > 0$ is the rate of time preference, subject to the dynamics of the capital stock and the emission stock (12). The capital stock dynamics can be described in the following way. Assume a constant returns to scale Cobb-Douglas specification for the production function (1):

$$Y = K^{a_1} H^{a_2} (AL)^{a_3} (BE)^{a_4} \tag{14}$$

where and $E(t) = \phi(Z(t))$ as defined above. Expressing output in per worker terms we obtain:

$$y = e^{\zeta t} k^{a_1} h^{a_2} E^{a_4},\ \zeta = xa_3 + a_4(b-n)$$

where labor augmenting technical change grows at the constant rate $x$, energy augmenting technical change grows at a constant rate $b$, and as usual $y = \frac{Y}{L}$, $k = \frac{K}{L}$, $c = \frac{C}{L}$, $h = \frac{H}{L}$ and $e_L = \frac{E}{L}$ are expressed in per capita (or per worker) terms. Following Barro and Sala-i-Martin, we assume equality of depreciation rates and equality of marginal products between manufactured and human capital in equilibrium. Then, the social planner's problem can be written as:[11]

$$\max_{\{\hat{c}(t),E(t)\}} \int_0^\infty e^{-\omega t} U(\hat{c}, S)\, dt \;, \omega = \rho - n - (1-\theta)\,\xi \tag{15}$$

subject to: (16)

$$\dot{\hat{k}} = f\left(\hat{k}, E\right) - \hat{c} - p_E \hat{e}_L - (\eta + \delta + \xi)\,\hat{k},\;\; f\left(\hat{k}, E\right) = s\tilde{A}\hat{k}^{\beta} E^{a_4} \tag{17}$$

$$\dot{S} = \psi(E) - mS,\;\; Z = \psi(E) \tag{18}$$

with $k = \hat{k}e^{\xi t}$, $h = \hat{h}e^{\xi t}$, $c = \hat{c}e^{\xi t}$ and $e_L = \hat{e}_L e^{\xi t}$, where $\left(\hat{k}, \hat{h}, \hat{c}, \hat{e}\right)$ denotes per effective worker magnitudes and $\xi = \frac{\zeta}{1-a_1-a_2}$. The current value Hamiltonian for this problem is:

$$\mathcal{H} = U(\hat{c}, S) + p\left[f\left(\hat{k}, E\right) - \hat{c} - p_E \hat{e}_L - (\eta + \delta + \xi)\,\hat{k}\right] + \lambda\left(\psi(E) - mS\right) \tag{19}$$

The optimality conditions implied by the maximum principle are:

$$U_{\hat{c}}(\hat{c}, S) = p \;,\; U_{\hat{c}\hat{c}}(\hat{c}, S)\,\dot{\hat{c}} + U_{\hat{c}S}(\hat{c}, S)\,\dot{S} = \dot{p} \tag{20}$$

$$p\left[f_E\left(\hat{k}, E\right) - p_E \hat{l}\right] = -\lambda \psi'(E)\,,\; \hat{l} = \frac{e^{-\xi t}}{L} \tag{21}$$

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{1}{\theta}\left[f_{\hat{k}}\left(\hat{k}, g\left(\hat{k}, \lambda, U_{\hat{c}}(\hat{c}, S), \hat{l}\right)\right) - \rho - \delta - \theta\xi\right] - \frac{U_{\hat{c}S}(\hat{c}, S)}{U_{\hat{c}\hat{c}}(\hat{c}, S)}\dot{S} \tag{22}$$

$$\dot{\lambda} = (\omega + m)\,\lambda - U_S(\hat{c}, S) \tag{23}$$

The system of (22), (23) along with the two differential equation below:

$$\dot{\hat{k}} \;=\; f\left(\hat{k}, g\left(\hat{k}, \lambda, U_{\hat{c}}(\hat{c}, S), \hat{l}\right)\right) - \hat{c} - p_E \hat{e}_L - (\eta + \delta + \xi)\,\hat{k} \tag{24}$$

$$\dot{S} \;=\; \psi\left(g\left(\hat{k}, \lambda, U_{\hat{c}}(\hat{c}, S), \hat{l}\right)\right) - mS \tag{25}$$

form a dynamic system, which along with the appropriate transversality con-

[11] For the derivation see Appendix 1.

ditions at infinity (Arrow and Kurz 1970) characterizes the socially optimal paths of $\left(\hat{c}, \hat{k}, \lambda, S, E\right)$.

As is well known the costate variable $\lambda(t)$ can be interpreted as the shadow cost of the emission stock $S(t)$. Using this interpretation of $\lambda(t)$, it can be shown by comparing (21) with the profit maximizing conditions implied by (7) that if a time dependent tax $\tau(t) = -\frac{-\lambda \hat{l}}{p}$ is chosen, then firms will choose the socially optimal amount of energy as input. Then the energy share can be written as:

$$s_E = \frac{\left[p_E + \tau\psi'(E)\right] E}{Y}, \text{ with } \tau = \frac{-\lambda\hat{l}}{p} = \frac{-\lambda\hat{l}}{U_c} \tag{26}$$

Thus the share of energy in output along the optimal path consists of two parts. The first is associated with the market price of the energy used in production, while the second part is associated with the tax imposed on the emissions created by the use of energy as a factor of production. This second part reflects the social cost of externality associated with the use of energy in production. Under the optimal emission tax it can be shown that the solution of the competitive equilibrium will coincide with the social planners solution.

#### 2.1.2 Competitive equilibrium

The representative consumer considers the stock of pollution as exogenous and chooses consumption to maximize lifetime utility, or:

$$\max_{c(t)} \int_0^\infty e^{-(\rho-n)t} U(c, S)\, dt \tag{27}$$

subject to the budget flow constraint:

$$\dot{a} = w + ra - c - na + \tau z \tag{28}$$

where $a$ is per capita assets, $c$ per capita consumption, $w$, $r$ the competitive wage rate and interest rate respectively and $\tau z$ are per capita lump sum transfers due to environmental taxation, where $z = Z/L$ per capita emissions. The representative firm maximizes profits and in equilibrium $a = k + h$. Then the following proposition can be stated.

**Proposition 1** *Under an optimal tax* $\tau = \frac{-\lambda \hat{l}}{p}$ *of the emission content of energy used, the paths* $\left(\hat{c}(t), \hat{k}(t), S(t), Z(t)\right)$ *of a decentralized competitive equilibrium coincide with the socially-optimal paths.*

For proof see Appendix 2.

## 3 Estimating an Externality-Adjusted TFPG

The theoretical framework developed above suggests that in order to obtain the correct share of energy in output for TFPG measurements, the cost of environmental externality should be properly accounted for. However, when we seek empirical estimates of this "correct share" of emissions in $CO_2$ (or $CO_2$ equivalent greenhouse gases) created by the use of energy, these emissions do not have a "price" in the absence of environmental policy[12]. Thus in applied TFPG measurements we might not account for the contribution of the part of the energy input which is associated with the generation of the environmental externality and which remains unpaid if the price of energy

[12] A "price" in this case could be an environmental tax for the period we analyze, a traditional permit system with a well defined emission permit price or a binding emission limit. Such a type of 'price' did not emerged untill Kyoto.

does not include an environmental tax (optimal or not) or any other policy instrument. Therefore, traditional TFPG measurements can be biased. If emissions were taxed at a rate $\tau > 0$, environment could be regarded as a paid factor of production and the externalities created by the use of energy would be, at last partly, internalized. If however emissions are not subject to any regulatory policy (which has been the most usual case in reality) environment is an unpaid factor of production and we need independent estimates of the shadow cost of emission, $\lambda(t)$ to adjust TFPG measurements.

To further study the possible bias in TFPG and the nature of the externality adjustment we use again the Cobb-Douglas production function (14), under constant returns to scale in the log linear specification:

$$\ln y = a_0 + (xa_3 + ba_4)t + a_1 \ln k + a_2 \ln h + a_4 \ln e_Z, \ \sum_{i=1}^{4} a_i = 1 \tag{29}$$

where:

$$xa_3 + ba_4 = \gamma_E = TFPG \tag{30}$$

In (30), $\gamma_E$ is TFPG defined in (9) which includes both labor augmenting $(xa_3)$ and energy augmenting $(ba_4)$ technical change. Thus in principle TFPG can be obtained by estimating the parameters of (29). As shown in the previous section under an optimal environmental policy the energy share is defined as:

$$s_E = \frac{(p_E + \tau\psi')\, E}{Y} \tag{31}$$

When there is no environmental policy then $\tau = 0$ and the energy share is simply:

$$s_E = \frac{p_E E}{Y} \tag{32}$$

When input elasticities are estimated from (29), it is clear that if data correspond to a period where no policy with respect to GHGs was present, then the estimated energy share, $a_4$, will be (32) and not the correct share (31). Thus TFPG estimates will be biased. The estimates of (29) can be used however to estimate an externality-adjusted TFPG.

Let $(\hat{\gamma}_E, \hat{s}_K, \hat{s}_H, \hat{s}_E) = (xa_3 + ba_4, a_1, a_2, a_4)$ the TFPG and the elasticity estimates obtained from (29), then using (10) and (31), (32) the externality-adjusted TFPG can be obtained as:

$$\gamma_E^A = \frac{\dot{y}}{y} - \hat{s}_K \frac{\dot{k}}{k} - \hat{s}_H \frac{\dot{h}}{h} - \hat{s}_E \frac{\dot{e}}{e} - \frac{\tau \psi' E}{Y} \quad (33)$$

$$\gamma_E^A = \hat{\gamma}_E - \frac{\tau \psi' E}{Y} \quad (34)$$

Estimates of (29) are usually obtained from panel data so that an overall estimate of TFPG is obtained through (33) or (34). Individual country estimates can be obtained by using the estimated shares $\hat{s}_K, \hat{s}_H, \hat{s}_E$ from (29) and the average growth rates of output and inputs per worker for each one of the countries in the panel. The individual country estimates for $i = 1, ..., n$ countries in the panel can be obtained as:

$$\hat{\gamma}_{iE} = \overline{\left(\frac{\dot{y}}{y}\right)}_i - \hat{s}_K \overline{\left(\frac{\dot{k}}{k}\right)}_i - \hat{s}_H \overline{\left(\frac{\dot{h}}{h}\right)}_i - \hat{s}_E \overline{\left(\frac{\dot{e}}{e}\right)}_i \quad (35)$$

Then the individual country externality-adjusted TFPG estimate, will be obtained as:

$$\gamma_{iE}^A = \hat{\gamma}_{iE} - \frac{\tau \psi' E_i}{Y_i} \quad (36)$$

## 3.1 Results

Our estimates of the externality-adjusted TFPG are obtained in two steps. In the first step factor shares are estimated from (29), while in the second step the adjustments indicated by (33) or by (36) are carried out. Our data refer to a panel of 23 OECD countries for the years 1965-1990. Although the data set is not very recent, it represents a period where no $CO_2$ policy was present and therefore over this time period energy can be assumed as an externality generating input without any internalization, which is exactly the concept we are using in the development of our theoretical model.[13] Thus, this not so recent data set, can be regarded as an appropriate data set for testing the hypothesis that some of the output growth attributed to technological progress, for the period 1965-1990, should be attributed to the uninternalized environmental externality. The estimates of the production function are shown in table 1.

**Table 1:** Production Function Estimation

[13] We used data on real GDP, Capital per worker, Population and Real GDP per worker from the Penn Tables v5.6. Data on $CO_2$ emissions in kt were obtained from the World Bank *World Development Indicators* (2002). Primary energy data measured in mtoe were obtained from the International Energy Agency. We used as proxy for $H$ an index constructed from education data. This index is defined as $H_{it} = \exp(\phi(\epsilon_{jt}))$. Where $\epsilon_{jt}$ is average years in education in country $i$ at year $t$, and $\phi$ is a piecewise linear function with zero intercept and slope 0.134 for $\epsilon_{jt} \leq 4$, 0.101 for $4 < \epsilon_{jt} \leq 8$, and 0.068 for $\epsilon_{jt} > 8$.(see Hall and Jones (1999); Henderson and Russel (2005)). Data on education were obtained from the World Bank, *World Development Indicators* (2002).

All data in physical units were transformed to indexes. For the construction of the index for each variable the mean value for this variable over the whole sample was used as the base, or $x_{it} = y_{it}/\bar{y}$, $\bar{y} = \frac{1}{n+T}\sum_{i=1}^{n}\sum_{t=1}^{T} y_{it}$, where $y$ is variable in physical units and $x$ is the corresponding index.

| Parameters and Statistics | Estimates* |
|---|---|
| $a_1 = \hat{s}_K$ | 0.298 |
| $a_2 = \hat{s}_H$ | 0.027 |
| $a_4 = \hat{s}_E$ | 0.16 |
| $Traditional\ TFPG, \hat{\gamma}_E$ (%) | 1.21 |
| $R^2$ | 0.99 |
| $DW$ | 2.08 |

*All estimated parameters are highly significant.

The results suggest that physical capital's share in output is 29.8%, the corresponding share of energy as an input at 16% and the corresponding share for education which is used as a proxy for human capital is 2.7%. The estimate of the overall total factor productivity growth ($\hat{\gamma}_E$) is 1.21%.

It should be noted that the estimation of (29) represents estimation of a primal model, that might suffer from endogeneity associated with inputs. This would implying inconsistency in the estimates of the production function. However as it has been shown by Mundlak (1996, proposition 3) under constant returns to scale OLS estimates of a $k$-input Cobb-Douglas production function in average productivity form with regressors in inputs-labour ratio, are consistent. This is however exactly the type of production function we have in our model.

To estimate (29) we adopt a panel estimation approach with "fixed effects" to allow for unobservable "country effects" (e.g. Islam (1995). As shown by Mundlak (1996) this estimator applied to the primal problem is superior to the dual estimator which is applied to the dual functions. Furthermore the "fixed effects" estimator addresses the problem of correlation between the constant term $\gamma_E$, which is the TFPG estimate, with the regressors[14]. The estimation was performed using weighted least squares (WLS) in order to take into account both cross-section heteroscedasticity and contemporaneous correlation among countries in the sample.[15]

The overall average traditional TFPG obtained in table 1 is adjusted for the uninternalized environmental externality using (34). In order to perform the adjustment indicated by (34) we need the parameters $\psi'$ and $\tau$. We obtain $\psi'$ as the coefficient of the relationship $Z = \sigma E$, where $\sigma = \psi'$. The value for this parameter was obtained by a regression of energy on $CO_2$ emissions with all variables measured in physical units, (mtoe for energy and ktn for $CO_2$ emissions)[16]. Parameter $\tau$ should be interpreted as the cost of externality. To approximate this parameter we used an estimate of the marginal damage cost of $CO_2$ emissions, which was $\tau = 20\$/tCO_2$ (Tol, 2005). Thus, to obtain the externality-adjusted TFPG, we approximate the environmental policy parameter $\tau$ by the marginal damage of $CO_2$ emissions. Using the overall sample averages for energy ($E$) and output ($Y$) for the 23 countries the results from the adjustment are shown in table 2.

**Table 2:** Traditional $(\hat{\gamma}_E)$ and Externality-Adjusted $\left(\gamma_E^A\right)$ TFPG (%)

[14] This correlation has been regarded as one of the disadvantages of the regression approach in TFPG measurement (Barro 1999, Barro and Sala-i-Martin 2004).

[15] The estimation is carried out in two steps. In the first step the model is estimated via simple OLS. Using the obtained residuals the conditional country specific variance is calculated and it is used to transform both the dependent and independent variables of the second-stage regression. Specifically for each country, $y_i$ and each element of $x_i$ (independent variables) are divided by the estimate of the conditional standard deviation obtained from the first-stage. Then a simple OLS is performed to the transformed observations expressed as deviations of their means. This procedure results in a feasible generalized least square estimator described by Wooldridge ( 2000, Ch. 8) and Greene (2003, Ch. 11). EViews panel estimation with "cross-section SUR" option was used for estimating the production function.

[16] The value of the coefficient is 2.43 and this value is highly significant with $R^2 = 99\%$. Correction for first order autoregression of the residuals was performed. The first order autoregressive coefficient was significant.

| | |
|---|---|
| Traditional TFPG<br>(1) | 0.012 |
| Adjustment for Externality : $\frac{\tau\psi'(E)E}{Y}$<br>(2) | 0.019 |
| Externality-Adjusted TFPG<br>(3) = (1) − (2) | −0.007 |

As seen in table 2, the adjustment for the externality exceeds the traditional TFPG estimate and therefore the overall externality-adjusted TFPG is negative. This result suggests that, if the externality associated with energy use is internalized at a cost of 20$/tCO$_2$ then the part of output growth attributed to technological change, for the period 1965-1990 vanishes, or to put it differently the positive contribution of technological change to output growth during 1965-1990 was counterbalanced by the negative externality generated in the process of output growth during the same period. The impact of externality however is realized only when this externality is internalized.

The results of the individual country externality-adjusted TFPG, obtained by using the estimated shares of the production function and the average values of each type of capital for each one of the countries in our sample, are summarized in table 3. The second column shows the traditional TFPG estimates obtained by using (35), while the third column shows the externality-adjusted TFPG estimates obtained by using (36).

**Table 3**: Traditional $(\hat{\gamma}_{iE})$ and externality-adjusted $\left(\gamma_{iE}^{A}\right)$ TFPG (%)

| **Countries** | **Traditional TFPG** | **externality-adjusted TFPG** |
|---|---|---|
| *CANADA* | 0.670 | -1.979 |
| *U.S.A.* | 0.275 | -2.206 |
| *AUSTRIA* | 0.635 | -0.779 |
| *BELGIUM* | 1.079 | -1.039 |
| *DENMARK* | 0.321 | -1.289 |
| *FINLAND* | 1.144 | -1.107 |
| *FRANCE* | 0.705 | -0.778 |
| *GREECE* | 0.831 | -0.479 |
| *ITALY* | 1.537 | 0.387 |
| *LUXEMBOURG* | 1.699 | -2.580 |
| *PORTUGAL* | 1.690 | 0.649 |
| *SPAIN* | 0.415 | -0.695 |
| *SWEDEN* | -0.040 | -2.028 |
| *SWITZERLAND* | -0.059 | -1.122 |
| *U.K* | 0.859 | -0.896 |
| *JAPAN* | 1.646 | 0.235 |
| *ICELAND* | 0.473 | -2.533 |
| *IRELAND* | 1.638 | -0.172 |
| *NETHERLANDS* | 0.489 | -1.414 |
| *NORWAY* | 1.564 | -0.247 |
| *AUSTRALIA* | 0.567 | -1.226 |
| *MEXICO* | 0.330 | -0.814 |
| *TURKEY* | 1.420 | 0.214 |
| **Average** | **0.865** | **-0.952** |

The pattern is very similar to the result obtained in table 2. When the externality is internalized at 20\$/t$CO_2$ only four externality-adjusted TFPG estimates remain positive. Sensitivity analysis was performed using two arbitrary values for $\tau$, $\tau = 10\$/tCO_2$ and $\tau = 5\$/tCO_2$. The results indicate that the externality-adjusted TFPG estimates are positive for all countries when $\tau = 5\$/tCO_2$.

## 4 Concluding Remarks

This paper seeks to extend the traditional measurement of TFPG by taking into account the use of the environment, proxied by the use of energy, as an input in the production process which is not paid its social cost in the absence of environmental policy. We obtain externality-adjusted TFPG estimates by subtracting from output growth, the contribution of the unpaid part of energy which is associated with $CO_2$ emissions created during the production process, but which are not accounted for in the traditional TFPG measurements due to the lack of environmental policy. We use estimates of the marginal damages from $CO_2$ emissions to value the uninternalized part of energy. Our results indicate that our externality-adjusted TFPG measurements could be significantly different from traditional TFPG estimates depending on the the marginal $CO_2$ emission damages. If this value is close to 20\$/t$CO_2$ then the TFPG takes negative values during the sample period. That is, when each input, including environment, used in the production process is fully paid for its contribution to total output growth, then no TFPG can be detected. Thus our result suggests that uninternalized environmental externalities at a global level might be another reason for having negative TFPG estimates along with institutional changes and conflicts suggested by Baier et al. (2006). Our results seems therefore to support the idea that part of what has been regarded as TFPG could be the "unpaid" part of the environment use in production. This effect counterbalances the positive impact of technology and knowledge accumulation. Whether this effect is sufficiently large so that TFPG is non existent for a certain time period is an issue that largely depends on the estimates of environmental damages. Nevertheless, there seems to be strong empirical support to the idea that at least part of what has been thought as TFPG is the unaccounted use of the environment in the growth process.

**Appendix 1**

*Derivation of the Social Planner's Problem*

Net investment is total output minus consumption, energy cost, and depreciation of human and man made capital. Capital accumulation in per worker terms, assuming that the two capital goods depreciate at the same constant rate $\delta$, (Barro and Sala-i-Martin, 2004) is given by:

$$\dot{k} + \dot{h} = y - c - p_E e_L - (\eta + \delta)(k + h) \tag{37}$$

where $p_E$ is the price of energy in terms of consumption. Set $k = \hat{k}e^{\xi t}$ and $h = \hat{h}e^{\xi t}$, $c = \hat{c}e^{\xi t}$ and $e_L = \hat{e}_L e^{\xi t}$ so that $\dot{k} = \dot{\hat{k}}e^{\xi t} + \xi \hat{k} e^{\xi t}$ and $\dot{h} = \dot{\hat{h}}e^{\xi t} + \xi \hat{h} e^{\xi t}$. Substituting $\dot{k}$ and $\dot{h}$ in (37) and dividing by $e^{\xi t}$ we obtain:

$$\dot{\hat{k}} + \dot{\hat{h}} = e^{(\zeta - \xi + a_1 \xi + a_2 \xi)t} \hat{k}^{a_1} \hat{h}^{a_2} E^{a_4} - \hat{c} - p_E \hat{e}_L - (\eta + \delta + \xi)(\hat{k} + \hat{h})$$

to make the above equation time independent we choose $\xi$ such that $\zeta - \xi + a_1\xi + a_2\xi = 0$ or $\xi = \frac{\zeta}{1-a_1-a_2} = \frac{xa_3 + a_4(b-n)}{1-a_1-a_2}$. Then,

$$\dot{\hat{k}} + \dot{\hat{h}} = \hat{k}^{a_1} \hat{h}^{a_2} E^{a_4} - \hat{c} - p_E \hat{e}_L - (\eta + \delta + \xi)(\hat{k} + \hat{h}) \tag{38}$$

We assume that the allocation between physical and human capital is such that the marginal products for each type of capital are equated in equilibrium if both forms of investment are used (Barro and Sala-i-Martin, 2004).[17] The equality between marginal products implies a one to one relationship between physical and human capital, or:

$$a_1 \frac{\hat{y}_t}{\hat{k}_t} - \delta = a_2 \frac{\hat{y}_t}{\hat{h}_t} - \delta,\ \hat{h} = \frac{a_2}{a_1}\hat{k},\ \dot{\hat{h}} = \frac{a_2}{a_1}\dot{\hat{k}} \tag{39}$$

Using (39) in (38) we obtain:

$$\begin{aligned} \dot{\hat{k}} &= \tilde{A}\hat{k}^{\beta}E^{a_4} - \alpha\hat{c} - p_E \alpha \hat{e}_L - (\eta + \delta + \xi)\,\hat{k}, \qquad (40) \\ \tilde{A} &= \left(\frac{a_2^{a_2} a_1}{a_1^{a_2}(a_1 + a_2)}\right),\ \beta = a_1 + a_2,\ \alpha = \left(\frac{a_1}{a_1 + a_2}\right) \end{aligned}$$

By slightly abusing notation and in order to simplify relationships we keep using in the text $\hat{c}$ and $\hat{e}_L$, instead of $\alpha\hat{c}$ and $\alpha\hat{e}_L$ in the capital accumulation equations similar to (40) since the results are not affected. Considering a utility function $U(c,S) = \frac{1}{1-\theta}c^{1-\theta}S^{-\gamma} \quad \theta, \gamma > 0$ we obtain using the substitution $c = \hat{c}e^{\xi t}$.

$$\begin{aligned} U(c,S) &= \frac{1}{1-\theta}c^{1-\theta}S^{-\gamma} = \frac{1}{1-\theta}\left(\hat{c}e^{\xi t}\right)^{1-\theta} S^{-\gamma} = \qquad (41) \\ &= e^{(1-\theta)\xi t}\frac{1}{1-\theta}\hat{c}^{1-\theta}S^{-\gamma} = e^{(1-\theta)\xi t}U(\hat{c},S) \end{aligned}$$

Using (13), (41), (12), and (40) the social planners problem can be written as (15)

## Appendix 2

*Consumers*

[17]This substitution is convenient since by adopting it we do not need a separate state equation for human capital. It does not however affect the basic results regarding the interpretation of the unpaid part of energy associated with emissions generated by energy use.

Defining the current value Hamiltonian for the problem as:

$$H = U(c, S) + \pi (w + ra - c + na + \tau z) \tag{42}$$

standard optimality conditions imply:

$$U_c(c, S) = \pi \ , \ U_{cc}(c, S)\,\dot{c} = \dot{\pi} \tag{43}$$

$$\dot{\pi} = (\rho - r)\,\pi \text{ or} \tag{44}$$

$$\frac{\dot{c}}{c} = \frac{1}{\theta}(r - \rho) \tag{45}$$

*Firms*

The representative firm maximizes profits (7) assuming that physical capital, human capital and loans are perfect substitutes as stores of value we have $r = R_K - \delta = R_H - \delta$.The profit function for the firm can be written in per worker terms, using the Cobb-Douglas specification and setting $k = \hat{k}e^{\xi t}$, $h = \hat{h}e^{\xi t}$, and $\zeta - \xi + a_1\xi + a_2\xi = 0$, $\xi = \zeta - a_1\xi - a_2\xi$ as:

$$\frac{\Pi}{L} = e^{\xi t}\left[f\left(\hat{k}, \hat{h}, E\right) - R_K\hat{k} - R_H\hat{h} - we^{-\xi t} - p_E\hat{e}_L - \tau\frac{\psi(E)}{L}e^{-\xi t}\right] \tag{46}$$

$$\zeta = \xi - a_1\xi - a_2\xi \tag{47}$$

In equilibrium firms take $R_K, R_H, w$, $p_E$ and $\tau$ as given and maximize for any given level $\hat{l} = Le^{\xi t}$ by setting:

$$f_{\hat{k}} = R_K = r + \delta \tag{48}$$

$$f_{\hat{h}} = R_H = r + \delta \tag{49}$$

$$f_E = \frac{p_E + \tau\psi'(E)}{\hat{l}} \Rightarrow f_E\hat{l} = p_E + \tau\psi'(E)^{18} \tag{50}$$

$$e^{\xi t}\left[f\left(\hat{k},\hat{h},E\right) - f_{\hat{k}}\hat{k} - f_{\hat{h}}\hat{h} - \left(p_E + \tau\psi'\left(E\right)\right)e_L e^{-\xi t}\right] = w, \tag{51}$$

$$\hat{e}_L = e_L e^{-\xi t} = \frac{E}{L}e^{-\xi t} \tag{52}$$

$$e^{\xi t}\left[f\left(\hat{k},\hat{h},E\right) - f_{\hat{k}}\hat{k} - f_{\hat{h}}\hat{h} - \left(f_E\hat{l}\right)e_L e^{-\xi t}\right] = w \tag{53}$$

Competitive equilibrium implies that profits are zero. By substituting (48)-(53) into (46) the zero profit condition implies:

$$\begin{aligned} & f\left(\hat{k},\hat{h},E\right) - R_K\hat{k} - R_H\hat{h} \\ & -e^{\xi t}\left[f\left(\hat{k},\hat{h},E\right) - f_{\hat{k}}\hat{k} - f_{\hat{h}}\hat{h} - \left(p_E + \tau\psi'\left(E\right)\right)e_L e^{-\xi t}\right]e^{-\xi t} - \\ & p_E e_L e^{-\xi t} - \tau\frac{\psi\left(E\right)}{L}e^{-\xi t} = 0 \end{aligned} \tag{54}$$

For the zero profit condition to hold it is necessary that:

$$\tau\psi'\left(E\right)\frac{E}{L}e^{-\xi t} = \tau\frac{\psi\left(E\right)}{L}e^{-\xi t}$$

which implies that

$$\psi'\left(E\right)E = \psi\left(E\right) \text{ or } \frac{d\psi\left(E\right)}{dE}\frac{E}{\psi\left(E\right)} = 1 \tag{55}$$

Therefore, existence of competitive equilibrium when the emissions embodied to energy are taxed, requires that the emission function has unit elasticity with respect to energy or that it can be written as $Z = \sigma E$.

*Equilibrium*

In equilibrium $a = k + h$ so $\hat{a} = \hat{k} + \hat{h}$ . Then the flow budget constraint:

$$\dot{a} = w + ra - c - na + \tau z \tag{56}$$

can be written as:

$$\dot{k} + \dot{h} = w + r\,(k + h) - c - n\,(k + h) + \tau z \tag{57}$$

Setting as before $k = \hat{k}e^{\xi t}$ and $h = \hat{h}e^{\xi t}$, $c = \hat{c}e^{\xi t}$, and taking the time derivatives of $k$ and $h$ we obtain:

$$\begin{aligned} &\dot{\hat{k}}e^{\xi t} + \xi\hat{k}e^{\xi t} + \dot{\hat{h}}e^{\xi t} + \xi\hat{h}e^{\xi t} = \\ &w + r\left(\hat{k}e^{\xi t} + \hat{h}e^{\xi t}\right) - \hat{c}e^{\xi t} - n\left(\hat{k}e^{\xi t} + \hat{h}e^{\xi t}\right) + \tau z \end{aligned} \tag{58}$$

substituting (48)-(51) into (58), and using in equilibrium $r = f_{\hat{k}} - \delta = f_{\hat{h}} - \delta$, $f_E\hat{l} = p_E + \tau\psi'(E)$, $\hat{l} = Le^{\xi t}$ we obtain:

$$\dot{\hat{k}} + \dot{\hat{h}} = f\left(\hat{k}, \hat{h}, E\right) - (n + \delta + \xi)\,(\hat{k} + \hat{h}) - \hat{c} - p_E\hat{e}_L - \tau\psi'\,(E)\,\frac{E}{L}e^{-\xi t} + \tau\frac{\psi\,(E)}{L}e^{-\xi t}$$

using (55) we have:

$$\dot{\hat{k}} + \dot{\hat{h}} = \hat{k}^{a_1}\hat{h}^{a_2}E^{a_4} - \hat{c} - p_E\hat{e}_L - (\eta + \delta + \xi)(\hat{k} + \hat{h}) \tag{59}$$

Using as above the assumption that in equilibrium the allocation between physical and human capital is such that the marginal products for each type of capital are equated if we use both forms of investment, we have as before $a_1\frac{\hat{y}_t}{\hat{k}_t} - \delta = a_2\frac{\hat{y}_t}{\hat{h}_t} - \delta$ and $\hat{h} = \frac{a_2}{a_1}\hat{k}$ , $\dot{\hat{h}} = \frac{a_2}{a_1}\dot{\hat{k}}$. Then (59) becomes

$$\dot{\hat{k}} = f\left(\hat{k}, E\right) - \alpha\hat{c} - \alpha p_E\hat{e}_L - (\eta + \delta + \xi)\,\hat{k}\ ,\quad f\left(\hat{k}, E\right) = s\tilde{A}\hat{k}^{\beta}E^{a_4} \tag{60}$$

which is the social planners transition equation (17).

Setting $c = \hat{c}e^{\xi t}$ and $\dot{c} = \xi\hat{c}e^{\xi t} + \dot{\hat{c}}e^{\xi t}$ into (45) we obtain

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{1}{\theta}\left[f_{\hat{k}}\left(\hat{k}, E\right) - \rho - \delta - \xi\theta\right] - \frac{U_{\hat{c}S}\left(\hat{c}, S\right)}{U_{\hat{c}\hat{c}}\left(\hat{c}, S\right)}\dot{S} \tag{61}$$

Under optimal taxation $\tau = -\lambda\hat{l}/p$. We have therefore, from the social planner's problem that $f_E\left(\hat{k}, E\right)\hat{l} = p_E - \left(\lambda\psi'\left(E, t\right)\hat{l}\right)/p$ with $p = U_{\hat{c}}\left(\hat{c}, S\right),\ E = g\left(\hat{k}, \lambda, p, \hat{l}\right)$ while from the firms problem, (46), we have $f_E\left(\hat{k}, E\right)\hat{l} = p_E + \tau\psi'(E)$. The optimality conditions for the choice of energy coincide. It should be noticed that $\tau/\hat{l} = -\lambda/p$, that is the tax per effective worker is equal to the shadow cost of emissions expressed in utility terms.

Substituting $E$ into (61) and (12) we obtain:

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{1}{\theta}\left[f_{\hat{k}}\left(\hat{k}, g\left(\hat{k}, \lambda, U_{\hat{c}}\left(\hat{c}, S\right), \hat{l}\right)\right) - \rho - \delta - \theta\xi\right] - \frac{U_{\hat{c}S}\left(\hat{c}, S\right)}{U_{\hat{c}\hat{c}}\left(\hat{c}, S\right)}\dot{S} \tag{62}$$

$$\dot{S} = \psi\left[g\left(\hat{k}, \lambda, U_{\hat{c}}\left(\hat{c}, S\right), \hat{l}\right)\right] - mS \tag{63}$$

The dynamic system (60), (62) and (63) determines the evolution of $\left(\hat{c}, \hat{k}, S\right)$ in a decentralized competitive equilibrium under optimal emission taxation. By comparing them with (22), (24) and (25) it is clear that the path of the decentralized competitive equilibrium under optimal emission taxation coincides with the socially optimal path.

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