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This paper provides a cradle-to-grave model for political union between two independent states. We show that in equilibrium merger agreement is path dependent and, in some cases, time inconsistent. We introduce negotiated, contested, and time-consistent constitutions, each addressing a larger class of merger problems. The possibility of contest constrains the set of optimal taxes and more importantly it enhances the stability of constitution. Ethnic and economic heterogeneity constrain the set of mutually desirable mergers, which in extreme cases could rule out merger altogether. Intra-state economic heterogeneity is shown to be more constraining than other forms of heterogeneity. We also show that the rent extracted by technologically advanced province in a union for transferring technology to the backward province is not only increasing in complexity of technology but also bounded from above.

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Key Words: Bargaining, Constitution, Contest, Path dependence, Union, Time inconsistency

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1.0 Introduction

Constitution is a commitment device which helps groups with conflicting interests within a state to commit to a scheme for distribution of power, which is underwritten by the actual distribution of means to power within the polity. A constitution can also be treated as a convention, a self-enforcing equilibrium under which everyone is better-off by conforming as long as all others do so.[1] Once the underlying distribution of means to power in the polity changes some players would be better-off by deviating from the behaviour prescribed by the convention and others would be obliged to follow the lead. A constitution is thus unlikely to outlive the socio-economic conditions under which it is a stable equilibrium and should thus provide for its own amendment. A period of transition from one constitutional arrangement to another is also known as constitutional quandary. There are only a few occasions when a state might face constitutional quandaries. The first and foremost occasion comes when states face major political changes, e.g., regime change (abolition of monarchy, independence from foreign rule, transition to or from communism/military dictatorship), territorial change (partition, union with another state), etc. The other occasion when even an otherwise stable state might face constitutional crisis is when the socio-economic structure of the polity changes, e.g., discovery of valuable mineral resources in some province, introduction of new technology benefiting only an ethnic and/or territorial fraction of the state, demographic change in favour of some group, etc. Unfavourable outcome in international wars might also lead to a constitutional quandary.

We are interested in the case involving political union between two independent states. Incidentally over the last two centuries disagreement over distribution of political power and/or resources torpedoed almost every non-coercive attempt to form a union out of sovereign states in the Third World. In this paper we try to address the question of optimal constitutional design in the run-up to formation of political unions. In the process we examine issues like path dependence, time inconsistency, and relation between constitution and contest.

Before proceeding it would be worthwhile to see how this paper relates to existing research. To be precise this paper relates to four lines of research. First of all it relates to the literature

[1] A self-enforcing equilibrium 'is called a convention *only if* one can imagine a different rule that would also be self-enforcing' (Little 2002: 94-95).

on game-theoretic modelling of constitutions. There is a very large literature focusing on economic implications, especially for fiscal policy, of constitutional design. We are, however, only tangentially concerned with such issues. In spirit this paper relates to Grossman (2002) who models a self-enforcing constitution that provides an alternative to civil war in an *established* state. Our analysis, even though it applies to trans-national context, is in agreement with the central insight in Grossman (2002), namely, constitution can check conflicts if and only if expected gains from outright victory are not too large. Its subject matter is however closer to Bordignon and Brusco (2001) who also present a cradle-to-grave model for union between two states. However, our model differs in significant respects from their model, which is primarily concerned with optimal secession rule. They provide a two period two state (both states identical ex ante) model of a union, which is expected to deliver trans-national public goods in future that cannot be produced within individual states. Driving force for union in our case is sharing different kinds of technologies and matching surplus factor inputs.[2]

Secondly, it adds a new perspective to the issue of make-up and break-up of states. The model presented is fashioned so as to rule out merger due to conventional reasons, e.g., trade, economies of scale, etc, and focuses on the twin issues of technology transfer and resource mobility. To the best of our knowledge these issues have not been discussed in the relevant literature in the general framework presented here.[3] Thirdly, this paper contributes to the literature on impact of demographic heterogeneity on political processes. More specifically we show that ethnic and economic heterogeneities severely constrain the set of mergers.[4] Finally our model is in agreement with one of the insights provided in Bulow and Rogoff (1989), namely, there are limits to sovereign debt recovery.

The rest of the paper is organized as follows. Section 2 introduces the setting in two steps. First we define a constitutional agreement and identify its independent components and then

[2] Chen and Ordeshook (1994) also model constitutional secession rules but for a three province union. Inman and Rubinfeld (2008) model a constitution, which supports transition to democracy in South Africa.

[3] Alesina and Spolaore (2005) provide an extensive review of literature on make-up and break-up of states, which more or less begins with Friedman (1979). In a related paper Yarborough and Yarborough (1994) argue that changes in state boundaries are required when cross-border contracts are not credible. But they do not provide a formal model. In a critical review, Young (2004) highlights the widely shared shortcomings of this literature. Also see Herbst (2000) in this regard. Noteworthy among models for specialized mergers is Chang (1995) who models a monetary union.

[4] See, for instance, McCormick and Owen (2004) on role of heterogeneity within the state and rebel camps in civil wars. Tsebelis (1988) deals with impact of intra-coalition heterogeneity on elections while Putnam (1988) explores the impact of intra-national heterogeneity on international agreements.

introduce the socio-economic setting within which the independent components are determined. Section 3 introduces the benchmark model, with homogeneous population. In Section 4 we discuss the impact of demographic heterogeneity on constitutional agreements. Section 5 concludes. Unless otherwise specified please refer Appendix I for proofs.

2.0 Background

2.1 *Outline*

Consider two independent states, State A and State B, unequally endowed with human and material resources and production technologies (specified in sub-section 2.2, *infra*), exploring the possibility of merging to form a union. After merger erstwhile independent states become provinces of a union. Within the union each province as a whole would like to safeguard its interests. For this purpose they negotiate a constitution, which is the primary concern of this paper. Within union unequally endowed states can resolve to jointly manage technology and resources (labour and capital).[5] In case of technology this implies that the better technology is implemented in backward province whereas in case of resources this implies resource mobility, which eliminates resource scarcities and surpluses within the union. These policies enhance the economic product of the union. If both labour and capital are mobile then mobility of one of the two would be redundant in the process of matching excess demand and supply for factor inputs. So without loss of generality we can work with the assumption that only one of the resources, say labour, is mobile. *Our conclusions remain unchanged if the possibility of mobility of labour and capital are interchanged*. Interchanging labels of the two resources in the model would suffice. In our model identity of mobile factor becomes important *only* in presence of ethnic heterogeneity.

Typology of Unions Unions can be classified according to extent and type of merger that brings states together. A merger of states leading to a *unitary state* is followed by implementation of common tax rate throughout the post-merger state whereas a merger of states leading to a *federal state* is followed by implementation of different tax rates in

[5] For importance of resource, especially labour, mobility and technology transfer for political unions see case studies in Bookman (1993) and Horowitz (1985 [2000]).

different provinces of the post-merger state.[6] If states agree to jointly manage either resources *or* technology in the union then the merger is referred to as *partial merger* whereas in a *full merger* states agree to jointly manage both resources and technology. Similarly, when states agree to jointly manage resources *and/or* technology in a limited way within the union then the merger is referred to as *incomplete merger* whereas in a *complete merger* states agree to jointly manage resources and/or technology to the fullest extent. In the event of merger limited technology transfer implies that advanced state offers advanced technology for a fraction of capital in backward state whereas limited resource mobility is obvious. A partial merger could be incomplete or complete and similarly an incomplete merger could be partial or full. It should be clear that the size of resources at the disposal of the union depends on the nature of merger agreement. Obviously an incomplete, partial merger would result in lesser economic output compared to a complete, full merger.

States can either choose to provide only for technology transfer (labour mobility) within the union under a TT-only (LM-only) merger. These are one issue-one step, i.e. partial, mergers. Alternatively, states can choose to provide for both technology transfer and labour mobility within the union under a full merger. Here states have three options: implement technology transfer and labour mobility simultaneously – TTLM merger, implement labour mobility after completing technology transfer – TTLM (TT) merger, and implement technology transfer after ensuring labour mobility – TTLM (LM) merger. The last two mergers can be interpreted as multi step-multi issue mergers addressing issues sequentially. TTLM merger is referred to as multi issue-one step merger. Given the large number of possibilities it is not difficult to see that the sequence of implementation of various components of a merger agreement will have a significant impact on the structure of union. Finally note the following tie-breaking property.

PROPERTY 1: (a) When indifferent between independence and union a state chooses the former. (b) When indifferent between contest and negotiation a state chooses the latter.

Constitution We propose a constitution addressing the basic structure of a state, which can apply to a union of any number of states. It is only when we set about to find an optimal constitution that we need to restrict the number of states due to modelling constraints. Choice

[6] Derivations for federal set-up are cumbersome due to different tax rates in provinces so we will focus on unitary unions in this paper. However, the results hold for an *equivalent* federal union as well after suitable transformation of the tax function. Details available from author.

of a two state model, however, needs some justification in addition to the usual quip that '*Three is a crowd in game theory'*. There is indeed empirical support for choosing a two state framework, namely, for the whole of 20th Century we know of no case of negotiated merger involving more than two sovereign states.[7] With this justification we can proceed with the two-state framework. Constitution consists of articles divided into two categories. *Part I* of the constitution contains the objective of union. *Part II* deals with instruments for implementing *Part I*. The articles of the constitution are defined below.

1. *Preamble* (C0) states the objective of union (type and degree of merger) and the necessary conditions for its existence.

$$C0 = (CC_n, \theta, SR);\ CC_n \in CC \tag{1}$$

$$CC = \{CC_1 = LM, CC_2 = TT, CC_3 = TTLM(LM), CC_4 = TTLM(TT), CC_5 = TTLM\}$$

We will show later in the paper that each element of CC relates to an equilibrium in our model. Constitution helps in implementing one of them. We can therefore refer to the elements of *CC* as constitutional conventions.

$$SR: (g, t) \rightarrow \{(a, b) |\ a \in \{\text{Exit, Continue}\}, b \in \{\text{War, No War}\}\} \tag{2}$$

where g is additional economic product generated due to union and t is tax rate

Secession Rule (SR) specifies the conditions under which the union can be dissolved. Whenever *SR* specifies "Exit" union dissolves peacefully else the secessionist province might be punished. The degree of merger is given by $\theta = (\theta_L, \theta_T)$. $\theta = (1,1)$ for complete, full mergers; (0,1) or (1,0) for complete, partial mergers; $\theta = (a, b)$ for incomplete, full mergers; (0,b) or (a,0) for incomplete, partial mergers; (0,0) for no merger, where $a, b \in (0,1)$.[8]

[7] The partially successful attempt to include Southern Yemen in an expanded United Arab Republic (a sovereign state formed after the merger of Egypt and Syria in 1958, which was dissolved in 1961) is the closest one gets to an attempt to achieve a merger between three sovereign states; even here the actual process unfolded in two stages each involving two states at a time. After the dissolution of Spanish American Empire at least half a dozen failed attempts were made in the 19th Century to unite different groups of successor states.

[8] Many countries have central and provincial lists of functions. $\theta_i = 1$ could be interpreted as issue i being on central list. Similarly, $\theta_i = 0$ could be interpreted as issue i being on provincial list because it implies that the union is not concerned with the issue, which is therefore left to respective provinces. Similarly $\theta_i \in (0, 1)$ could be interpreted as the issue being on a concurrent list. Another interpretation of θ is as follows. $\theta_L = 1$ can be interpreted as individual right to free mobility within the union whereas $\theta_T = 1$ can be interpreted as obligation of state to ensure overall technological improvement in the backward province.

2. (a) *Seat Sharing Rule* (C1) governs seat sharing in the union legislature. State *i* is assigned λ_i fraction of seats in the union legislature.

$$C1 = \left\{ \lambda_i \mid \lambda_i \in (0,1),\ \sum_i \lambda_i = 1 \right\}_{i \in \{A,B\}} \tag{3}$$

2. (b) *Revenue Sharing Rule* (C1a) governs the rate of taxation, *t*, and share of State *i* in the tax revenue of union, β_i .

$$C1a = \left\{ (t\ ,\beta_i) \mid \beta_i \in (0,1),\ \sum_i \beta_i = 1,\ t \in \left[t^{\min}, t^{\max}\right] \right\}_{i \in \{A,B\}} \tag{4}$$

On the one hand tax rate cannot exceed a level, $t^{\max}$, beyond which individuals are denied subsistence. On the other hand the minimum tax rate, $t^{\min}$, has to be positive otherwise the state cannot function and will thus cease to exist. The cost of functioning could include provision of public goods. However, we are not concerned with the composition of basket of public goods. The limits to taxation are revealed by nature and apply to all types of states. Later on we will show that an optimal secession rule specifies dissolution of unions that require taxation beyond these limits. Allowing different tax limits for different states will complicate the expressions without adding any new insights.

3. *Revision Rule* (C2) governs rule-based revision of the initial agreement.

$$C2 = \left\{ \Psi \mid Z(\Gamma+1) = \Psi(Z(\Gamma))\ \ iff\ \ \exists\ i\ \ s.t.\ \ \eta_i(\Gamma+1) \neq \eta_i(\Gamma) \right\}^{9} \tag{5}$$

where $\eta_i(\Gamma) =$ bargaining power of State *i* in period Γ and $Z \in \{C0, C1, C1a, C3\}$

4. (a) *Constitution Amendment Rule* (C3) governs the procedure for amending the *constitution*. This rule is given by α, which specifies the quorum to effect a constitutional change, e.g., if $\alpha = 2/3$ support of at least one more than 2/3rd of the legislators is necessary.

$$C3 = \{\alpha \mid \alpha \in (0.5,1)\} \tag{6}$$

4. (b) *Inviolability Clause* (C3a) declares if the original scope of merger (C0) is mutable or not. INV $\in$ {0, 1}, where C0 is immutable if INV = 0.

[9] Z(Γ=Γ′) or Z(Γ′) is the value of Z in period Γ = Γ′ , where Γ = 0, 1, 2… |***X***| is the cardinality of the set ***X***.

We end the introduction to constitution with a few remarks. First, INV is exogeneously fixed. If the union is beneficial to one of the states, which is able to compensate the loser, or both states then INV = 0 does not bind. Else INV = 0 binds because the union cannot be restructured to generate additional surpluses, if possible. Second, a revision rule can only revise certain articles according to a pre-determined formula and is not self-referential while an amendment rule covers *all* articles of the constitution (unless constrained by INV) in the sense that a group of legislators fulfilling the requirement of quorum can change the entire constitution including the amendment rule itself. Third, we require the constitution amendment rule to meet the following criteria: *Stability* (should discourage frivolous amendments), *Inclusiveness* (should not dissuade a weaker state from joining the union), and *Flexibility* (should be as liberal as possible). Having defined the articles of constitution we can now define a constitution as $C = (C0, C1, C1a, C2, C3, C3a)$.

Independent articles of constitution Merger negotiations can be decomposed into a number of parts dealing with different components of the constitution, which reminds of multi-issue bargaining.[10] However, we have to adopt a different approach because in our case the issues (articles), as we will show shortly, are not independent. In fact, a closer look at the various articles of constitution would suggest that rather than representing different *pies* these articles address different aspects of the same *pie*. We will isolate the independent articles of the constitution in two steps and argue that the order in which they are taken up does not require any bargaining over agenda. First of all note that even though in principle corresponding to each type of merger we can entertain the possibility of incomplete mergers, i.e. θ_T and/or $\theta_L < 1$, the following result shows that such mergers, $\theta_T, \theta_L \in [0,1)$, are not possible.

PROPOSITION 1: (a) For any type of merger complete mergers are strictly preferred to incomplete mergers. (b) An incomplete merger of one type is never preferred to a complete merger of another type.[11]

Secondly, we argue that a state's share in union revenue is non-decreasing in its share of seats in union legislature and that a weaker state would be unwilling to join a union under a relaxed

[10] See Fershtman (1990) for agenda in complete information bargaining games.

[11] It is well known that incomplete mergers are not uncommon, at least in South Asia. In India peripheral provinces, which are substantially different from the "mainstream" in terms of ethnic identity, were incorporated under incomplete and/or partial merger arrangements. In our model incomplete mergers could arise if we introduce demographic heterogeneity (*cf.* Section 4.1 *infra*).

amendment rule.[12] The following result formalizes these points. Later on we will show that union tax rate depends on seat sharing rule.

LEMMA 1: For a given equilibrium seat sharing rule, λ_i (a) $\beta_i = \lambda_i$ is the equilibrium revenue sharing rule and (b) $\alpha = \max(\lambda_i, \lambda_j)$ is the equilibrium constitutional amendment rule satisfying the requirements of flexibility, stability, and inclusiveness.

Lemma 1 implies that an agreement on C1 also fixes C1a and C3. C3a is known to be exogenously fixed. The next result isolates the independent articles of constitution.

LEMMA 2: CC_n and C1 are the only independent (sub-)articles in *Part I* and *Part II* of the constitution, respectively.

Note that C1 itself depends on CC_n so that ultimately the entire constitution can be reduced to CC_n, which is not surprising as the latter is an expression of the objective of merger. But it is *through* C1, not directly, that CC_n determines the rest of the constitution (C1a, C2, and C3).[13] Now it is easy to see that bargaining on agenda is not required since in any case preamble (CC_n) has to be fixed before C1.

2.2 *Specifics*

Human Resources and Material Resources: Our world consists of two states. L_i and K_i, respectively, are the shares of labour and capital held by state i.[14] A capital (labour) constrained state's share in aggregate or worldwide capital (labour) falls short of its share in aggregate labour (capital). It should not be difficult to see that between a pair of states one would be capital constrained while the other would be labour constrained. In other words the upper limit of availability of one of the factors is fixed in each state due to skewed distribution of resources. Relative distribution of resources across states is assumed to be time invariant. Within each state assume equal division of capital and labour resources, available in the state, among all individuals, who are endowed with identical preferences. Depending on merger agreement labour might be mobile. Once labour mobility is allowed it leads to instantaneous

[12] In a number of cases Third World minorities were promised equal treatment at the time of independence but later on the majorities easily amended the constitution to the disadvantage of the former.
[13] It is not for nothing that in real life merger negotiations seat sharing is the most contentious issue.
[14] Normalization used in the paper, namely, $K_A + K_B = 1$ and $L_A + L_B = 1$ is without loss of generality. One could as well use $K_A + K_B = K'$ and $L_A + L_B = L'$.

and optimal reallocation of labour between states. *As pointed out above all our results hold if instead of labour capital or both labour and capital are mobile.* We assume that resource mobility does not lead to permanent redistribution of resources so that labour and capital continue to belong to the state of origin irrespective of where they are employed. In Section 4.4 we provide a rationale for this assumption.

Technology: Consider the following production function for both states:

$$\text{Output, } Q_i = X_i \min(q_L L_i,\ \ q_K K_i), \text{ where } X_i \in \{A,\ \ B\} \tag{7}$$

For sake of simplicity assume $q_L = q_K = 1$ so that one unit each of labour and capital are required to produce X units of output.[15] Let *A* and *B,* respectively, represent productivity of technology in State A and State B. The state that is initially relatively technologically advanced is also referred to as advanced state while the other is referred to as backward. The same good is produced in both states and its price is normalized to unity.[16] We assume that individuals in both states have identical subsistence requirements, which can be fulfilled using the backward technology even at the maximum rate of taxation, $t^{\max}$.

Technologies differ in terms of time (*T**) required to transfer them to a new user. Technologies requiring infinite time to transfer ($T^* \to \infty$) are referred to as Type I technologies while the remainder are referred to as Type II technologies.[17] $\gamma \in \{I, II\}$ denotes the type of technology. *T** is in fact a proxy for complexity of technology. Transfer of Type I technology involves only the transfer of *use rights* while in case of Type II technology transfer implies that after a gestation period the ability of new user to use the technology is no longer dependent on the continued support of the original owner. We assume that productivity in backward province increases to the maximum possible level as soon as agreement for technology transfer takes place even though assimilation/transfer of technology, if at all possible (as in case of $\gamma = II$), takes place over a longer period. The alternate assumption that

[15] To use the normalization $K_A + K_B = K'$, etc instead of $K_A + K_B = 1$, etc we need to assume $K'q_K = L'q_L = 1$.
[16] Without loss of generality we can interpret output as a basket of goods and services with the only restriction that the basket be same across states. We are not concerned with the composition of the basket. Further note that our setting differs from a Heckscher-Ohlin economy. In our case two factors are used to produce one output and states are endowed with different technologies and factors can be mobile.
[17] With Type I technologies TTLM (TT) merger is not possible because in such a merger LM has to be implemented after completion of technology transfer, which never happens with this type of technology. So a TTLM (TT) merger with Type I technology is effectively a TT merger.

productivity increases gradually over time only adds complexity to the analysis without affecting any of our results.

Division of economic product: Given production function (Eqn 7 with $q_L = q_K = 1$) labour and capital receive equal returns. In case of excess availability of an input in state *i*, say labour, some units $(L_i - K_i)$ are left completely unutilised or all units of that input are equally under-utilized, i.e. each unit of labour works at K_i / L_i fraction of its potential. However, this distinction does not matter for us because within an independent state or province returns to a factor are divided equally among all the owners. More specifically we do not differentiate between owners of employed and unemployed resources.[18] The share of product accruing to capital is divided equally among all capital holders and that accruing to labour is divided equally among all labour owners. So, within a state all individuals, who happen to own equal capital and labour by assumption, receive equal shares from the economic product and are thus equally well-off.

State taxes individuals uniformly and spends the entire revenue on public goods. Pre-merger states can have different tax rates, $\left(t_i^{independent}, t_j^{independent}\right)$, but within a unitary union tax rate is same across provinces, $t_i^{union} = t^{union}$. Pre-merger tax rate is immaterial since everything is spent within the same political entity. However, post-merger tax rate has implications for redistribution of income across provinces. We assume that there is no deadweight loss associated with inter-province tax based transfers. However, all our results can be easily derived even in case of non-zero deadweight losses, i.e. for every unit of transfer from a province only a fraction reaches the recipient province. Furthermore our results remain unaffected if incomes from labour and capital resources are taxed differently. For simplicity we assume that income generated from employment of resources outside the province of origin is taxed in the home province. The substance of this paper remains unchanged if instead we provide for taxation of income generated by mobile resources in the province of employment or partly in the province of employment and partly in the province of origin.

Players: Since each state/province has individuals endowed with identical preferences and material resources we can treat a state/province as an internally undifferentiated monolith. So

[18] If agents are uncertain about whose resources are going to be left unused then organization into a cooperative serves as *unemployment* insurance. If we add risk aversion to our model then such behaviour is more plausible.

within an independent state/province aggregate and individual optimization problems are equivalent. In absence of union independent states are the players otherwise provinces of the union are the players. Policy-makers of state/representatives of provinces in union legislature are chosen at random from among their respective populations. Alternatively all individuals within a state/province contribute equal time to the task of policy making, which can now be interpreted as part of the basket of goods and services produced in the concerned territory. In either case we do not face principal-agent problem, which also rules out merger driven by empire builders. Finally assume the players to be risk neutral.

Pay-offs: Economic (share of seats in legislature) and political components of pay-off to a state are additive. The pay-off function is given by:

$$U_i^s = E_i\left(Q_i^s, t_i^s\right) + P_i\left(\lambda_i^s, t_i^s, Q^s\right);\ s \in \{independence, union\} \tag{8}$$

Q_i^s and λ_i^s are, respectively, state *i*'s economic product and share in legislature. Independent states enjoy full control over levying and utilization of taxes, $\lambda_i^{independent} = 1$, but the size of state resources is smaller. The share of provinces in union legislature is necessarily limited, $\lambda_i^{union} = \lambda_i < 1$, but the size of state resources is larger than before so that there is a possibility of trade-off between the two components of pay-off. Pay-off to a province in a unitary union from share of seats in union legislature is equivalent to its share in union taxes and is given by $\lambda_i^{union}(tQ)^{union}$, where $\beta_i^{union} = \lambda_i^{union}$ by virtue of Lemma 1 and $(tQ)^{union}$ is the total tax revenue of union. Economic component of pay-off consists of economic product net of taxation, i.e. $Q_i^{union}\left(1 - t^{union}\right)$. The pay-off functions for a state under different conditions are provided below.

$$U_i^{indep} = \left(1 - t_i\right)Q_i + t_i Q_i = Q_i \tag{9}$$

$$U_i^{union} = \left(1 - t^{union}\right)Q_i^{union} + \lambda_i t^{union} \sum_i Q_i^{union} \tag{10}$$

Post-merger Intra-Union Arrangement: If labour from one province migrates to the province where output is labour constrained the migrants send back remittances proportionate to their labour input while the capital rich province gets additional income due to employment of hitherto unutilized capital. Migrant labour is not discriminated against and is paid as much as the native labour. Relaxing this assumption affects the details but not the substance of the

paper. It is trivial to establish that whenever free labour mobility is agreed upon migration takes place only from labour surplus to labour constrained province and that the upper limit to migration is provided by fraction of surplus labour (which can be loosely referred to as *unemployment rate*) in labour surplus province. This holds true since labour force and technology is not differentiated by skill levels. If technology transfer has been agreed upon then after merger better technology is implemented in the backward province. Recall that we do *not* allow permanent resource migration, which would imply irreversible resource transfer and consequent loss of bargaining power of the "donor" state/province.

Depending on initial material and technological endowments, i.e. gross economic product, each province, erstwhile state, exercises some control over policy making in the union, which is denoted by λ_i^{union}. λ_i^{union} might not coincide with population/resource shares in the union. Having described at length the political (constitution) and the economic primitives of the model we can now proceed to examine the actual process of merger. But before that few words on what happens in the event of lack of consensus.

Contest In case the two sides fail to settle issues via negotiation they choose between contest and status quo. The latter is obvious. For the former we assume the following contest success function after Skaperdas (1996):

$$p_i\left(e_i, e_j\right) = f\left(e_i\right) \Big/ \sum_{j \in I} f\left(e_j\right); \; f\left(e_i\right) = \nu \cdot e_i^{\,r}, \text{ where } r \geq 0 \text{ and } \nu > 0 \qquad (11)$$

where p_i is the probability state i emerges successful, $r \geq 0$ measures the effectiveness of additional effort in a contest, and e_i denotes the resources state i can mobilize for a contest. (Actually we do not need the specific functional form given in Eqn 11.) In the event of contest states incur a fixed cost C_i irrespective of the outcome.[19] F captures the advantages of contest strategy. In case state i strikes first $F_i > 0$, which brings down the cost of contest to state i. $F_i = 0$ when both attack simultaneously. Also $F_i > 0 \Rightarrow$, $F_i < 0$, i.e. when state i strikes first state j incurs additional costs for being defensive, which in other words adds to the cost of contest incurred by state j. A non-zero first strike advantage ensures *contest* is never a strictly

[19] Cost of contest to a state could be endogenous in that it could depend on its economic activity. Higher productivity means more efficient war making, which saves cost but it also means higher opportunity cost. Higher economic product likewise means that cost of contest as a fraction of whole economy might be smaller but in absolute terms it might be higher than that of a state with lesser economic output. Another source of endogeneity could result from loss of productive assets in a contest.

dominated strategy. Assume $|F_i| < C_i$ holds for both states. In our model the process of negotiation is assumed to be costless relative to contest. There are two categories/technologies of contest, $\kappa \in \{I, II\}$, and cost of contest and first strike advantage increase with the degree of appropriation involved:

1. Category I: Winner extracts entire additional economic production possible due to a merger, which leaves the loser with its No Merger economic product.
2. Category II: Winner implements most favourable tax rate within the set of mutually agreeable bargaining solutions for constitutional union. (Within a union this type of contest can be interpreted as an electoral contest over tax policy.)

Under an interesting possibility ignored here a state could indulge in destruction or permanent appropriation of a counterpart's productive resource base to increase its bargaining power as in Skaperdas (2006). This is tantamount to assuming that the cost of permanent appropriation of resources is prohibitive, i.e. $C_i\ (permanent) > t^{\max} \cdot \max(A, B) \cdot \min(L_i, K_i)$. It is indeed very easy to overlook the fact that without savings or reasonably large wealth endowment costly wars cannot be financed (more on impact of savings in Section 4).[20] Alternatively, the presumption could be that the present distribution of resources has been arrived at after all possibilities of appropriation of factors of production have been exhausted. Finally note the difference between Anbarci *et al* (2002) and Skaperdas (2006) and our approach. Their focus is on how outcome varies as bargaining norm changes for a given type of contest. We are concerned with how outcomes change with type of contest for a given bargaining norm.

3.0 Constitutions

In the run-up to merger agreement on independent articles, namely, type of merger (CC_n) and seat sharing rule (λ), is the primary concern of negotiation. The ultimate objective of the negotiation is not to actually divide the unionwide economic product. Rather the intention is to use the shares obtained from a hypothetical partition of the unionwide economic product generated by a given type of merger to arrive at λ, the share in seats in the union legislature. λ derived below have to be interpreted in this spirit. Ideally a full and complete merger leading to simultaneous technology transfer and labour mobility (TTLM) should be the

[20] Colomer (2007: 35) draws attention to the fact that some of the 20th Century wars cost more than 100% GDP to the participants. Incidentally these wars involved efforts to change existing borders. In other words it is easier to snatch part of current output than to appropriate productive resources permanently.

preferred outcome under which total product is maximized. However, the two sides can choose from among several different kinds of complete mergers, including both partial and full types. Recall that by virtue of Proposition 1 incomplete mergers are never contracted. Given the large number of ways of achieving merger we briefly discuss the issue of choice of type of merger and conflict resolution technology and then introduce the actual negotiation process.

A plausible solution, perhaps not satisfactory, could be that we allow nature to choose the set of desirable mergers for each state, which is made known to both states.[21] Let $\mathcal{T}(i)$ denote the set of desirable types of merger chosen by nature for State *i*. At the beginning of period $\Gamma = 0$ nature chooses **N** = (K_i, L_i, A, B, γ, κ, $\mathcal{T}(i)$, $\mathcal{T}(j)$, INV), i.e., material endowments, type of technology and technological gap, type of contest, set of desirable mergers, and whether preamble is mutable or not. $\mathcal{T}(i) \cap \mathcal{T}(j)$ is the set of mutually agreeable merger types. States decide the type of merger to be negotiated through a normal form game $\mathcal{CoM}$, in which each state has $|\mathcal{T}(i) \cap \mathcal{T}(j)|$ strategies. If $|\mathcal{T}(i) \cap \mathcal{T}(j)| = 1$ then the outcome is unique else $\mathcal{CoM}$ resembles a pure coordination game and has $|\mathcal{T}(i) \cap \mathcal{T}(j)|>1$ pure strategy Nash equilibria and no mixed strategy equilibria. But each side would like to join the union with as favourable a seat share, etc possible. States can choose a conflict resolution technology from $\mathcal{CRT}$ = {*status quo*, *contest*, *negotiate*} to resolve the conflict.

3.1 *Game of Merger, $\mathcal{M}$*

We model the game of merger with rational players and complete information using an alternating offers bargaining model with endogenous inside options and contest as outside option. Pay-off functions, rationality of each player, and all relevant parameters of the model are common knowledge. In such a model if outside option of *both* players is dominated by outcome generated by bargaining outcome based on inside option then outside option has no impact on solution of the problem. If, however, outside option dominates the bargaining outcome for even one player it dictates the solution to the game as if inside options were not

[21] Earlier we saw that TTLM (TT) mergers are not possible with Type I technologies (*cf.* footnote 17, *supra*). Hence by fixing type of technology nature can constrain the set of desirable mergers. Historical experience can also play the role of nature. A state that had a bad experience with labour mobility in past ignores mergers involving labour mobility thereafter. However, to account for such possibilities we need to introduce at least one more state and/or history (prior to the onset of present round of merger negotiations). Alternatively, we can constrain the set of desirable mergers by introducing demographic heterogeneity as in Section 4.1 (*infra*).

there (Muthoo 1999: Section 6.4). In other words we have two separable regimes, under the first bargaining dominates ($\mathcal{N}$). In the second contest dictates the solution – there are two structurally identical sub-cases depending on whether contest dominates negotiation for state *i* or *j* ($\mathcal{CN}$). We will show below that in our model outside option cannot dominate bargaining for both players simultaneously. We can therefore model the problem dichotomously.

Negotiation Game, $\mathcal{N}$

We use the alternating offers bargaining model with endogenous inside options[22] to model the process of bargaining.[23] The 'pie' under negotiation is the present discounted value of the total economic product of the union, $Q^{Union}/r = Q(CC_n)/r = \sum_i Q_i^n/r$, where CC_n signifies type of merger. In each period of disagreement the inside options of the states are determined endogenously. Endogeneity results from the fact that the inside options depend on the choice made by players within the process of bargaining through a non-cooperative game. Gains from merger are the present discounted values of difference between the total economic product of the union and the sum of inside options. We assume that both players have identical discount rates $(r_A = r_B = r)$, so their evaluations of gains from a given type of merger are identical. Discount factors $(\delta_A \neq \delta_B)$, applicable to bargaining, differ due to different response times, $\Delta_A \neq \Delta_B$. Δ_i has two interpretations. One, it is the time, Δ_{ii}, which state *i* takes to make an offer after rejecting an offer of state *j*. Two, it is the time, Δ_{ij}, after which state *j* allows state *i* to make a counter-offer after the latter rejects an offer of the former. Ideally $\Delta_i = \max(\Delta_{ii}, \Delta_{ij})$. We will assume $\Delta_{ii} < \Delta_{ij}$ so that $\Delta_i = \Delta_{ij}$.

Disagreement game, $\Theta(n)$ The actions of players when in disagreement in the process of negotiation are captured by a disagreement game, which is represented by a simultaneous move one-shot game of complete information, $\Theta(n) = [\{U_i\}, \{S_i(n)\}]_{i\in\{A,B\}}$, where

[22] Busch and Wen (1995) extend alternating offers model presented in Rubinstein (1982) to account for endogenous inside options. We follow the slightly modified version of their model presented in Muthoo (1999).

[23] In a repeated bargaining framework the stream of surplus generated is bargained over every period. By assuming that states are committed to a bargaining solution (which is what a constitution is all about) we differentiate our treatment of the issue at hand from repeated bargaining framework, even though the commitment in the present case is conditional upon stability of underlying disagreement game. Only when this game changes renegotiation is considered otherwise the agreement holds perpetually and any deviation invites punishment. Even this last alibi for renegotiation does not arise because in our model states agree in advance upon revision and secession rules.

$U_i : \Pi_{i\in\{A,B\}} S_i(n) \rightarrow \Re$ and $S_i(n)$ is set of strategies available to state *i*, identical for both states, in the event of disagreement in the course of negotiation for *n*-type merger. $\hat{\Theta}(n)$ is the set of Nash equilibria of $\Theta(n)$. In the event of disagreement the two sides choose actions in the disagreement game according to which they enjoy interim benefits till they reconvene for negotiations.

In one issue-one step (partial) mergers, TT or LM, the two sides choose to remain independent in case of disagreement over merger. In case of multi issue-multi step mergers, TTLM (LM) or TTLM (TT), in the event of disagreement in each step the two sides have only one option. If there is a disagreement during the first step the two sides can only choose not to merge whereas if there is a disagreement in latter steps the two sides can only choose not to proceed with further merger. In other words multi-step mergers are structurally akin to one issue-one step mergers. In multi issue-one step mergers, TTLM, in the event of disagreement over full, complete merger the two sides can choose from among three different options – no merger (NM), merger with only technology transfer (TT), and merger with only labour mobility (LM). TTLM merger cannot be reduced to a one issue-one step merger or a combination of two such mergers.

$S_i(n) = \{NM\}$ in case of one issue-one step mergers whereas in case of multi issue-one step mergers $S_i(n) = \{NM, LM, TT\}$. The case involving $|S_i(n)| = 1$ is trivial. It is to cases involving $|S_i(n)| > 1$ that we turn next. Disagreement game with multiple strategies, as in case of TTLM mergers, can be depicted using a normal form game shown in Figure 1. For description of pay-offs see Appendix II. A number of cases can be obtained for interaction of two states depending on different combinations of technology and resource distribution. Table 1 summarizes the entire range of possibilities. The disagreement game shown in Figure 1 is provided only as an aid to understanding. Otherwise the following analysis relates to the general case introduced in sub-section 2.2 (*supra*).

With rational players and complete information alternating offers bargaining with endogenous inside options yields an agreement in the initial period with $|\hat{\Theta}|$ stationary subgame perfect equilibria (SSPE), where $|\hat{\Theta}|$ is the number of Nash equilibria of Θ (Muthoo 1999). For each essentially one issue-one step merger there is one SSPE corresponding to one Nash

equilibrium of Θ, namely, (NM, NM). In case of multi issue-one step merger there are four SSPEs corresponding to the four Nash equilibria of Θ, namely, (LM, LM), (TT, TT), (NM, TT) and (NM, NM). The last two equilibria are not only pay-off equivalent but also identical in terms of outcome, namely, complete independence in absence of agreement on merger. Therefore, we consider only (LM, LM), (TT, TT), and (NM, NM) and the corresponding equilibria are referred to as TTLM-LM, TTLM-TT, and TTLM-NM. The following result provides the equilibrium seat sharing rule. The proof follows in straightforward manner from a slight modification of similar results available in literature (Busch and Wen 1995, Muthoo 1999) for the case $(r_i \neq r_j)$ to our case where $(r_i = r_j = r)$ and $(\Delta_i \neq \Delta_j)$.

LEMMA 3: In the limit as $\Delta_i \to 0$ and $\Delta_j \to 0$ (but $\Delta_i \neq \Delta_j$) the unique SSPE seat sharing rule for states $i, j \in \{A, B\}$ is given by $(\lambda_i(m,n \mid \Theta(n)), 1-\lambda_i(m,n \mid \Theta(n)))$, where $d_i(m,n)$ is the pay-off accruing to State *i* as inside option in the *m*th Nash equilibrium of $\Theta(n)$, the disagreement game for *n*-type merger, and $g(m,n)$ is the rate of gain from *n*-type merger when strategies corresponding to the *m*th Nash equilibrium of $\Theta(n)$ are chosen in the event of disagreement. Since we are interested in structurally independent mergers $n \in \{1, 2, 5\}$. $m \in \{1, \ldots \mid \hat{\Theta}(n) \mid\}$ and $i \neq j$.

$$\lambda_i(m,n \mid \Theta(n)) = \left(\frac{d_i(m,n)}{r} + \eta_i \frac{g(m,n)}{r} \right) \Big/ \frac{Q(CC_n)}{r} \tag{12}$$

$$g(m,n) = Q(CC_n) - d_i(m,n) - d_j(m,n)$$

$$\eta_i = \Delta_j / (\Delta_i + \Delta_j)$$

Note a few things here. One, using the limiting solution of the negotiation model saves us from making unreasonable assumption about the identity of the first mover in the negotiation game. Two, $\lambda_i(m,n \mid \Theta(n))$ is equivalent to the asymmetric Nash bargaining solution to our problem. Three, our model does not have mixed strategy SSPEs or non-Markov perfect equilibria.[24] Finally, we can say that Δ_i, the time for which state *i* is forced to wait before making its offer after rejecting state *j*'s offer, is non-decreasing in the resources state *j* can

[24] The model has infinite Markov perfect equilibria, many of them pay-off equivalent, in which agreement is reached in the initial period. Markov SPEs do not add much to our understanding of the present problem other than drawing attention to the fact that there is no limit to variation in equilibrium power sharing agreements.

mobilize.[25] Since, the only difference between our players is their resource and technological endowments (i.e. economic product), their relative bargaining power has to depend on relative economic product. We assume a linear dependence between the inside option of state *i* and the delay state *i* can cause before the next round of bargaining whenever state *j* rejects its offer.

$$\forall i, j \;\; \Delta_j(n,m) = a \cdot d_i(n,m) \tag{13}$$

so that relative bargaining power of state *i* is given by

$$\eta_i(n,m) = \frac{\Delta_j}{\Delta_i + \Delta_j} = \frac{d_i(n,m)}{d_i(n,m) + d_j(n,m)}, \text{ where } \sum_i \eta_i = 1 \text{ and } \eta_i \in (0,1) \tag{14}$$

The next result, which relates seat sharing and relative bargaining power, follows directly from (12) – (14).

LEMMA 4: $\eta_i(n,m) = \lambda_i(n,m) \;\; \forall n, m(n)$.

Tax Adjustment Now we will address the issue of omission of tax adjustments in Θ. Whenever $|S_i(n)| > 1$ disagreement game supports the possibility of partial mergers pay-offs in Θ need to be adjusted for tax transfers within the *ad hoc* union. This is required only in case of multi issue-one step TTLM mergers so we will confine ourselves to disagreement games of the type shown in Figure 1. Using (8) – (10) we can arrive at tax adjusted pay-offs. The corresponding tax adjusted disagreement game, Θ', is provided in Figure 2. The next result establishes the equivalence of Θ and Θ'.

LEMMA 5: Θ', the tax adjusted disagreement game, is strategically equivalent to Θ'', the reduced form disagreement game, $\forall \;\; t \in (0,1)$.

[25] We can arrive at (12) using a more rigorous model in which state *i* has capacity to cause additional delay of $\Delta_i - \Delta$ before the next round of bargaining whenever its offer is rejected, where $\Delta = \Delta_{ii} = \Delta_{jj}$. We can show that in a model with inside options given that state *j* delays whenever state *i*'s offer is rejected the latter resorts to delay tactics if and only if certain conditions are fulfilled and that these conditions necessarily hold for both states in the limiting case. See Avery and Zemsky (1994) and Muthoo (1999) for bargaining with delays.

The following result establishes equivalence between equilibrium seat sharing determined using tax adjusted and unadjusted disagreement games. It follows from *Lemma* 4 and 5 because negotiation games induced by Θ' and Θ are strategically equivalent

LEMMA 6: $\lambda_i(m,n\,|\,\Theta) \approx \lambda_i(m,n\,|\,\Theta') = \lambda_i(m,n\,|\,\Theta'')\ \ \forall t$.

The next result formalizes the idea that the net recipient of transfers within the union would prefer higher union tax rate and vice versa.

LEMMA 7: Unequally endowed states have opposed preferences over union tax rate with state *i* favouring higher taxes if and only if $\lambda_i^n \geq E_i^n / \left(E_i^n + E_j^n\right)$ or $\lambda_i^n g^n - x_i^n \geq 0$, where E_i^n represents economic product of state *i* after *n*-type merger, x_i^n represents additional product generated within state *i* after merger, and $g^n = \sum_i x_i^n$ represents the total gain from merger.

Contested Negotiations, $\mathcal{CN}$

As suggested above states can use one of the following CRTs, namely, *status quo*, *contest*, and *negotiate*, to resolve the conflict. We have already derived results for negotiations without contests while the case involving *status quo* as a strategy is trivial. In fact, it was subsumed as a disagreement strategy, namely, No Merger, within the negotiation game. Since contest cannot be subsumed as an inside option we introduce it as an outside option in the negotiation game. See Muthoo (1999) for difference between inside and outside options. We assume that the winner of the contest imposes its favourite solution, which varies with the category of contest, κ, under consideration. SQ_i denotes the returns from maintaining status quo. The following expressions provide total gains from merger and returns to state *i* from negotiations and contest, respectively, where we omit the superscript, *n*, for type of merger. Without loss of generality we assume $\lambda_i g > x_i$.

Gains from merger: $g = x_i + x_j$ (15)

Negotiation: $U_i = (1-t)E_i + t\lambda_i\left(E_i + E_j\right) = E_i + t\left(\lambda_i g - x_i\right)$ (16)

Contest (κ=I) : $R_i^I = p_i\left(E_i + x_j\right) + \left(1 - p_i\right)SQ_i - \left(C_i - F_i\right) = p_i g + SQ_i - \left(C_i - F_i\right)$ (17)

$$\text{Contest } (\kappa = \text{II}): R_i^{II} = p_i U_i\left(t^{\max}\right) + (1 - p_i) U_i\left(t^{\min}\right) - \left(C_i - F_i\right) \tag{18}$$

$$\text{Or } R_i^{II} = E_i + \rho\left(\lambda_i g - x_i\right) - \left(C_i - F_i\right)$$

where $\rho = \left(p_i t^{\max} + (1 - p_i) t^{\min}\right) \in \left(t^{\min}, t^{\max}\right)$ and use is made of Lemma 7 following which we can say that if $\lambda_i g > x_i$ then state *i* prefers $t^{\max}$ and state *j* prefers $t^{\min}$. Note that *g* captures the overall hike in economic output due to a given type of a merger, which is different from what individual provinces gain from merger. In the following result we show the limits to possibility of contest. In the proof of this proposition we specify conditions under which different conflict resolution strategies dominate.

PROPOSITION 2: For a given type of merger *contest* cannot dominate *negotiate* for both states simultaneously.

The following is immediately apparent from the proof of Proposition 2.

COROLLARY 1: Set of feasible bargains is never empty in case of Category II contests, which essentially involve contests to relocate along the bargaining frontier.

In the next proposition we identify the set of CRTs, seat sharing rule, and tax rate under different conditions.

PROPOSITION 3: Given Property 1 and $|F_i| < C_i$

1. *Sovereign Peace*: If $|\mathcal{T}(l)| = 0$, $l \in \{i, j\}$, in equilibrium there is no conflict, and hence no contest, between states. $\mathcal{CRT}$ = {*status quo*}. The states remain independent and $\lambda_i^{independent} = 1$. States are free to choose tax rates, $t \in [t^{\min}, t^{\max}]$.
2. *Negotiated/Contested Peace*: If $|\mathcal{T}(i) \cap \mathcal{T}(j)| \geq 1$ in equilibrium there is conflict between the states. For sufficiently large gains from merger and/or small cost of contest SQ is strictly dominated and $\mathcal{CRT}$ = {*contest*, *negotiate*}. *n*-type merger takes place in the initial period, where $CC_n \in \mathcal{T}(i) \cap \mathcal{T}(j)$.
    1. *Negotiated Constitutional Union*: $t(i) \cap t(j) \neq \phi$, i.e. *Negotiate* dominates *Contest* for both states. $\lambda_i^0 = \lambda_i^0\left(m \mid \Theta'(n)\right)$ is given by Lemma 6. Post-merger tax rate is given by

$$t^*(\kappa = I) \in \left[\frac{p_i\, g - x_i - (C_i - F_i)}{\lambda_i\, g - x_i}, \frac{p_i\, g - x_i + (C_j - F_j)}{\lambda_i\, g - x_i} \right] \cap \left[t^{\min}, t^{\max} \right]$$

$$t^*(\kappa = II) \in \left[\rho - \frac{C_i - F_i}{\lambda_i\, g - x_i}, \rho + \frac{C_j - F_j}{\lambda_i\, g - x_i} \right] \cap \left[t^{\min}, t^{\max} \right].$$

2. *Contested Constitutional Union*: $t(i) \cap t(j) = \phi$, i.e. *Negotiate* dominates *Contest* for only one of the states, say, *j*. $\lambda_i\left(m \mid \Theta'(n)\right) = \frac{R_i^{\kappa}}{\sum_i E_i^n}$, where R_i^{κ} is given by (17) or (18) depending on type of contest, κ. Post-merger tax rate is given by $t^* = t^{\max}$ if $\lambda_i\, g > x_i$ else $t^* = t^{\min}$.
3. *Mutual War*: If $|\mathcal{T}(i) \cap \mathcal{T}(j)| = 0$ and $|\mathcal{T}(l)| \neq 0$ for at least one state there is conflict between the states. $\mathcal{CRT}$ = {status quo, contest} iff cost of contest is sufficiently high else $\mathcal{CRT}$ = {contest}. Contest takes place in every period.

In words whenever a merger is mutually desirable and gains are modest relative to costs of contest states opt for the purely negotiated solution. Otherwise state for which contest is the dominant conflict resolution strategy dictates the solution. The following should be immediately apparent.

COROLLARY 2: (a) The set of all possible pure strategy equilibria of Game of Merger, $\mathcal{M}$, consists of $1+\sum_n \sum_m \left|\hat{\Theta}''(n,m)\right| + |\hat{g}|$ equilibria corresponding to different combinations of merger types and contest categories, where $\hat{g}$ is the set of mergers for which gains from merger are not identical and for which *negotiate* dominates *contest* for only one of the states. *n* and *m* refer to the type of merger and disagreement strategy, respectively. (b) The equilibria of Game of Merger, $\mathcal{M}$, are path dependent, $\lambda_i\left(m \mid \Theta'(n)\right) = f(\mathbf{P})$, where $\mathbf{P} = (n,\ m(n))$ is the *path of merger*.

Path of merger refers to the choice of type of merger and the ad hoc arrangement, if any, during the period of disagreement during negotiation for the merger. Different values of $\lambda_i(m,n)$ relate to different merger types. The shares of the states in the union vary with type of equilibria even though the type of merger might be same. In other words we have shown that for the same initial distribution of resources and technology gap the states end up with

different shares in a union depending on the path of merger. We provide a numerical example below to illustrate this point for equilibria of $\mathcal{N}$.

Numerical Example Consider the following values for endowment parameters in the example discussed in Appendix II. Technology: A = 2B = 2, Resources: $K_A = 0.70$, $K_B = 0.30$, $L_A = 0.80$, and $L_B = 0.20$. In Table 2 we provide share of state B in union legislature for different types of mergers. Irrespective of the route chosen backward state's share in policy-making is far from the ideal distribution of power based on share in economic product of the union. So why does a backward state choose to merge? One reason why backward state might agree for such a merger is that full unification, in the present example, leads to 150 % hike in its absolute product whereas the corresponding figure for State A is merely 7.14 %. In this example the advanced state is driven towards merger by greater share of an expanded tax base whereas higher income drives the backward state to merge. Disproportionate share gained by advanced state, which entitles it to transfer some of State B's newly generated wealth to its regions via taxation, can be interpreted as the price for technology transfer paid by backward state.

In the next result we show the time inconsistency of constitutional equilibria involving Type II technology, which can be transferred over a finite time. Time inconsistency in behaviour refers to mismatch between preferred ex post and optimal ex ante strategies. The intuition behind the result is that once technology transfer is over the erstwhile backward state would seek revision of existing arrangement, something not accounted for in Proposition 3.

PROPOSITION 4: If a merger involves technology transfer then irrespective of the type of merger the constitutional equilibria supported by Proposition 3 are time consistent iff $\gamma = I$ (Type I technology).

Following Proposition 4 it is easy to see that for $\gamma = I$ revision and secession rules are trivial. Further irrespective of type of technology deriving revenue sharing rule $\beta_i = \lambda_i$ and amendment rule $\alpha = \max(\lambda_i, \lambda_j)$ from seat sharing rule is straightforward. We turn our attention to revision and secession rules, which brings us face to face with the problem of time inconsistency in merger agreements. The technologies, which can be transferred over a finite period, i.e., Type II technology, are associated with time inconsistency.[26] However, since in most of the cases technology is Type II it is, indeed, surprising that TT mergers have not been uncommon in the 20th Century. In other words, TT merger equilibrium is not superfluous. We

[26] In case of TTLM mergers the number of possible routes to merger and associated SSPEs are reduced post-technology transfer since TT is no longer a strategy in the redefined disagreement game.

now provide optimal revision and secession rules for $\gamma = II$, to address the problem of time inconsistency.

PROPOSITION 5: Given complete information and Property 1 states desirous of forging a union under the shadow of contest commit to the following *Secession Rule* (*SR*), which is sub-game perfect:

(1) Type I Technology and pre-technology transfer Type II Technology: $SR(g \leq 0$ or $t \notin [t^{\min}, t^{\max}]) =$ (Exit, No War) and $SR(g>0$ and $t \in [t^{\min}, t^{\max}]) \in$ {(Continue, No War), (Exit, War)}. For Category I contest *SR* is sub-game perfect iff $p_l g > (C_l - F_l)\ \forall l \in \{i, j\}$. For Category II contest *SR* is sub-game perfect iff $\rho(\lambda_l g - x_l) > (C_l - F_l) - x_l\ \forall l \in \{i, j\}$.

(2) Post-technology transfer Type II Technology: $SR(g, t|\Gamma \geq T^*) =$ {Exit, No War} if $CC_n^{\Gamma=0} =$ {TT} *and* INV = 0 or $\mathcal{T}(i) \cap \mathcal{T}(j) =$ {TT}, where T^* is the time required for technology transfer, else Case (1) applies.

The secession rule is optimal in the sense that as long as union generates additional benefits from synergy none of the province secedes otherwise the rule allows for peaceful dissolution. Further even though in equilibrium there is no secession still if it so happens that if one of the states has an infinitesimal tendency to stray from the equilibrium in some period then the other state would punish the deviant as long as conditions specified in Proposition 5 are satisfied for both states. If, however, the conditions are fulfilled only for one of the states, say *j*, then when state *i* secedes in contravention of secession rule then state *i* has to face contest immediately in the same period but not vice versa. In other words technology of contest provides limited internal capacity to the constitution to check non-equilibrium secessionist behaviour, if any.[27] Finally we state the optimal revision rule.

PROPOSITION 6: Given complete information states desirous of forging a union under the shadow of contest commit to C2*, which is the unique sub-game perfect *Revision Rule*, where $Z \in \{CC_n, SR, \lambda_i, t\}$ and $Z(\Gamma = 0) = Z(0)$.

1) $C2^*(\Gamma < T^*, \gamma \in \{I, II\})$ and $C2^*(\Gamma \geq T^*, \gamma = I) : Z(\Gamma + 1) = \Psi(Z(\Gamma)) = Z(0)$
2) $C2^*(\Gamma \geq T^*, \gamma = II)$:

[27] Here one point bears mentioning. If $\exists l \in \{i, j\}$ such that $\mathcal{T}(l) \backslash \{TT\} \neq \phi$ then whenever INV=0 and $CC_n^{\Gamma=0} =$ {TT} or $\mathcal{T}(i) \cap \mathcal{T}(j) =$ {TT} post-demerger states find themselves under Case 3 of Proposition 3, which implies endless contests.

a. If INV = 0 *and* $CC_n(\Gamma = 0) = \{TT\}$ or $\boldsymbol{T}(i) \cap \boldsymbol{T}(j) = \{TT\}$: $Z(\Gamma) = \phi$. De-merger occurs under *SR* and Proposition 3, Case 1 applies.
b. Else INV = 1 and $\boldsymbol{T}(i) \cap \boldsymbol{T}(j) \backslash CC_n(\Gamma < T^*) \neq \phi$: $Z(\Gamma + 1) = \Psi(Z(\Gamma)) = Z(\Gamma = T^*)$, where $CC_n(\Gamma \geq T^*) \in \boldsymbol{T}(i) \cap \boldsymbol{T}(j) \backslash \{CC_n(\Gamma < T^*)\}$, which in turn determines rest of the articles through $\lambda(\Gamma \geq T^*)$ following Proposition 3, Case 2.

The important point about Proposition 5 and 6 is that irrespective of type of technology neither of them is time-inconsistent, which means that we are not using time-inconsistent revision and secession rules to resolve the time inconsistency of other articles. To summarize our findings so far note that whenever possible in equilibrium merger takes place in the initial period itself[28] without any contest under the *time consistent contested constitution*, *C** = (C0* - proposition 1 and 5, C1* - proposition 3, C1a* - proposition 3, C2* - proposition 6, C3* - proposition 3) and there is no secession in contravention to secession rule.

One last point before we close this section. Despite complete information and threat of contest there are limits to what an advanced state can extract for transferring technology. In the next proposition we specify the upper limit in case of TT mergers. Results for other types of mergers involving technology transfer can be obtained similarly.

PROPOSITION 7: The fraction of additional product generated due to technology transfer in a TT merger, which advanced state, say *i*, can extract through tax transfers is bounded from above by $\lambda_i(\Gamma = 0) \cdot t^{\max} < 1$.

4.0 Extensions and Qualifications

4.1 *Demographic heterogeneity*: We suggested that demographic heterogeneity could play a role like nature or history in constraining the set of desirable mergers. Heterogeneity can be of different types, say, ethnic, economic, or both. Another way of looking at heterogeneity is to differentiate between inter-state and intra-state heterogeneity. Table 3 summarizes the entire range of possibilities in this regard. Type III (inter-state economic) heterogeneity has been addressed in the benchmark model in Section 3. Results of Section 3 also apply to Type II

[28] With some modifications to this section we can also construct equilibria which support delayed mergers.

(intra-state ethnic) heterogeneity without any changes because in our setting intra-state pay-offs are independent of ethnicity. Of the remaining types only Type I (inter-state ethnic heterogeneity) and Type IV (intra-state economic heterogeneity) are structurally independent. We will therefore discuss only Type I and IV heterogeneities below assuming intra-group homogeneity and absence of cross-border collaboration between groups.[29]

Type IV (intra-state economic) heterogeneity Assume intra-state economic heterogeneity in the form of population divided into two classes, namely, labour and capital. In absence of resource mobility return to a factor in a state is inversely proportional to its availability in that state. The exact relationship is immaterial as long as it implies inverse proportionality so that owners of scarce resource earn scarcity rents. The identity of scarce resource is immaterial. All we need is scarcity of a resource and a group that can extract rents. Disproportionate division of the product captures the relative powers of the two groups and is also the source of conflict between them, which can be eliminated only when resource mobility is permitted. The group holding scarce input has a vested interest in maintaining the asymmetry. Consequently owners of scarce resource have a disproportionate power if every merger proposal has to be cleared by each group within a state before it is negotiated with other state. The game for choice of merger within states transforms into a nested or two-level game with the first round being played within each state followed by another round between states.

The key insights from the above model are as follows. Mergers involving resource mobility are never agreed to domestically as long as each economic interest group has a veto within a state. The veto of the group having excess resources is, in fact, redundant because in equilibrium it does not use its veto. The owners of scarce resource in a backward state would support a full merger with technology transfer and resource mobility only if technology gap is sufficiently large. Otherwise this group would agree to technology transfer and limited resource mobility. In the limiting case it would veto any merger that includes more than technology transfer. In this latter case if there is an apprehension that the union might impose resource mobility in future even technology transfer becomes a dominated strategy for owners of scarce resources. In an advanced state the owners of scarce resource veto every type of merger proposal. In other words a merger which might be good for the states would not be implemented once we introduce even minimal economic heterogeneity in population of at

[29] Details available from author.

least one state, with each interest group having a veto over choice of merger options. More importantly this result is irrespective of the identity of mobile resource.

Type I (inter-state ethnic) heterogeneity Assume that states are ethnically different. For a while assume that only labour is mobile. In addition to earning returns to their respective unemployed factor inputs immigrant labour from labour surplus province and owners of excess capital in capital surplus province both suffer some disutility from engaging with ethnic strangers. For sufficiently large ethnic antipathy labour mobility becomes a dominated strategy for at least one of the states. It is in such cases that the identity of mobile resources matters in our model because now states could bargain on a third dimension namely capital mobility (CM). Now TTLMCM is full merger while TTLM, TTCM, LMCM, LM, CM, and TT are partial mergers. Disagreement strategies now include CM, in addition to TT, LM, and NM. Since our analysis holds for the case involving n-types of mergers and $m(n)$ disagreement strategies we can very easily introduce capital mobility into the model without affecting the structure of the model.

The first casualty in both cases above is labour mobility, which becomes less favoured once we introduce ethnic or economic heterogeneity. With introduction of either ethnic or economic heterogeneity the model can support incomplete mergers contrary to Proposition 1 (*cf.* footnote 11, *supra*). But there is a difference as well. In case of ethnic heterogeneity capital mobility is never a dominated strategy even though labour mobility might be whereas in case of economic heterogeneity any kind of resource mobility could be dominated. To conclude in our model intra-state ethnic heterogeneity and inter-state economic heterogeneity are not restrictive while inter-state ethnic heterogeneity and intra-state economic heterogeneity can constrain the set of desirable mergers. Of the two the latter is more restrictive.

Three possible extensions not implemented till now come to mind. One, interacting demographic heterogeneity with labour force and/or technology differentiated by skill levels; intuitively the key change because of this would be that unemployment rate would no longer provide an upper limit to resource migration. Two, interacting divergent institutional preferences (unitary/federal, majoritarian/proportional, official language and religion, etc) with heterogeneity; here the problem is especially acute with time inconsistent mergers, where the recipient of technology might accept certain repugnant institutions as a temporary

expedient. In both the set of mutually desirable mergers, $\mathcal{T}(i) \cap \mathcal{T}(j)$ would be constrained. In fact, lexicographical preferences, with institutions being prior, can completely rule out merger. Three, replacing the veto with a policy contest between labour and capital within states. In this case the only difference is that with a positive probability the veto of owners of scarce resources is void. Rest of the analyses follows Section 3.

4.2 *Role of information*: Whenever nature does not reveal **N** (*cf*. Section 3.0 *supra*) fully to at least one of the states there is asymmetric information and consequently there would be incentives to misrepresent the same under certain conditions. States could potentially use private information to seek larger share in union. However, even if a state were to gain some advantage in this way the other state would figure out the discrepancy without much delay after formation of the union[30] and seek revision through C2, C3, or outright contest. While misrepresenting private information to its advantage a state has to take into account such future setbacks. So the act of misrepresenting has to be dynamically consistent and efficient. It is possible that asymmetric information could lead to more contests after formation of unions in case of pooling equilibria.

4.3 *Saving* Introducing common saving rates across states does not affect relative bargaining power in negotiations and rate of success in contests over time. It, however, makes expensive contests affordable and necessitates inclusion of more categories of contest. Introducing more destructive or appropriate types of contest raises a number of questions. What if a backward state emerges winner in a contest and manages to capture some labour and capital of advanced state. Has the backward state managed to appropriate the advanced technology as well? Even if we were to ignore this savings might help a backward state to buy advanced technology through lump sum transfer. However, the amassed wealth can itself be targeted by the advanced state undercutting the backward state's ability to follow such a course of action. Advanced state's action would be governed by two motivations, namely, greed and fear. The former is obvious while the latter arises because if it does not save as much as backward state it might end up losing a future contest when the latter would be able to organize more resources for contest. Possibly in equilibrium we would be back to the situation where both sides agree to a merger in initial period leading to payment through tax transfers rather than

[30] This might not be possible always. Numerous instances of stalled or rigged census exercises across the globe are a case in point (Kumar and Ghosh 2006 and Horowitz 1985 [2000]). Likewise it might not be feasible to carry out adequate survey of capital resources in troubled provinces of a union.

outright purchase at some future date. Unequal savings rates are though more problematic even under complete information because this new feature destabilizes the basic process of bargaining itself by changing relative bargaining power with time.

4.4 *Time inconsistency and permanent resource mobility*: Irreversible resource mobility changes the disagreement game. For this very reason a state that is going to lose bargaining power should not agree for permanent migration of its surplus resources. Consider two cases: (a) labour surplus backward state and (b) labour surplus advanced state. Recall that individuals own both labour and capital endowments. In the latter case moving to backward state with capital is not profitable. So surplus labour of an advanced state would relocate to the backward, labour deficient state while retaining capital in home state where productivity is higher and capital is scarce. In the former case, labour surplus backward state, migrants could profit from migrating along with their share in capital whereas in case of sufficiently high technology gap even migration sans capital is beneficial. In this case to rule out the possibility of permanent migration with or without capital it is sufficient (for the purpose of Section 3) to assume that states do not allow permanent transfer of capital resources and the technology gap between the two states falls below a certain threshold so that it does not make sense to migrate permanently leaving behind income from capital. Further work is required to establish the conditions under which a state would agree to irreversible resource transfer.

5.0 Concluding Remarks

This paper provides a model for optimal design of constitution for political union between unequally endowed states. We propose a constitution and then find the conditions under which negotiation equilibrium exists for a merger leading to a unitary state. We introduced a contested constitution to take into account the changes in negotiated solution under the threat of contest. The impact of contest is twofold. Firstly, it constrains the set of optimal taxes. Secondly, and more importantly, it increases the stability of constitution by making the punishment strategy in secession rule credible. The equilibrium in case of contested constitutional agreement is shown to be path dependent and, in some cases, time inconsistent. To address the latter issue we introduced a time consistent contested constitution. We also show that the rents extracted by the advanced state in the union for transferring technology to the backward state are increasing in complexity of technology and that there are limits to such rents. We then show that demographic heterogeneity constrains the set of mutually desirable

mergers. Intra-state economic heterogeneity is shown to be more constraining than other types of heterogeneity. Finally we draw attention to a few unresolved issues, e.g., impact of savings and permanent resource migration on inter-state mergers.

Appendix I: Proofs of results

Proof of Proposition 1: It is advisable to go through this proof after reading Sections 2 and 3. (a) $\theta_T \in [0,1]$ and $\theta_L \in [0,1]$ denote, respectively, fraction of capital in backward state for which advanced technology is provided and fraction of unemployed labour force of labour surplus state permitted to work in labour deficient state. Pay-off to state *i* from *n*-type merger is given by the following, where M_l^n is economic product of state *l* under *n*-type merger and $l \in \{i, j\}$

$$U_i^n = M_i^n(1-t) + t\lambda_i\left(M_i^n + M_j^n\right) = M_i^n + t\left(\lambda_i\left(M_i^n + M_j^n\right) - M_i^n\right)$$

We know that $SQ_l \le M_l^n \quad \forall l \in \{i, j\}$ and $\forall n$, where SQ_l is pay-off to state *l* from No Merger. We can substitute $SQ_l + x_l^n$ for M_l^n, where $x_l^n \ge 0$ is the additional economic product of state *l* in case of *n*-type merger and $x_l^n > 0$ for at least one *l*.

$$U_i^n = (SQ_i + x_i^n)(1-t) + t\lambda_i\left(SQ_i + x_i^n + SQ_j + x_j^n\right)$$

$$U_i^n = (1-t)SQ_i + t\lambda_i\left(SQ_i + SQ_j\right) + \left(t\lambda_i\left(x_i^n + x_j^n\right) + x_i^n(1-t)\right)$$

First consider an incomplete LM only merger (*n* = 1). Excess labour from one state works with excess capital of the other and the economic product is divided equally between the two. Thus, additional economic product accruing to each state when labour mobility is unlimited is given by $x_i^1 = x_j^1$. When only θ_L fraction of labour is allowed mobility then the additional economic product accruing to each state is given by $\theta_L x_i^1 = \theta_L x_j^1$. The pay-off to state *i* and *j* in this case is given by the following, both strictly increasing in θ_L.

$$U_i^1 = (1-t)SQ_i + t\lambda_i\left(SQ_i + SQ_j\right) + \left(t\lambda_i\left(\theta_L x_i^1 + \theta_L x_j^1\right) + \theta_L x_i^1(1-t)\right)$$

$$U_j^1 = (1-t)SQ_j + t\lambda_j\left(SQ_i + SQ_j\right) + \left(t\lambda_j\left(\theta_L x_i^1 + \theta_L x_j^1\right) + \theta_L x_j^1(1-t)\right)$$

Now consider an incomplete TT only merger (*n* = 2). Let *i* be the advanced state. Since in this type of merger the advanced state *i* merely transfers technology there is no increase in its economic product, $x_i^2 = 0$, even though it benefits from tax based transfers. The pay-off to state *i* and *j* in this case is given by the following both of which are strictly increasing in θ_T.

$$U_i^2 = (1-t)SQ_i + t\lambda_i(SQ_i + SQ_j) + (t\lambda_i(\theta_T x_j^2))$$
$$U_j^2 = (1-t)SQ_j + t\lambda_j(SQ_i + SQ_j) + (t\lambda_j(\theta_T x_j^2) + \theta_T x_j^2(1-t))$$

Now it is easy to show that pay-off to state from multi-issue mergers is also strictly increasing in both θ_T and θ_L. Note that we have not assumed any particular distribution of resources and the assumption of state *i* being advanced is without loss of generality. It remains to be pointed out that in absence of any other constraints, which is the case in our model, both states irrespective of their technological and material endowments strictly prefer a complete merger ($\theta_T = 1$ and/or $\theta_L = 1$) to incomplete merger ($\theta_T \in$ [0, 1) and/or $\theta_L \in$ [0, 1)) $\forall n$, which establishes the result. *QED*

(b) Let us assume the contrary to be true. If so then an incomplete merger of n_i-type would be preferred to a complete merger of n_j-type. However, if an incomplete merger of n_i-type is preferred we know from the first part of this proposition that then a complete merger of n_i-type would be strictly preferred to both a complete merger of n_j-type as well as an incomplete merger of n_i-type. So the two states would conclude a merger agreement for a complete merger of n_i-type rather than an incomplete merger of n_i-type, which leads to the claim made in the proposition. *QED*

Proof of Lemma 1: (a) Suppose $\beta_i = \lambda_i$ is not the equilibrium. Instead assume $\beta_i^{'} > \lambda_i$ to be the equilibrium. $\beta_i^{'} > \lambda_i \Rightarrow \beta_j^{'} = 1 - \beta_i^{'} < \lambda_j = 1 - \lambda_i$. However, λ_i is by definition the seat sharing equilibrium and is therefore reflective of the relative bargaining power of the states. Cognizant of its real bargaining power State *j* would renegotiate and achieve at least $\beta_j \geq \lambda_j \Rightarrow \beta_i = 1 - \beta_j \leq \lambda_i$. Similarly, if $\beta_i^{'} < \lambda_i$ then State *i* would renegotiate and achieve $\beta_i \geq \lambda_i$. Combining the two we conclude that in equilibrium $\beta_i^{'} \neq \lambda_i$ cannot be true. *QED*

(b) Suppose α is not the equilibrium. Instead assume $\alpha^{'} < \min(\lambda_i, \lambda_j)$ to be the equilibrium. Since the two states are unequally endowed $\lambda_i \neq \lambda_j$, so that $\sum_i \lambda_i = 1 \Rightarrow \lambda_i \neq 0.5$ $\Rightarrow \min(\lambda_i, \lambda_j) < 0.5$. In this case $\alpha^{'}$ is more relaxed than simple majority voting rule and violates the requirement of stability because partisans of both provinces can unilaterally amend the constitution. Next assume $\alpha^{'} \in [\min(\lambda_i, \lambda_j), \max(\lambda_i, \lambda_j))$ to be the equilibrium. If this is true then immediately after merger State *i* with higher λ can amend the constitution, including C1 where λ is specified, to its advantage. This contradicts the fact that λ is by definition equilibrium seat sharing

rule. Here α' violates the requirement of inclusiveness because a priori the weaker state would find it sub-optimal to join the union because it is completely defenceless against moves to reduce its seat and revenue share in union. The weaker state would therefore not join the union the moment α' is known to be the initial amendment rule.

Thus the set of equilibrium amendment rules satisfying inclusiveness is given by $C3^{Incl} = \{\alpha \mid \alpha \geq \max(\lambda_i, \lambda_j)\}$. Under such an amendment rule the stronger province of the union would need the support of at least one legislator from the weaker province to successfully implement an amendment advantageous to it. Any legislator from weaker province who votes for the amendment does so if and only if the change is beneficial for him. Since individuals in each state are endowed with identical preferences and resources (*cf.* Section 2.2) it ensures that such a change would be beneficial for all others in his province as well. Further, it is easy to see that only amendment rules that satisfy inclusiveness also satisfy the requirement of stability. So, $C3^{Incl} = C3^{Stable}$. To account for flexibility we choose the most liberal amendment rule. Thus in equilibrium constitutional amendment rule is given by $C3^* = \min(C3^{Incl} \cap C3^{Stable}) = \max(\lambda_i, \lambda_j)$. *QED*

Proof of Lemma 2: SR depends on two factors: the stream of additional economic product generated by virtue of merger (CC_n, C1 and C1a) and the ability of union to restructure via C2 and C3 in face of crises. But we have shown in Lemma 1 that C1a and C3 are pegged to agreement on C1. Further revision rule C2 depends on C1 since the extent of revision required depends on the initial arrangement (C1, C1a, C3, SR, CC_n). Now it is easy to see that CC_n and C1 are the only independent (sub-)articles, respectively, in Part I and II of constitution. *QED*

Proof of Lemma 5: The result is trivially true for $|S_i(n)| = 1$. For $|S_i(n)| > 1$ let $V = \hat{\Theta}'(n) \setminus \{NM, NM\}$, i.e. there are $|V| = |\hat{\Theta}'(n)| - 1$ equilibria of disagreement game which are different from (NM, NM) and support some kind of partial merger. Key to establishing the equivalence is that pay-off in Θ' corresponding to vth disagreement equilibrium is a weighted average of two quantities with one of them necessarily greater than the (NM, NM) pay-off while the other is greater than or equal to the latter depending on the strategy chosen. Recall that there is one tax rate across the unitary union, say, t. The equivalence suggested in the lemma follows if the following requirement is satisfied $\forall l \in \{i, j\}$ and $\forall v$, where revenue share $\beta_i = \lambda_i$ vide Lemma 1.

$$SQ_l < M_l^v(1-t) + t\lambda_l\left(M_l^v + M_{\{i,j\}\setminus l}^v\right)$$

where SQ_l is No Merger disagreement pay-off of state l and M_l^v is the economic product of state l under vth equilibrium of disagreement game, Θ', where $v \in \{1, 2, \ldots |V|\}$.

We know that the following always holds, $SQ_l \leq M_l^v \;\; \forall l \in \{i, j\}$ and $\forall v$. We can substitute for M_l^v with $SQ_l + x_l^v$, where $x_l^v \geq 0$ is the additional economic product of state *l* and $x_l^v > 0$ for at least one *l*. The above condition can now be recast as follows:

$$SQ_l < (SQ_l + x_l^v)(1-t) + t\lambda_l\left(SQ_l + x_l^v + SQ_{\{i,j\}\setminus l} + x_{\{i,j\}\setminus l}^v\right)$$

This last condition can be reduced to the following using the fact that $\lambda_l = SQ_l / \left(SQ_l + SQ_{\{i,j\}\setminus l}\right)$

$$0 < x_l^v(1-t) + t\lambda_l\left(x_l^v + x_{\{i,j\}\setminus l}^v\right)$$

which is true for $\forall\, t \in (0,1)$, $\forall\, l$, and $\forall\, v$. Hence, all equilibria of normal form disagreement game $\Theta'(n)$ lie along the diagonal. The justification for the substitution $\lambda_l = SQ_l / \left(SQ_l + SQ_{\{i,j\}\setminus l}\right)$, which is not immediately apparent would bear some elaboration. The *v*th equilibrium of disagreement game corresponds to either TT or LM merger. The substitution for λ_l suggested is actually the corresponding λ_l for TT or LM merger derived using Lemma 3. And here it is very important to note that we are *not* proceeding along a circular loop. For TT and LM mergers, $|S_i(n)| = 1$, the tax adjustment is trivial and tax adjusted game is *identical*, and not merely strategically equivalent, to the unadjusted game as indicated at the very beginning of this proof. So when we carry out the above substitution we are not using unadjusted seat shares to show equivalence between adjusted and unadjusted disagreement games.

To obtain Θ'' we need to add an infinitesimal quantity $\varepsilon(>0)$ to the pay-off of State A corresponding to the equilibrium (TT, TT) in Θ to preserve strict preferences in Θ'. (Without this addition we get an additional, but pay-off *and* also outcome equivalent, equilibrium.) With this adjustment it is easy to see that the strategic choices available to players in disagreement games Θ' and Θ'' are identical.

QED

Proof of Lemma 7: In case of unequal endowments following *Lemma* 4 $\lambda_i \neq \lambda_j$. The total pay-off of state *i* from *n*-type merger is given by the following expression:

$$U_i^n = E_i^n(1-t) + t\lambda_i\left(E_i^n + E_j^n\right) = E_i^n + t\left(\lambda_i\left(E_i^n + E_j^n\right) - E_i^n\right)$$

which is strictly non-decreasing in *t* if and only if $\lambda_i^n \geq E_i^n / \left(E_i^n + E_j^n\right)$, the condition stated in the lemma. Equivalence between this condition and $\lambda_i^n g^n - x_i^n \geq 0$ arises from the following

substitutions $E_i^n = d_i^n + x_i^n$ and $\lambda_i^n = d_i^n / \left(d_i^n + d_j^n\right)$. d_i^n is the disagreement pay-off. For such a state its contribution to union tax revenue is less than what it gets back and thus it prefers highest possible tax rate and vice versa. *QED*

Proof of Proposition 2: Recall that $|F_l| < C_l$, where $l \in \{i, j\}$. Assume without loss of generality that $\lambda_i g - x_i > 0$. Nature chooses a contest category for the states. We will first determine the conditions under which certain CRT(s) dominate other CRTs for a state for a given type of merger. Using (15) - (18) it is easy to show that the following hold:[31]

a) *Status Quo* is strictly dominated by *Contest* for state l for κ = I and II if $p_l g > \left(C_l - F_l\right)$ and $\rho\left(\lambda_l g - x_l\right) + x_l > \left(C_l - F_l\right)$, respectively.

b) *Negotiate* always dominates *Status Quo*.

c) *Negotiate* dominates *Contest* for state i, state j, and both if $t(i) \neq \phi$, $t(i) \neq \phi$, and $t(i) \cap t(j) \neq \phi$, respectively.

κ = I: $t(i) = \{t \mid t \geq t_i \;\; \& \;\; t_i \leq t^{\max}\}$, where $t_i\left(\lambda_i g - x_i\right) = p_i g - \left(C_i - F_i\right) - x_i$

$t(j) = \{t \mid t \leq t_j \;\; \& \;\; t_j \geq t^{\min}\}$, where $t_j\left(\lambda_i g - x_i\right) = p_i g + \left(C_j - F_j\right) - x_i$

κ = II: $t(i) = \{t \mid t \geq t_i \;\; \& \;\; t_i \leq t^{\max}\}$, where $t_i = \rho - \left(C_i - F_i\right) / \left(\lambda_i g - x_i\right)$

$t(j) = \{t \mid t \leq t_j \;\; \& \;\; t_j \geq t^{\min}\}$, where $t_j = \rho + \left(C_j - F_j\right) / \left(\lambda_i g - x_i\right)$

Now we proceed in two steps. (1) First we establish that $t(i) \cap t(j) = \phi$ is impossible with κ = II. (2) Then we show that with κ = I $t(i) \neq \phi$ and $t(j) \neq \phi$ cannot be simultaneously true.

(1) For Category II contests $t(i) \cap t(j) = \phi$ is possible only if one of following holds: a) $\rho + \dfrac{C_j - F_j}{\lambda_i g - x_i} < \rho - \dfrac{C_i - F_i}{\lambda_i g - x_i}$, b) $t^{\max} < \rho - \dfrac{C_i - F_i}{\lambda_i g - x_i}$, or c) $\rho + \dfrac{C_j - F_j}{\lambda_i g - x_i} < t^{\min}$. Given that $\rho \in \left[t^{\min}, t^{\max}\right]$ is always true none of the above hold and therefore c*ontest* is always dominated by *negotiate* for both states.

(2) For Category I contests $t(i) \cap t(j) = \phi$ is possible only if one of following holds: a) $\dfrac{p_i g - \left(C_i - F_i\right) - x_i}{\left(\lambda_i g - x_i\right)} > \dfrac{p_i g + \left(C_j - F_j\right) - x_i}{\left(\lambda_i g - x_i\right)}$, b) $\dfrac{p_i g + \left(C_j - F_j\right) - x_i}{\left(\lambda_i g - x_i\right)} < t^{\min}$, or c)

[31] Note that in each case possibility of contest increases with increase in size of potential surplus generated through cooperation or increase in probability of success of one of the states.

$\frac{p_i g - (C_i - F_i) - x_i}{(\lambda_i g - x_i)} > t^{\max}$. Now assume that contest dominates negotiation for state *i*, which implies the following $t(\lambda_i g - x_i) < p_i g - (C_i - F_i) - x_i$. Contrary to the assertion in the lemma further assume that the same is true about the state *j* as well. This implies $t(\lambda_j g - x_j) < p_j g - (C_j - F_j) - x_j$. Combining the two we obtain $0 > \sum_i (C_i - F_i)$, which cannot be true. This implies that c) cannot hold whenever b) holds and vice versa. Further it is easy to see that a) does not hold under any condition, which leads to the claim that for this category of contests *contest* dominates *negotiate* at most for one of the sides. *QED*

Proof of Proposition 3: 1) This case is straightforward. Here according to nature's draw neither is interested in any kind of merger.

2) First note that by virtue of Proposition 2 we do not have a case where *Negotiate* is dominated by *Contest* for both states.

1. We are in regime $\mathcal{N}$ since neither state's outside option is more attractive than their respective inside options. It is as if contests are unknown to the world. Equilibrium seat sharing rule λ_i is given by Lemma 6 while tax rate is determined using λ_i subject to $t(i) \cap t(j) \neq \phi$ (see proof of Proposition 2, *supra*).

2) In this case outside option of one state (here *i*) dominates the strategic interaction so that the other state (here *j*) makes an offer so as to make the former indifferent between contested and negotiated solutions. Then by virtue of Property 1 (b) state *j* settles for negotiated solution dictated by its outside option. Note that $\lambda_j \geq d_j(m,n) / \sum_j E_j^n$ for Category I contests as long as $g > p_i g - (C_i - F_i)$, which always holds implying that state *j* is better off accepting $\lambda_j = 1 - \lambda_i(m, k \mid \Theta'(n))$ in case *Negotiate* dominates *Contest* only for state *j*.

3) State with $|\boldsymbol{T}(l)| \neq 0$ will precipitate contest every period because the other state is unwilling to consider any type of merger. Given that first strike advantage is non-zero *Contest* is never strictly dominated. So the other state would at least try to avoid loss due to defensive contest strategy and will resist every period. *QED*

Proof of Proposition 4: After technology transfer is complete disagreement game changes in favour of erstwhile backward state, say *j*, which implies $\eta_j^{\Gamma \geq T^*} > \eta_j^{\Gamma < T^*}$. Assume that contrary to the claim in the proposition equilibria described in Proposition 3 are also time consistent for Type II technology,

$\kappa = II$. If true this implies that erstwhile backward state chooses not to seek revision of sharing rules. In other words it prefers to continue with the existing arrangement despite the fact that its winning probabilities and bargaining power have increased. But the pay-off function (see Eqn 10) is strictly increasing in shares in legislature seats and revenue, and therefore the economic product, accruing to a state, which brings us to a contradiction. Therefore, after completion of technology transfer the initial agreement has to be revised. If not the challenger withdraws from the union and in a fresh round of merger negotiation/contest it emerges better-off. Hence the claim in the proposition that constitution for mergers not involving technology transfers at all or involving Type I technology are trivially time-consistent. In passing note that this result implies that whenever Type II technology is involved in merger initial agreement has to be revised at T^* because the initial arrangement is no longer sub-game perfect in all periods. In this case only an arrangement with a time-consistent revision and secession rule can be sub-game perfect. *QED*

Proof of Proposition 5: (1) With Type I technology post-merger gains from merger, g, are never zero so if a merger agreement takes place it remains in place till perpetuity. It is easy to verify that agreements characterized by Proposition 3 are trivially self-enforcing since neither state has an incentive to withdraw from the union. From Proposition 4 we know that with Type II technologies equilibria described in Proposition 3 are not sub-game perfect because the underlying disagreement game changes starting from $\Gamma = T' + T^*$, where T' is the period in which merger agreement is arrived at (= 0 in our case), which will be addressed later.

Assume $\lambda_i g - x_i > 0$, i.e. i is the net recipient of transfers, without loss of generality. There are only two possibilities. One, $g = x_i + x_j > 0$ and $x_l > 0 \quad \forall l \in \{i, j\}$. Province j retains $x_j - t(\lambda_i g - x_i) > 0$ of the additional product generated within its domain even after tax based transfers to the other province. So as long as $g > 0$ neither i nor j has any incentive to leave the union. Two, $g = x_j > 0$ and $x_i = 0$. Share of province i in additional product generated is $t\lambda_i x_j > 0$ while province j retains $x_j + t(\lambda_j x_j - x_j) = x_j - t\lambda_i x_j > 0$. Once again, as long as $g > 0$, neither has any incentive to leave the union. So if tax rate falls within the specified range the union is viable. So whenever SR does not permit secession none of the states has an incentive to secede.

In case of deviation by province, say, i, we need to check if the punishment strategy prescribed in SR is optimal. Province j would contest an attempt to secede by province i iff the cost of contest is

sufficiently low. Parallel requirement emerges for *j* as well.[32] Now we check if the rule is optimal for *g* $\leq$ 0. Finally, note that as long as the cost of contest is positive whenever $g \leq 0$ contest is a dominated strategy. Hence the result stated in the proposition. [33]

(2) With Type II technologies for $\Gamma \in [1, T^*]$ the above applies. Otherwise if union cannot reinvent itself because INV = 0 or $\boldsymbol{T}(i) \cap \boldsymbol{T}(j) = \{TT\}$ we have $g = 0$ so that dissolution is optimal and *contest* is a dominated strategy. Else the union restructures itself and fresh shares are determined as per Proposition 3. Any deviation is punished as per the secession rule (Case 1 above). *QED*

Proof of Proposition 6: The intuition is simple. Even after technological convergence states need each other to harness excess factor inputs so that rational players in complete information set up would avoid loss of benefits from cooperation and instantly adjust the agreement as and when relative bargaining power changes. (1) Following Proposition 4 agreement is time-consistent for the cases $(\Gamma < T^*, \gamma \in \{I, II\})$ and $(\Gamma \geq T^*, \gamma = I)$, which implies that the optimal revision rule specified is unique sub-game perfect revision rule because $\eta_j^{\Gamma} = \eta_j^{\Gamma=0} \quad \forall \Gamma > 0$. (2) Regarding the other case $(\Gamma \geq T^*, \gamma = II)$ first note that in Case (a) union is dissolved so that the revision rule is the unique sub-game perfect rule. In Case (b) merger type is revised to $CC_n^{\Gamma \geq T^*} \in (\boldsymbol{T}(\mathrm{i}) \cap \boldsymbol{T}(\mathrm{j}) \setminus \{CC_n^{\Gamma < T^*}\}$ and part (1) of this proposition applies thereafter. *QED*

Proof of Proposition 7: The advanced state, say *i*, can at most reap $\Pi = \min(K_j, L_j)(X_i - X_j)\sum_{\Gamma=0}^{\Gamma=T^*} \mu^{\Gamma} \lambda_i(\Gamma) \cdot t(\Gamma)$ for transferring technology to the backward state under TT merger, where μ is the discount factor. Following Propositions 3 - 6 we can say that $t(\Gamma)$ = $t^{\max}$ and $\lambda_i(\Gamma) = \lambda_i(\Gamma = 0)$ for $\Gamma \in [0, T^*]$. $\partial\Pi / \partial T^* > 0$, that is the more complex a technology (larger T^*) higher the technology rents accruing to the advanced state. $T^* \to \infty$ relates to the most complex technology. The upper limit specified in the proposition is given by $\lim_{T^* \to \infty} \Pi \Big/ \left(\min(K_j, L_j)(X_i - X_j)\sum_{\Gamma=0}^{\Gamma=T^*} \mu^{\Gamma} \right)$. *QED*

[32] Note that we do not use grim trigger strategies to support punishment equilibria. Incidentally in a world without contests or too expensive contests grim trigger strategies might be required to support punishment equilibria but in our model players would find it impossible to commit themselves to such strategies.

[33] It can be shown that in our framework if g = 0 for whatever reasons, exogenous or endogenous, then a merger cannot be salvaged through federalization of the state.

Appendix II: Description of Disagreement Game (Fig 1)

Assume that the initial distribution of capital and labour is such that production in State A is capital constrained whereas the production in State B is labour constrained. In a labour constrained State B by definition $K_B > L_B$. Given that $K_A + K_B = 1$ and $L_A + L_B = 1$ hold this implies $K_A < L_A$. Further assume $A > B$, which implies technological preponderance of State A. So we are dealing with an advanced capital constrained State A facing a backward labour constrained State B. Recall our production function $X_i \min(L_i, \ K_i)$, where $X_i \in \{A, B\}$, and that we divide economic product equally between labour and capital inputs and also that migrant labour is not discriminated against.

In case states adopt different disagreement strategies or both adopt No Merger (NM) as their disagreement strategy then they obtain their respective No Merger or status quo pay-offs. Under No Merger State A's pay-off is AK_A: $0.5AK_A$ from use of its capital and another $0.5AK_A$ from commensurate use of its labour because it is capital constrained. Likewise State B's pay-off is BL_B: $0.5BL_B$ from use of its labour and another $0.5BL_B$ from commensurate use of its capital because it is labour constrained.

If labour mobility (LM) is the disagreement strategy of both states then State A gets additional income, $0.5B(L_A - K_A)$, due to employment of previously unemployed share of its population whereas State B gets additional income due to utilization of its excess capital. Pay-offs to State A and B are $AK_A + 0.5B(L_A - K_A)$ and $0.5B(K_B + L_B)$, respectively. If technology transfer (TT) is the disagreement strategy of both states then State B obtains additional income, $(A - B)L_B$, due to improved productivity. State A receives only its No Merger pay-off, AK_A, as its unemployed labour is not permitted to work in State B state. The corresponding pay-off for State B is AL_B.

Finally, if agreement is reached upon both technology transfer and labour mobility, i.e. a TTLM merger is agreed upon then State B obtains additional income due to improved productivity and full utilization of its capital within its state whereas State A gets additional income due to full utilization of its labour force. Pay-offs to State A and B are $0.5A(L_A + K_A)$ and $0.5A(L_B + K_B)$, respectively. Also note that to obtain the disagreement game for one issue one step mergers (e.g., TT and LM) ignore the strategies TT and LM in Fig 1, which leaves behind a 1x1 matrix where both players have only one strategy, namely, NM.

References

Alesina, Alberto and Enrico Spolaore (2005), *The Size of Nations*, The MIT Press

Anbarci, N., S. Skaperdas, and C. Syropoulous (2002), 'Comparing Bargaining Solutions in the Shadow of Conflict: How Norms against Threats Can Have Real Effects', *Journal of Economic Theory*, 106, 1-16

Bookman, M. Z. (1993), *Economics of Secession*, St. Martin's Press

Bordignon, M. and S. Brusco (2001), 'Optimal secession rules', *European Economic Review*, 45, 1811-1834

Busch, L. and Q. Wen (1995), 'Perfect Equilibrium in a Negotiation Model', *Econometrica*, 63, 545-565

Bulow, J. and K. Rogoff (1989), 'A Constant Recontracting Model of Sovereign Debt', *Journal of Political Economy*, 97, 155-178

Chang, Roberto (1995), 'Bargaining a Monetary Union', *Journal of Economic Theory,* 66, 89-112

Chen, Yan and Peter G. Ordeshook (1994), 'Constitutional Secession Clauses', *Constitutional Political Economy*, 5 (1), 45-60

Colomer, Josep (2007), *Great Empires, Small Nations: The Uncertain Future of the Sovereign State*, Routledge

Fershtman, Chaim (1990), 'The Importance of Agenda in Bargaining', *Games and Economic Behavior*, 2 (3), 224-238

Friedman, David (1977), 'A Theory of the Size and Shape of Nations', *Journal of Political Economy*, 85 (1), 59-77

Grossman, Herschel I. (2002), 'Constitution or Conflict?', *NBER Working Paper No.* 8733

Herbst, Jeffrey (2000), *States and Power in Africa: Comparative Lessons in Authority and Control*, Princeton University Press

Horowitz, Donald L. (1985 [2000]), *Ethnic Groups in Conflict*, University of California Press

Inman, R. P. and D. L. Rubinfeld (2008), 'Federal Institutions and the Democratic Transition: Learning from South Africa', *NBER Working Paper No.* 13733

Kumar, Vikas and Kunal Ghosh (2006), 'Anomaly in "Greater Nagaland" Demand Along Assam Border: Role of Lingua Franca, Ethnic Cleansing and Historical Deception – Part I & II', *Manuscript*

Little, IMD (2002), *Ethics, Economics, and Politics*, Oxford University Press

McCormick, Gordon. H., and Guillermo. Owen (2004), 'Factionalism, Violence, and Bargaining in Civil Wars,' *Homo Œconomicus*, 20 (4), 361-390

Muthoo, Abhinay (1999), *Bargaining Theory and Applications*, Cambridge University Press (Cambridge)

Putnam, Robert D. (1988), 'Diplomacy and domestic politics: the logic of two level games', *International Organization*, 42 (3), 427-460

Rubinstein, Ariel (1982), 'Perfect Equilibrium in Bargaining Model', *Econometrica*, 50, 97-110

Skaperdas, Stergios (1996), 'Contest Success Functions', *Economic Theory*, 7, 283-290

Skaperdas, Stergios (2006), 'Bargaining versus Fighting', *Defence and Peace Economics*, 17 (6), 657-676

Tsebelis, George (1988), 'Nested Games: The Cohesion of French Electoral Coalitions', *British Journal of Political Science* 18 (2), 145-170

Yarbrough, B. V. and R. M. Yarbrough (1994), 'International Contracting and Territorial Control: The Boundary Question', *Journal of Institutional and Theoretical Economics*, 150 (1), 239-64

Young, Robert (2004), 'Secession as Revolution', *Homo Œconomicus*, 21 (2), 372-395

Table 1: Distribution of Material and Technological Endowments

Cases[34]	Material Endowments		Technology Gap
	State A	State B	
I: Resource constraint, unequal technology	$K_A \neq L_A$	$K_B \neq L_B$	$A/B \neq 1$
II: Resource sufficiency, unequal technology	$K_A = L_A$	$K_B = L_B$	$A/B \neq 1$
III: Resource constraint, identical technology	$K_A \neq L_A$	$K_B \neq L_B$	$A/B = 1$
IV: Resource sufficiency, identical technology	$K_A = L_A$	$K_B = L_B$	$A/B = 1$

Table 2: Numerical Example (Capital Constrained State A vs. Labour Constrained State B)

Technology: A = 2B = 2, Resources: $K_A = 0.70$, $K_B = 0.30$, $L_A = 0.80$, $L_B = 0.20$

Type of Merger	**Share of State B**	
	in legislature of union	**in economic product of union**
Partial mergers	*One issue-one step*	
TT	0.125	0.222
LM	0.125	0.147
Full merger	*One issue-multi step*	
TTLM-NM	0.125	0.250
TTLM-LM	0.147	0.250
TTLM-TT	0.222	0.250
	Multi issue-multi step	
TTLM(LM)	0.147	0.250
TTLM(TT)	0.222	0.250

Table 3: Types of Demographic Heterogeneity

Type	*Inter-state*	*Intra-state*
Ethnic	Type I	Type II
Economic	Type III	Type IV
Ethnic and Economic	Type V	Type VI

[34] Example discussed at length in Appendix II belongs to Case I. All other cases are special cases of this Case I.

Figure 1: Disagreement Game, Θ : (Capital Constrained Advanced State A vs. Labour Constrained Backward State B)

		State B		
		LM	TT	NM
State A	LM	$(AK_A + 0.5B(L_A - K_A), 0.5B(K_B + L_B))$	$(AK_A, \quad BL_B)$	$(AK_A, \quad BL_B)$
	TT	$(AK_A, \quad BL_B)$	$(AK_A, \quad AL_B)$	$(AK_A, \quad BL_B)$
	NM	$(AK_A, \quad BL_B)$	$(AK_A, \quad BL_B)$	$(AK_A, \quad BL_B)$

Figure 2: Tax Adjusted Disagreement Game, Θ': (Capital Constrained Advanced State A vs. Labour Constrained Backward State B)

		State B		
		LM	TT	NM
State A	LM	$\begin{pmatrix} (AK_A + 0.5B(L_A - K_A))(1 - t^{LM}) + \lambda_A t^{LM}(AK_A + BK_B), \\ 0.5B(K_B + L_B)(1 - t^{LM}) + \lambda_B t^{LM}(AK_A + BK_B) \end{pmatrix}$	(AK_A, BL_B)	(AK_A, BL_B)
	TT	(AK_A, BL_B)	$\begin{pmatrix} AK_A(1 - t^{TT}) + \lambda_A t^{TT} A(K_A + L_B), \\ AL_B(1 - t^{TT}) + \lambda_B t^{TT} A(K_A + L_B) \end{pmatrix}$	(AK_A, BL_B)
	NM	(AK_A, BL_B)	(AK_A, BL_B)	(AK_A, BL_B)

No merger pay-off: $((AK_A(1 - t^A) + AK_A t^A), \quad (BL_B(1 - t^B) + BL_B t^B)) = (AK_A, \quad BL_B)$

Figure 3: Reduced Form Disagreement Game, Θ'': (Capital Constrained Advanced State A vs. Labour Constrained Backward State B)

		State B		
		LM	TT	NM
State A	LM	$(AK_A + 0.5B(L_A - K_A), 0.5B(K_B + L_B))$	$(AK_A, \quad BL_B)$	$(AK_A, \quad BL_B)$
	TT	$(AK_A, \quad BL_B)$	$(AK_A + \varepsilon, AL_B)$	$(AK_A, \quad BL_B)$
	NM	$(AK_A, \quad BL_B)$	$(AK_A, \quad BL_B)$	$(AK_A, \quad BL_B)$

$\varepsilon > 0$