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Bush Meets Hotelling: Effects of  
Improved Renewable Energy Technology  
on Greenhouse Gas Emissions

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# Bush Meets Hotelling: Effects of Improved Renewable Energy Technology on Greenhouse Gas Emissions

## **Abstract**

Fossil fuels are non-renewable carbon resources, and the extraction path of these resources depends both on present and future demand. When this “Hotelling feature” is taken into consideration, the whole price path of carbon fuel will shift downwards as a response to the reduced cost of the renewable substitute. An implication of this is that greenhouse gas emissions in the near future may increase as a response to the reduced cost of the renewable substitute. If this is the case, increased climate costs may outweigh the benefits of reduced costs of a substitute, thus reducing overall social welfare.

# 1 Introduction

The Bush Administration has argued that a Kyoto type agreement will not achieve much unless it covers almost all countries in the world. Instead, it has proposed support for R&D directed towards lowering costs of alternative energy sources. However, there are many reasons why such a focus on technology development as an alternative to policies and agreements focusing directly on emissions will not give significant reductions of greenhouse gas emissions. The present paper discusses one such reason: When the supply side of fossil fuels is taken into consideration, fossil fuel prices may decline as a consequence of improved renewable energy technology.

The most important contribution to the climate problem is CO<sub>2</sub> from the combustion of fossil fuels. The climate problem is thus to a large extent caused by extracting carbon resources and transferring them to the atmosphere. Logically, any discussion of the climate problem therefore ought to be intimately linked to a discussion of the extraction of carbon resources. In spite of this obvious fact, surprising little of the literature makes this link. However, there are important exceptions. Early contributions making this link include Sinclair (1992), Ulph and Ulph (1994) and Withagen (1994). However, most of this literature and more recent literature makes this link in the context of discussing optimal climate policies<sup>1</sup>, in spite of the fact that a broad and globally cost-effective international climate agreement seems to be unlikely in the near future. There is little work making the link between climate policies and exhaustible resources when policies are non-optimal or international agreements are incomplete. Again, there are exceptions: Peter Bohm (1994) was probably the first to point out carbon leakage effects via the carbon resource market when only a subset of countries participate in a climate agreement. A similar issue is treated by Hoel (1994), but only within a static framework. Sinn (2008) has recently shown that when the

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<sup>1</sup>See e.g Hoel and Kverndokk (1996), Tahvonen (1997), Chakravorty et al. (2006).

exhaustibility of carbon is taken into consideration, climate policies that are not optimally designed may increase emissions instead of reducing them. Finally, while various effects of technology change have recently been discussed in the context of climate policy, most of this literature ignores the fact that such technology change may have consequences for the supply of carbon. An exception is Strand (2007), who shows that a technology agreement that will make carbon redundant in the future may increase present carbon emissions. The present paper makes a less drastic assumption about technology improvement: Although technology improvement will lower the costs of renewable energy, carbon resources will still have lower costs than the substitute. The consequences of such a technology improvement are analyzed for a situation where different countries (or groups of countries) have climate policies of differing ambition levels, but where there does not exist an efficient global climate agreement. An important insight from this analysis is that climate costs may increase as a consequence of the improved technology of renewable energy.

The rest of the paper is organized as follows. In section 2 it is shown that even if a given amount of carbon is eventually extracted, the time profile of the carbon extraction is important from a climate point of view. Any postponement in extraction is likely to lower climate costs, even if total emissions over time are given by the available carbon resources.

The model for the market of fossil fuels is presented in Section 3. On the supply side, there is a given and known stock of carbon resources (fossil fuels) that are supplied competitively. Moreover, there exists a perfect substitute for fossil fuels, with a constant unit cost and supplied competitively. On the demand side, each country is assumed to have some willingness to pay (WTP) for reducing carbon emissions, and each country sets a carbon tax equal to its WTP. (Alternatively, each country could have a domestic quota system giving a quota price equal to its WTP.) There will thus be a distribution of carbon taxes (or quota prices) across countries. This substitute will be

adopted by countries for which the fuel price plus the carbon tax exceeds the cost of producing the substitute. However, in countries that have a lower WTP the fuel price including the carbon tax will be lower than the cost of the substitute, and these countries will not adopt the substitute. These properties of the demand side are used to determine the market equilibrium for carbon extraction.

Section 4 analyzes the effects of an improvement of the technology for producing the substitute, thus lowering its cost. If the fossil fuel price were unaffected, this cost reduction would induce some countries to switch from fossil fuels to the substitute, so that global carbon emissions would decline. However, fossil fuels are non-renewable, and the competitive supply gives a price path of the fuel which depends both on present and future demand. When this “Hotelling feature” is taken into consideration, the whole price path of the carbon resource will shift downwards as a response to the reduced cost of the substitute. An implication of this, in combination with the absence of an efficient climate agreement, is that it is no longer obvious that greenhouse gas emissions decline in the near future. I show that carbon emissions are more likely to increase in the near future the higher is the elasticity of demand for the sum of carbon resource and the substitute and the scarcer the carbon resource is.

In a first-best social optimum, reduced costs of a substitute will always increase overall welfare. The welfare of fossil fuel suppliers will go down, but the increased welfare of all others will be larger than the welfare loss of the fuel suppliers. Section 4 demonstrates that without an efficient climate agreement, climate costs may increase so much that overall social welfare declines as a consequence of reduced costs of a substitute.

Section 5 shows that the results from Section 4 remain valid under the important extensions of the substitute being imperfect and extraction costs rising with accumulated extraction.

Section 6 concludes: An important policy implication is that technolog-

ical improvement in the production of renewable energy cannot in itself be trusted as a good mechanism to reduce greenhouse gas emissions. While technology improvement may be an important feature of international climate cooperation, it is important that this cooperation also focuses directly on emission reductions.

## 2 Climate costs and carbon resource extraction

In the subsequent analysis, it is assumed that the total amount of carbon resources are given, and that all of this carbon will eventually be extracted and thus emitted into the atmosphere.<sup>2</sup> Total emissions over all future years are thus given. In spite of this, the profile of the carbon extraction is important from a climate point of view. A rapid increase of carbon in the atmosphere will gradually decline over time, as it is transferred to other sinks. However, a significant portion (about 25% according to e.g. Archer, 2005) remains in the atmosphere for ever (or at least for thousands of years). Thus if a fixed amount of carbon, denoted  $C_0$ , is extracted over any time period, this will give a long-run increase of about  $C_0/4$  in the atmosphere. With a sufficiently slow rate of carbon extraction, carbon in the atmosphere will grow gradually and monotonically until its long-run level  $S^*$  is reached (asymptotically if carbon extraction declines asymptotically towards zero). This is illustrated by curve A in Figure 1, where  $S(0)$  is the amount of carbon in the atmosphere at our initial date 0 (so  $S^* \approx S(0) + C_0/4$ ). Clearly, such a development of carbon in the atmosphere will be associated with a gradually changing climate. With a higher rate of extraction, the carbon in the atmosphere will increase more rapidly, and will overshoot its long-run value  $S^*$ , as curve B in Figure 1. This will give a considerably faster climate change, probably

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<sup>2</sup>The implications of relaxing this assumption are discussed in Section 5.2.

with temperatures above the slow extraction path for several centuries. One can argue strongly that the climate costs associated with the rapid extraction path are much higher than the climate costs associated with the climate development associated with the slow extraction path, even if discounting is ignored. This seems particularly likely if some effects of climate changes are irreversible, and if the speed of climate change is also an important consideration.<sup>3</sup> Appendix I gives a formal treatment of the effects of postponing emissions under the frequently used assumption that climate costs at any time depend only on the stock of carbon in the atmosphere via its affect on the state of the climate. The climate cost at time  $\tau + \ell$ , where  $\ell$  is an exogenous time lag, is given as a function of the stock of carbon in the atmosphere at  $\tau$ , denoted  $D(S(\tau))$ . In Appendix I it is shown that a sufficient condition for climate costs to decline if emissions are postponed from  $t$  to a later date is that

$$\max_{\tau} \left[ \frac{D'(S(\tau))}{D'(S(t))} \right] < \frac{r + \delta}{(1 - \alpha)\delta} \quad (1)$$

for all  $\tau > t$ . In (1)  $\alpha$  is the share of carbon that remains in the atmosphere for ever, and  $\delta$  is the depreciation rate of the remaining share of carbon (see Appendix I for details). Condition (1) is more likely to hold the larger is the interest rate  $r$ . Consider therefore the case of a relatively small interest rate,  $r = 0.02$ . Moreover, let  $\alpha = 0.25$  and  $\delta = 0.013$  (cf. Appendix I). For these values the r.h.s. of (1) is 3.4. If  $D'(S(\tau))$  along the initial emission path never exceeds  $3.4 \cdot D'(S(t))$ , climate costs are therefore always reduced by a postponement in emissions. If e.g.  $D(S) = bS^2$ , this means that  $S(\tau) < 1.8 \cdot S(t)$  for all  $\tau > t$  is a *sufficient* condition for climate costs to decline as emissions are postponed. If we had  $r = 0.03$  instead of  $r = 0.02$ , the condition would instead be  $D'(S(\tau)) < 4.4 \cdot D'(S(t))$ , which with  $D(S) = bS^2$  would imply  $S(\tau) < 2.1 \cdot S(t)$  for all  $\tau > t$ . While these numbers suggest that the

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<sup>3</sup>Tahvonen (1995), Hoel and Isaksen (1995), and Hoel and Kverndokk (1996) explicitly consider the speed of climate change in their analyses.

sufficient condition (1) is likely to hold, it is not obvious that this is the case. However, the details in Appendix I suggest that a postponement of emissions may reduce climate costs even if this sufficient condition does not hold. In the subsequent sections I therefore assume that a postponement of emissions reduces climate costs.

### 3 The market for fossil fuels

The market for fossil fuels is modeled as a market for a homogeneous non-renewable carbon resource, given in fixed supply and with no extraction costs. The resource is supplied by competitive owners of the resource, and the equilibrium producer price  $p(t)$  therefore rises at the interest rate  $r$  as long as there are any remaining reserves.

The demand for carbon is given as the sum of demand from several countries. There is a perfect substitute for the carbon resource, supplied competitively at its unit cost  $b$ . Countries have identical gross utility functions depending on the sum of the use of carbon and the substitute,  $u(x+y)$ , where  $x$  and  $y$  are the use of carbon and the substitute, respectively.<sup>4</sup> If there were no environmental considerations, each country would therefore choose its  $x$  and  $y$  to maximize  $u(x+y) - px - by$ , subject to non-negativity constraints. However, countries have a WTP for reducing  $x$  that is equal to  $w$  per unit of  $x$ . This WTP may be equal to the marginal climate cost of the country, but it may also contain some elements of altruism so that the country has a WTP that exceeds the direct damage of its own emissions on itself. It is probably difficult to explain the relative high WTP of some European countries unless such an altruistic element is included.

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<sup>4</sup>Introducing differences in the  $u$ -functions across countries would complicate notation without changing any results.

Given a WTP equal to  $w$  per unit of emissions, the choice of  $x$  and  $y$  for a country maximizes  $u(x + y) - px - wx - by$ , implying that if  $p > b - w$ , the country uses only the substitute and not the carbon resource. If  $p < b - w$ ,  $x = D(p + w)$ , where  $D(\cdot) \equiv (u')^{-1}(\cdot)$ . The value of  $w$  is assumed to vary across countries.<sup>5</sup> This variation is modeled by a distribution function  $F(w)$  over  $[0, W]$ , giving us the proportion of countries that have WTP for reduced carbon use that does not exceed  $w$ .  $F(b - p)$  is therefore the proportion of that countries that use the carbon resource, while the proportion  $1 - F(b - p)$  use the substitute. Notice that if  $F(b) < 1$ , some countries will not use the carbon resource no matter how low its price  $p$  is.

Normalizing the size of countries so their total mass is one, the aggregate demand for carbon is given by

$$X = \int_0^{b-p} D(p + w) dF(w) = X(p, b) \quad (2)$$

Straightforward derivations reveal that

$$X_b(p, b) = D(b)F'(b - p) \quad (3)$$

and

$$X_p(p, b) = -X_b(p, b) + \int_0^{b-p} D'(p + w) dF(w) \quad (4)$$

Since  $F'$  is non-negative,  $X_b$  must also be non-negative, and strictly positive if  $F' > 0$ . The second term in (4) is negative since  $D' < 0$ , implying that  $X_p < 0$ . The aggregate demand function for carbon thus has the standard property that it is downward sloping as a function of the price. Moreover, this demand function will shift inwards if the cost of producing the substitute is reduced (unless  $F' = 0$ ).

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<sup>5</sup>Instead of the interpretation above of  $w$ , we could simply interpret  $w$  as an a carbon tax, determined through political processes in each country, and generally differing across countries.

The initial stock of unextracted carbon is denoted  $C_0$ . The Hotelling rule for the development of  $p(t)$  gives us the following two equilibrium conditions:

$$\int_0^T X(p(0)e^{rt}, b) = C_0 \quad (5)$$

$$p(0)e^{rT} = b \quad (6)$$

Equation (5) tells us that the sum of demand over all periods cannot exceed the available carbon resources, and (6) defines  $T$  as the time point when the producer price of carbon reaches the cost of producing the substitute.<sup>6</sup> These two equations determine  $T$  and  $p(0)$ , and once these two variables are determined the whole paths of  $p$  and  $X$  follow. Obviously, the equilibrium depends on the value of  $b$ .

## 4 Effects of lower costs of producing the substitute

Before giving a formal analysis, it is useful to give a graphical illustration of the effects of lowering  $b$ . In Figure 2, the initial value of  $b$  is  $b'$ , with a corresponding price path  $p'(t)$  and exhaustion date  $T'$ . Let  $b$  be reduced to  $b''$ , and imagine hypothetically that the date of exhaustion stays constant at  $T'$ . If this were the case, the new price path would be  $p^*(t)$ . But for  $p^*(t)$  to be the new equilibrium price path, the carbon resource must be exhausted exactly at  $T'$ . Is this possible? Along the path  $p^*(t)$  carbon demand will differ from the demand along the original path  $p'(t)$  for two reasons: First, it will be *higher* since the new price path is lower. Second, it

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<sup>6</sup>If  $F(w) = 0$  for  $w < \varepsilon$ , the demand for carbon will drop to zero when the carbon price reaches  $b - \varepsilon$ . This possibility is consistent with the equilibrium conditions given the way  $X(\cdot)$  is defined.

will be *lower* since  $b$  is lower, and  $b$  has a direct effect on carbon demand. If these two effects are equal, the hypothetical price path  $p^*(t)$  will also be the actual new equilibrium price path, and the date of exhaustion will be  $T'$  as before. Obviously, the two effects on demand will generally differ. If the price effect dominates, demand will be higher along  $p^*(t)$  than along  $p'(t)$ , so the resource will be exhausted before  $T$  is reached. To restore equilibrium in this case, the new equilibrium price path  $p''(t)$  must lie above  $p^*(t)$ , implying a new exhaustion date  $T'' < T'$ . Conversely, if the direct effect of reduced  $b$  dominates, demand will be lower along  $p^*(t)$  than along  $p'(t)$ , so there would be carbon resources remaining at  $T$ . To restore equilibrium in this case, the new equilibrium price path  $p''(t)$  must lie below  $p^*(t)$ , implying a new exhaustion date  $T'' > T'$ .

#### 4.1 Two special cases

Two special cases illustrate the two possibilities. Consider first the case in which the distribution function  $F(w)$  has the property that a share  $\gamma$  of the countries have  $w = 0$ , while the remaining countries have  $w > b$ . In this case there is no direct effect of  $b$  on demand, so that the price effect discussed above dominates. In this case  $T$  must therefore go down when  $b$  goes down. Formally, (5) can in this case be rewritten as

$$\int_0^T \gamma D(p(0)e^{rt}) = C_0$$

Together with (6), it is clear that a reduction in  $b$  must give a reduction in  $p(0)$  and in  $T$ .<sup>7</sup> This implies that the whole price path is moved down, and that the path for carbon extraction is shifted up (until the new  $T$ ). This latter effect is illustrated in Figure 3. The extraction path is originally given by  $X'(t)$ , with the carbon resource being depleted at  $T'$ . After  $b$  is

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<sup>7</sup>This result is well-known and can be found in most textbooks in resource economics, see e.g. xxx

reduced the new path is  $X''(t)$ , with the carbon resource being depleted at  $T''$ . As a consequence of the reduced cost of producing the substitute, some carbon extraction - and emissions - is thus moved from A to B in Figure 3. Notice that this result does not depend on the assumed heterogeneity across countries. If instead all countries had identical and small (or zero) values of  $w$ , we would get the same result: As long as  $w$  is sufficiently small, there will be no immediate reduction in demand as  $b$  becomes smaller. Therefore the extraction path must shift upward as illustrated in Figure 3 also in this case.

Consider next the special case in which the demand for carbon plus the substitute is independent of  $p$ . In this case there is no effect from the reduced price path from  $p'(t)$  to  $p^*(t)$  in Figure 1 on carbon demand. However, demand goes down as a consequence of  $b$  being reduced. In this case the new equilibrium price path  $p''(t)$  must therefore lie below  $p^*(t)$ , giving  $T'' > T'$ , as illustrated in Figure 4.<sup>8</sup> The changed extraction path implies that the extraction of some carbon has been moved from  $A$  to  $B$ , i.e. postponed.

## 4.2 The general case

We now turn to a more formal analysis to see how the exhaustion date  $T$  is affected by a reduction in  $b$ . Inserting (6) into (5) gives

$$\int_0^T X(be^{-r(T-t)}, b)dt = C_0 \quad (7)$$

which may conveniently be rewritten as

$$\int_0^T X(be^{-rz}, b)dz = C_0 \quad (8)$$

where  $z$  denotes time remaining until  $p$  reaches its upper limit  $b$ . Using the simplified notation  $X_p(z)$  and  $X_b(z)$  for  $X_p(be^{-rz}, b)$  and  $X_b(be^{-rz}, b)$ ,

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<sup>8</sup>This Figure illustrates the case of  $F(0) = 0$ , so that demand at  $p = b$  is zero. If instead  $F(0) > 0$ , i.e. some countries have zero WTP for reduced emissions, the extraction paths in Figure 4 would jump from positive values to zero at  $T'$  and  $T''$ , like in Figure 3.

straightforward derivations gives the following expression for  $\frac{dT}{db}$ :

$$\frac{dT}{db} = - [X(be^{-rT}, b)]^{-1} \int_0^T [X_p(z)e^{-rz} + X_b(z)] dz \quad (9)$$

Define  $I(z)$  as

$$I(z) = - \int_0^{b-be^{-rz}} D'(be^{-rz} + w)dF(w) \quad (10)$$

This expression gives the response of aggregate demand for the countries using carbon to a price change (measured positively). Using (10) and inserting (4) into (9), we obtain

$$\frac{dT}{db} = [X(be^{-rT}, b)]^{-1} \int_0^T [e^{-rz}I(z) - (1 - e^{-rz})X_b(z)] dz \quad (11)$$

The first term in square brackets in (11) is positive (except for a limiting case discussed below), while the second term (including the sign) is negative (except for a limiting case discussed below). The sign of  $\frac{dT}{db}$  is thus generally ambiguous.

In the special case illustrated by Figure 3 , demand for carbon was unaffected by a change in  $b$  for all  $p < b$ . For this special case we therefore had  $X_b = 0$ , implying that the second term in square brackets was zero. For this case it thus follows from (11) that  $\frac{dT}{db} > 0$ , as was previously shown. For the special case illustrated by Figure 4,  $D' = 0$  for all  $p < b$ . For this case  $I(z) = 0$  for all  $z$ , and it follows from (11) that  $\frac{dT}{db} < 0$ . A decline in  $b$  in this case therefore increases  $T$ .

We now turn to the general case. We have already shown that if  $D' = 0$  everywhere,  $T$  will increase as a response to a reduction in  $b$ . By continuity, the same must be true for  $D'$  sufficiently small. In Appendix II it is formally shown that for a given level of demand for the substitute for  $p = b$ ,  $\frac{dT}{db}$  will be positive if demand (for carbon plus the substitute) is sufficiently price sensitive for  $p < b$ . With a sufficiently high price sensitivity,  $T$  will therefore

go down as  $b$  is reduced.

The sign of  $\frac{dT}{db}$  depends not only on the distribution function  $F(\cdot)$  and the demand function  $D(\cdot)$ , but also on the amount of unextracted carbon ( $C_0$ ). For a very large value of  $C_0$ ,  $T$  will be very large, implying that the weight  $e^{-rz}$  in (11) will be close to zero for most the  $z$ -values in the integral in (11). The second term in this integral will therefore dominate, implying that  $\frac{dT}{db} < 0$ . This has a natural interpretation: For a very large unextracted resource stock, the resource rent will be close to zero. There is in this case not much scope in terms of a reduced resource price as a response to a lower value of  $b$ . In this case the direct affect of reduced  $b$  therefore dominates the aggregate demand response, implying that resource extraction is slowed down and thus lasts longer.

It follows from the discussion above that the date of resource exhaustion ( $T$ ) may either decline or increase as a consequence of a reduction in  $b$ . It seems reasonable to expect the extraction profile to change in a manner similar to Figure 3 if  $T$  is reduced (with or without a discontinuous jump at the date of exhaustion, cf. footnote 7), and similar to Figure 4 if  $T$  is increased (with or without a discontinuous jump at the date of exhaustion, cf. footnote 7). However, in Appendix II it is shown that it is possible for the extraction paths corresponding to two values of  $b$  to intersect more than once. This case seems less likely to occur the stronger the effect  $b$  has on  $T$ .

### 4.3 Welfare effects

When the cost of the substitute is reduced, there are three effects on social welfare. First, there is the direct effect of the reduction of  $b$ . The direct effect of the reduction of  $b$  is an increase in social welfare for all countries, since all countries sooner or later use the substitute. The second effect of reduced  $b$  is that this has an impact on the price path of the carbon resource: the lower is  $b$ , the lower is this price path. I return to this effect below.

Finally, there is the climate effect of the reduction in  $b$ . The previous sections demonstrated that a reduction in the cost of producing a substitute may either advance or postpone carbon extraction, and thus either contribute negatively or positively to the overall change in social welfare.

Let  $\Delta p(t) < 0$  denote the (small) change in the price path following from a (small) reduction in  $b$ . From the Hotelling Rule we have  $\Delta p(t) = e^{rt} \Delta p(0)$ . The value of the resource for the resource owner is  $p(0)C_0$ , so the change in this value is  $C_0 \Delta p(0)$ , which is negative.

The welfare level of a single country is

$$z = \int_0^{\infty} e^{-rt} [u(x(t) + y(t)) - p(t)x(t) - by(t)] dt$$

and the change in this welfare level due to the price change is (remembering that  $\Delta p(t) = e^{rt} \Delta p(0)$ )

$$\Delta z = \Delta p(0) \int_0^{\infty} [u'(x(t) + y(t)) - p(t)] \frac{dx(t)}{dp(t)} dt - \Delta p(0) \int_0^{\infty} x(t) dt \quad (12)$$

Consider first the second term in (12). The sum over all countries of the integral is simply  $C_0$ , which implies that the sum of the second term over all countries is equal to  $-C_0 \Delta p(0)$ . The sum of this term and the value change for the resource owner is thus equal to zero. Moreover, the sum of the first term in (12) over all countries would be equal to zero if all countries had chosen  $u' = p$ . In this case we would get the well-known result that price changes have no effect on aggregate welfare, although such price changes usually affect the distribution of social welfare. However, in our case the term in square brackets in (12) is equal to  $w$ , so that the first term in (12) is equal to

$$w \Delta p(0) \int_0^{\infty} \frac{dx(t)}{dp(t)} dt \quad (13)$$

which is positive since the derivative in the integral is negative. The overall

welfare effect of the price reduction caused by the reduction in  $b$  is therefore positive.

To conclude so far: Ignoring changes in climate costs, the reduction in  $b$  has an unambiguously positive effect on overall social welfare, although the owners of the carbon resource have a loss. If resource extraction is postponed, so that climate costs are reduced, this gives an additional increase in aggregate social welfare. However, if resource extraction is advanced as a consequence of the reduction in  $b$ , climate costs increase. In this case there are two positive effects on aggregate social welfare, and one negative. This negative effect may be so large that it dominates the two positive effects. This will be the case if there is a group of countries with sufficiently high WTP for reduced emissions (and so high that these countries are not using the carbon resource whatever the value of  $b$ ). In this case the sum of

the positive terms (13) may be small or zero, since this term is zero for the countries with the high WTP and  $x = 0$ . For sufficiently large increases in climate costs for some countries, these cost increases may be larger than the direct benefits for all countries of the reduction in  $b$ .

## 5 Extensions

The analysis was based on several simplifying assumptions. One of the most important assumptions was the assumption that there was a perfect substitute for the carbon resource so that there would be no demand for carbon at prices above  $b$ . A second important assumption was that there were no extraction costs. We shall see that the results derived also hold if these assumptions are relaxed.

## 5.1 An imperfect substitute

Carbon resources are used for many purposes, and it is likely to be easier to find good substitutes for some uses than for others. For users of electricity it makes no difference if the electricity is based on carbon or on alternatives such as wind, solar or nuclear. The choice between carbon and other energy sources in this case is thus simply a question of relative costs. A similar argument can be made for a significant part of other stationary energy uses. Finding good alternatives for direct use of natural gas in households and for oil in transportation is likely to be considerable more difficult. Even if one in the future has a good low-cost primary energy source, energy carriers such as electric batteries or hydrogen have several disadvantages compared with oil in the transportation sector. While the use of carbon might be eliminated as a large-scale stationary energy source in the future when the price of carbon is sufficiently high compared to its substitutes, there is likely to be continued demand for carbon resources for transportation and for some small-scale stationary uses even at quite high prices.

For the reasons given above, assume that each country in addition to its demand  $D(p + w)$  has some demand  $E(p + w)$  that will not be replaced by the substitute we are considering, no matter how low  $b$  is. Aggregating over all countries, this additional demand is

$$Y = \int_0^W E(p + w) dF(w) = Y(p) \quad (14)$$

With this change, the equilibrium condition in the market for carbon, previously given by (7), is changed. It is useful to first consider the equilibrium condition that must hold after the date  $T$  when the price of carbon reaches  $b$ . In the present case, the carbon price will continue to rise after this date, and since the equilibrium price cannot jump at  $T$ , it must be given by  $b^{r(t-T)}$  at any date after  $T$ .<sup>9</sup> Total resource use after  $T$ , denoted  $C_T$ , must

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<sup>9</sup>See e.g. Hoel (1984) for a more extensive discussion of market equilibria with this

therefore satisfy the equilibrium condition

$$\int_T^\infty Y(be^{r(t-T)})dt = C_T \quad (15)$$

which defines  $C_T$  as an decreasing function of  $b$ .

The equilibrium condition for the period  $[0, T]$  is given by the following slight modification of (7):

$$\int_0^T [Y(be^{-rz}) + X(be^{-rz}, b)] dz = C_0 - C_T(b) \quad (16)$$

This equation determines  $T$  as a function of  $b$ , and we now find

$$\frac{dT}{db} = Q \left\{ -C'_T(b) + \int_0^T [e^{-rz}(I(z) + J(z)) - (1 - e^{-rz})X_b(z)] dz \right\} \quad (17)$$

where

$$Q = [Y(be^{-rT}) + X(be^{-rT}, b)]^{-1} > 0$$

and  $J(z)$  is defined similarly to  $I(z)$ :

$$J(z) = - \int_0^W E'(be^{-rz} + w)dF(w) \quad (18)$$

The two new terms  $-C'_T(b)$  and  $J(z)$  are both positive, making  $\frac{dT}{db} > 0$  more likely. With an imperfect substitute it thus seems more likely that  $T$  will decline as a consequence of the reduction in  $b$  than in the case of a perfect substitute. Due to continuity (i.e. as  $Y(p)$  approaches zero for all  $p > b$ ), however, the possibility of  $\frac{dT}{db} < 0$  exists also in the present case.

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type of an imperfect substitute.

## 5.2 Extraction costs

Adding a constant unit cost of extraction will not change anything in our analysis. A more interesting case is when the unit cost of extraction is increasing in accumulated extraction, denoted  $c(A(t))$  where  $A(t)$  is accumulated extraction. If there is an absolute limit on total carbon extraction also in this case (*i.e.*  $A(t) \leq C_0$  for all  $t$ ), and this limit is binding, there will be no significant changes in our analysis (although the producer price development will be slightly different from the Hotelling rule of  $p(t)$  rising at the rate  $r$ ). A more interesting case is when the total amount extracted is determined by  $c(A) = b$ , as illustrated in Figure 5. In this case the resource price will be rising and lie between  $c(A)$  and  $b$  until  $p$  and  $c(A)$  reach  $b$  (see e.g. Heal (1976) and Hanson (1980)). The total extraction is determined by the equation  $c(A) = b$ , so that a reduction in  $b$  from  $b'$  to  $b''$  will reduce total extraction from  $A'$  to  $A''$ , see Figure 5. In this case the climate effect of reduced  $b$  is therefore more favorable than in the case treated previously. However, also in this case it is possible that a reduction in  $b$  will give an advancement in extraction and emissions, and that this may more than outweigh the positive effect of less total extraction. This is more likely the larger is  $c'(A)$  near  $c(A) = b$ , since a large value of  $c'$  implies that  $A$  is only slightly reduced when  $b$  declines. The limiting case of an absolute limit on  $A$  is simply that  $c'(A)$  approaches infinity.

## 6 Conclusions

Improved technology, of any kind, is obviously potentially good for a society. When resources are efficiently managed, one can always increase social welfare if technology is improved. However, with various types of market failures or regulatory failures, social welfare need not necessarily improve with improved technology. The present paper gives one example of this general feature. Without good global policies for managing the climate problem,

improved technology of renewable energy may increase climate costs. Moreover, the increased climate costs might even outweigh the direct benefits of the improved technology.

The political implication of this result is *not* that one should try to slow down technological progress. However, a belief that technological progress in itself can help solve the problem of climate change is much too optimistic, to say the least. Improved technology of various types of renewable energy can be an important ingredient of mitigating the climate problem, but cannot be expected to be an *alternative* to a good international climate agreement directed directly towards emission reductions. As a *supplement* to such a climate agreement, however, improved technology of renewable energy can play an important role.

## Appendix I: Climate costs

Let  $T(t)$  be the global average temperature change from some base year to time  $t$ . A standard formulation of the costs of climate change at  $t$  is that these are given by a function  $\tilde{D}(T(t))$  where  $\tilde{D}' > 0$ , and it is often also assumed that  $\tilde{D}'' > 0$ . Although I shall use this "standard" formulation, it is likely to be too simple to capture all concerns about climate change. In particular, the formulation ignores how fast climate change has occurred. Moreover, some consequences of climate change will be more or less irreversible and thus remain even if global temperature should decline after a rise. An obvious example is sea level rise due to melting of ice on Greenland or parts of Antarctica. Ignoring these complications, the present value of all costs associated with climate change is at the initial date 0 given by

$$V_0 = \int_0^{\infty} e^{-r\tau} \tilde{D}(T(\tau)) d\tau \quad (19)$$

Temperature change depends on the development of the amount of carbon

in the atmosphere. A long-run increase in the amount of carbon from its preindustrial level  $S_0$  to a stationary value  $S^*$  gives the following long-run increase in temperature (see e.g. xxx):

$$T^* = \frac{k}{Ln2} Ln \left( \frac{S^*}{S_0} \right)$$

where  $k$  is the so-called climate sensitivity. If e.g. the stock of carbon rises to twice its preindustrial level, the temperature will rise by  $k$ . According to IPCC (2007),  $k$  is "*likely*" to be in the range 2°C to 4.5°C, with a best estimate of about 3°C, and *very unlikely* to be less than 1.5°C".

Actual temperature responds to changes in the atmospheric concentration with quite a long time lag. Here I model this in the simplest possible way, namely by assuming a constant time lag  $\ell$  between  $S(t)$  and  $T(t)$ , implying

$$T(t) = \frac{k}{Ln2} Ln \left( \frac{S(t - \ell)}{S_0} \right)$$

Inserting this into (19) gives us

$$V_0 = \int_0^\infty e^{-r\tau} D(S(\tau - \ell)) d\tau \tag{20}$$

where

$$D(S(\tau - \ell)) = \tilde{D} \left( \frac{k}{Ln2} Ln \left( \frac{S(\tau - \ell)}{S_0} \right) \right)$$

is the climate damage at  $\tau$ , following from the amount of carbon in the atmosphere at  $\tau - \ell$ .

Consider next the climate damage caused by 1 ton of emissions at time  $t$ . I follow Archer (2005) and assume that a share  $\alpha$  remains in the atmosphere for ever, where  $\alpha$  is approximately 0.25. The remaining share  $1 - \alpha$  gradually depreciates at a rate  $\delta$ . The amount of 1 ton of carbon emissions at time  $t$  remaining in the atmosphere at  $\tau (> t)$  is thus  $\alpha + (1 - \alpha)e^{-\delta(\tau-t)}$ . If e.g.  $\delta = 0.013$  and  $\alpha = 0.25$ , 45 % of the original emissions will remain in the

atmosphere after 100 years, while 27 % still remains after 300 years. These numbers are roughly in line with what is suggested by Archer (2005) and others.

The total additional damage caused by 1 ton of carbon emissions at time  $t$  is the sum of additional damages at all dates from  $t + \ell$  to infinity caused by the additional stocks from  $t$  to infinity. To get from additional stocks at  $\tau$  to additional damages at  $\tau + \ell$  we must multiply the additional stocks at  $\tau$  by the marginal damage at  $\tau + \ell$ , which is  $D'(S(\tau))$ . The marginal damage of 1 additional ton of emissions at  $t$  is thus (discounted to 0) given by

$$v_0(t) = e^{-r(t+\ell)} \int_t^\infty e^{-r(\tau-t)} [\alpha + (1 - \alpha)e^{-\delta(\tau-t)}] D'(S(\tau)) d\tau \quad (21)$$

Notice that the time lag between changes in carbon concentration in the atmosphere and temperature change implies that carbon emissions at  $t$  only start changing the climate at  $t + \ell$ , this is the reason  $\ell$  occurs in the term in front of the integral.

The effect of a postponement of carbon extraction on climate damages is given by the properties of  $v_0(t)$ . If this function is declining in  $t$ , postponement of a given (small) amount of carbon extraction is an advantage in the sense that it reduces total climate costs (i.e. reduces  $V_0$  given by (19)). It follows from (21) that

$$v_0'(t) = -e^{-r(t+\ell)} D'(S(t)) + e^{-r(t+\ell)} \int_t^\infty (1 - \alpha)\delta e^{-(r+\delta)(\tau-t)} D'(S(\tau)) d\tau$$

which may be rewritten as

$$v_0'(t) = -e^{-r(t+\ell)} D'(S(t)) \left\{ 1 - (1 - \alpha)\delta \int_t^\infty e^{-(r+\delta)(\tau-t)} \frac{D'(S(\tau))}{D'(S(t))} d\tau \right\} \quad (22)$$

A postponement in extraction in any period with  $D'(S(t)) > 0$  will thus give a reduction in climate costs provided the term in curly brackets is positive. The second term in curly brackets satisfies

$$(1-\alpha)\delta \int_t^\infty e^{-(r+\delta)(\tau-t)} \frac{D'(S(\tau))}{D'(S(t))} d\tau \leq (1-\alpha)\delta \max_\tau \left[ \frac{D'(S(\tau))}{D'(S(t))} \right] \int_t^\infty e^{-(r+\delta)(\tau-t)} d\tau$$

or

$$(1-\alpha)\delta \int_t^\infty e^{-(r+\delta)(\tau-t)} \frac{D'(S(\tau))}{D'(S(t))} d\tau \leq \frac{(1-\alpha)\delta}{r+\delta} \max_\tau \left[ \frac{D'(S(\tau))}{D'(S(t))} \right]$$

A sufficient condition for the curly brackets in (22) to be positive is therefore that the r.h.s. of the inequality above is smaller than 1. In other words, when  $D'(S(t)) > 0$ , a sufficient condition for  $v'_0(t) < 0$  is that

$$\max_\tau \left[ \frac{D'(S(\tau))}{D'(S(t))} \right] < \frac{r+\delta}{(1-\alpha)\delta} \quad (23)$$

Even if the sufficient condition for  $v_0(t)$  to be declining does not hold, it may nevertheless be the case that  $v'_0(t) < 0$  for all  $t$ . Moreover, even if  $v'_0(t) < 0$  does not hold for all  $t$ , a postponement of extraction that shifts the curve B in Figure 1 towards the right and downwards may nevertheless give a reduction in climate costs as measured by  $V_0$ .

It is useful to consider the special case in which climate costs at any time are proportional to the stock of carbon in the atmosphere, i.e.  $D(S) = mS$ . In this case it follows from (21) that

$$v_0(t) = e^{-r(t+\ell)} \left[ \frac{\alpha}{r} + \frac{1-\alpha}{r+\delta} \right] m$$

which is declining in  $t$ . This expression gives the marginal cost at 0 of a unit of emissions at  $t$ . The marginal cost at  $t$  of a unit of emissions at  $t$  is

$$v_t(t) = e^{-r\ell} \left[ \frac{\alpha}{r} + \frac{1-\alpha}{r+\delta} \right] m$$

which is constant. One interpretation of the WTP  $w$  used in Section 3 and the subsequent sections is that  $w$  simply is  $v_t(t)$ , which varies across countries if  $m$  varies across countries.

## Appendix II: The effect of reduced $b$ on the extraction path

To illustrate the importance of the demand elasticity for the sign of  $\frac{dT}{db}$  given by (11), consider the class of demand functions

$$D(p) = \tilde{D}(p) + k \cdot (b - p)$$

where  $\tilde{D}'(p) < 0$  and  $k \geq 0$ . As  $k$  increases, demand becomes more sensitive to price changes, and demand also increases for  $p < b$ . With this demand function it follows from (4), in obvious notation, that

$$I(z) = \tilde{I}(z) + kF(b - be^{-rz})$$

From this equation it is clear that as  $k$  increases,  $I(z)$  will also increase. Moreover, as  $k$  increases demand also increases for all  $p < b$ , so the total extraction time  $T$  must decline. This means that as  $k$  increases, the lowest weight  $e^{-rT}$  in (11) increases. Together, these two effects imply that the integral in the expression for  $\frac{dT}{db}$  increases as  $k$  increases. It follows from this reasoning that  $\frac{dT}{db}$  must be positive for  $k$  sufficiently large.

In Figures 3 and 4 the extraction path after the decline in  $b$  intersects the initial extraction path only once. However, we cannot rule out the possibility

of the two extraction paths intersecting more than once. To see this, consider the case of  $\frac{dT}{db} = 0$ . Since  $T$  is independent of  $b$  in this case, a given value of  $z$  corresponds to the same time point independent of  $b$  in this case. It is straightforward to verify that differentiating carbon extraction  $X(be^{-rz}, b)$  with respect to  $b$  gives

$$\frac{dX(be^{-rz}, b)}{db} = [(1 - e^{-rz})X_b(z) - e^{-rz}I(z)]$$

where  $X_b(z)$  and  $I(z)$  are both positive. Consider this expression for a small value of  $z$ , i.e. at a date close to the date of exhaustion. If  $D'(p)$  is sufficiently close to zero for  $p \in [be^{-rz}, b]$ , it follows from (10) that  $I(z)$  will be "small" and thus  $\frac{dX}{db} > 0$ . If on the other hand  $F'(b - be^{-rz})$  is sufficiently small, it follows from (3) that  $X_b(z)$  will be "small" and thus  $\frac{dX}{db} < 0$ . In other words, at dates just prior to exhaustion extraction may go up or down as a response to reduced  $b$ , even if the exhaustion date is independent of  $b$ . By continuity, the same must be true even if  $T$  is affected by the reduction in  $b$ : Also in this case extraction may go up or down at a particular date as a response to  $b$  declining, independently of which direction  $T$  moves when  $b$  goes down. This proves the possibility of extraction paths for two values of  $b$  intersecting more than once.

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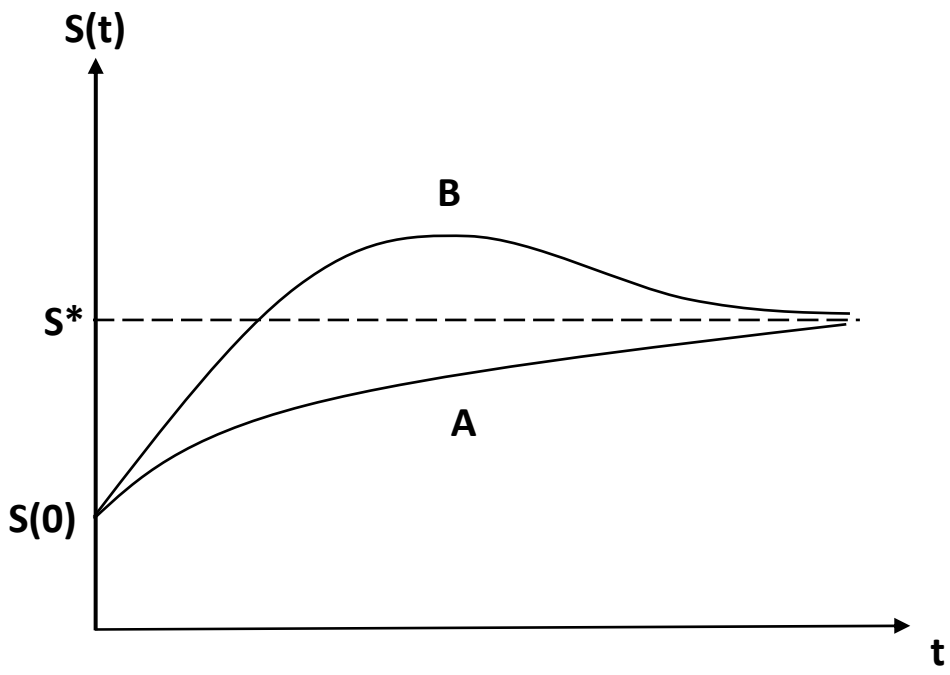


Figure 1

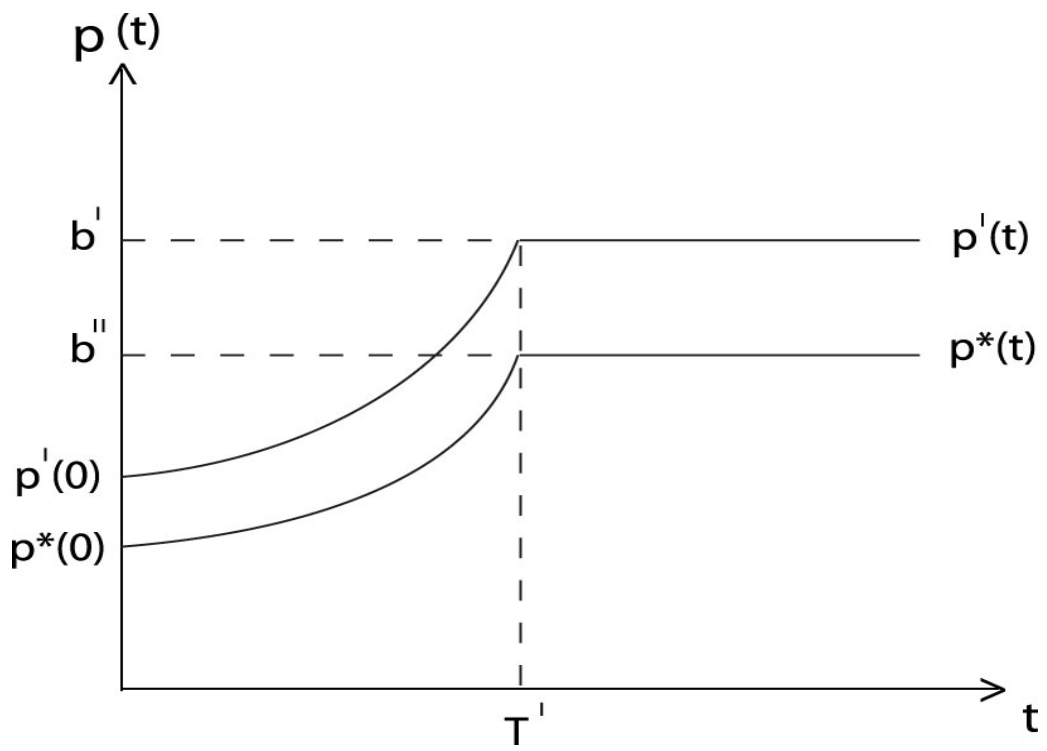


Figure 2

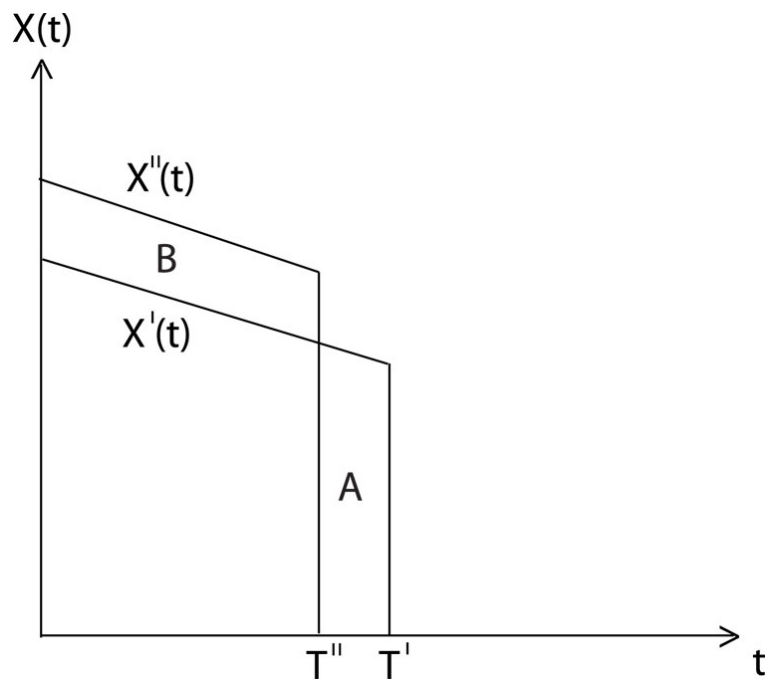


Figure 3

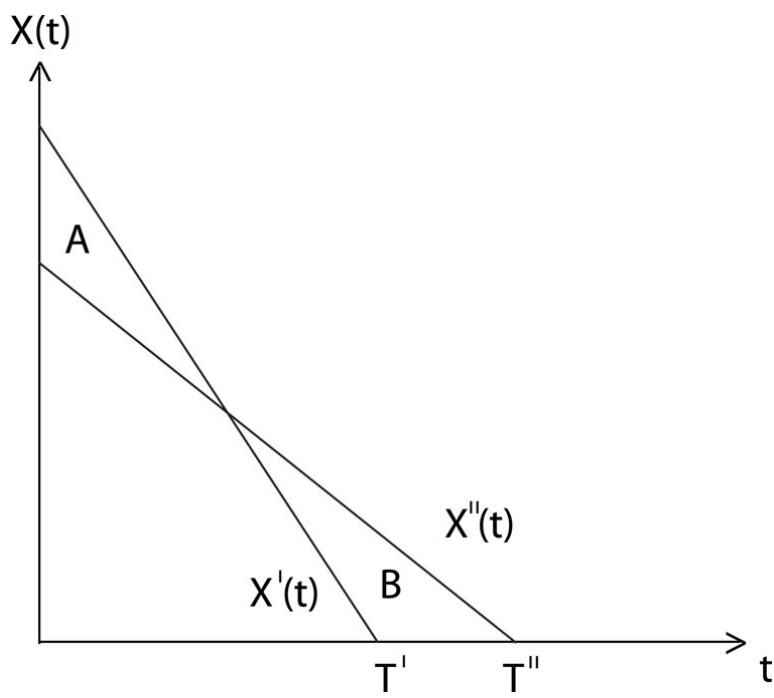


Figure 4

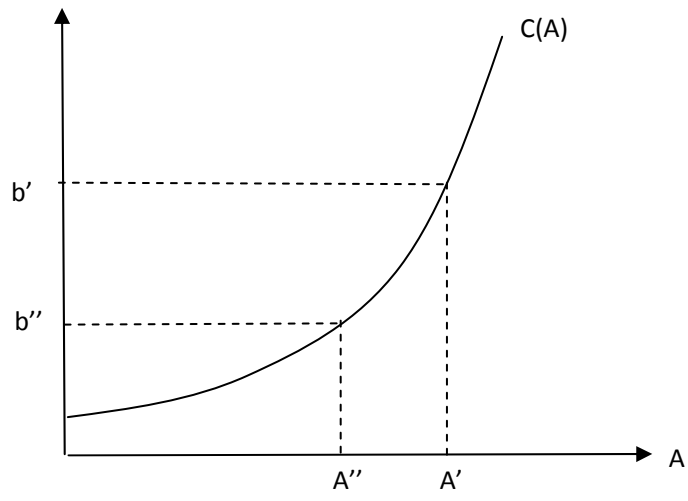


Figure 5