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Estimating Situational Effects on OPS

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Estimating Situational Effects on OPS*

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Abstract

‘What is the offensive value of Player A?’ Of all the metrics that sabermetricians have developed to attempt to answer that question, OPS (on base percentage plus slugging percentage) has been one of the first for the mainstream media to slowly embrace as an alternative to batting average. Looking at statistics for each team on ESPN.com, one sees that the batting statistics are sorted by OPS as the default sort. What if the question asked was ‘What is the offensive value of Player A in Situation B versus Situation C?’ In 1994, Jim Albert used the Gibbs sampler to estimate the effect different in-game situations had on batting average. One example of such a situation is a player’s breakdown statistics in home and away games. By employing the Gibbs sampler on each component of OPS, one can compute the situational effect on a player’s OPS. The data will consist of the hitting performance of major league regulars during the 2006 season who qualified for the batting title. Part of the appeal of OPS is that it is simpler to calculate than other more complicated metrics developed by sabermetricians; however, the raw value of OPS does have limitations such as not taking into consideration ballpark effects or the differences between the two leagues.

KEYWORDS: baseball, Gibbs sampler, OPS

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1 Introduction

Batting average, a hitter's number of hits divided by his number of at bats, has long been baseball's main measure of how good of a hitter a ball player is. Schwarz (2004) wrote that the National League in 1876 decided batting average would determine the league's batting champion, and the idea of .300 being the benchmark of whether or not a player was a great hitter soon followed.

During the 2006 season, Freddy Sanchez of the Pittsburgh Pirates won the National League batting title with a batting average of .344. His closest competitors, in terms of batting average, were the Florida Marlins' Miguel Cabrera (.339) and the St. Louis Cardinals' Albert Pujols (.331). If the only way to measure the success of a hitter was batting average, one might conclude that Sanchez was the better hitter compared to Cabrera or Pujols. Unfortunately for Freddy Sanchez, when a batter strides to the plate there are more outcomes than getting a hit or making an out.

On base percentage, or *OBP* for short, is a metric that measures how often a batter reaches base for any reason other than a fielding error, fielder's choice, fielder's obstruction, or catcher's interference. The equation for *OBP* looks like

$$OBP = \frac{H + BB + HBP}{AB + BB + HBP + SF},$$

where H is the number of hits, BB is the number of walks or bases on balls, HBP is the number of times a player was hit by a pitch, AB is the number of at bats, and SF is the number of sacrifice flies.

Table 1 compares the three hitters in the totals that make up a player's batting average and on base percentage. Suppose the definition of the most successful hitter was which one reached based with the highest percentage. Cabrera and Pujols would be neck and neck for the title of the most successful hitter.

Another factor to take into consideration when looking at batting averages and on base percentages is that not all hits are created equal. A player's slugging percentage, denoted as *SLG*, can be thought to measure his total power. The metric can be computed by

$$SLG = \frac{1B + 2 \times 2B + 3 \times 3B + 4 \times HR}{AB},$$

where $1B$ are the number of singles, $2B$ are the number of doubles, $3B$ are the number of triples, and HR are the number of home runs a player has hit. A simple interpretation of *SLG* is that it is a hitter's average number of bases per at bat. *SLG* ranges from 0 to 4.

Table 2 compares the three hitters in the totals that make up a player's batting average and on base percentage. If the criterion for the most successful hitter was his total power, Pujols would be considered the best of these three hitters.

<i>Name</i>	<i>H</i>	<i>BB</i>	<i>HBP</i>	<i>SF</i>	<i>AB</i>	<i>AVG</i>	<i>OBP</i>
Freddy Sanchez	200	31	7	9	582	.344	.378
Miguel Cabrera	195	86	10	4	576	.339	.430
Albert Pujols	177	92	4	3	535	.331	.431

Table 1: 2006 National League: *AVG* vs. *OBP*

<i>Name</i>	<i>1B</i>	<i>2B</i>	<i>3B</i>	<i>HR</i>	<i>AB</i>	<i>AVG</i>	<i>SLG</i>
Freddy Sanchez	139	53	2	6	582	.344	.473
Miguel Cabrera	117	50	2	26	576	.339	.568
Albert Pujols	94	33	1	49	535	.331	.671

Table 2: 2006 National League: *AVG* vs. *SLG*

Thorn and Palmer (1984) simply added a player's on base percentage and slugging percentage to obtain what is called *OPS*.

$$OPS = OBP + SLG$$

The thinking behind *OPS* is to combine the player's ability to get on base and to hit for power into one number, which measures a hitter's offensive value.

Like a player's batting average, it may be of interest to see how a player's *OPS* changes depending on the current in-game situation. For example, in 2006 Albert Pujols had an *OPS* of 1.141 at games played at home versus 1.062 on the road. Since Pujols is a right-handed hitter, a manager might want to know that Albert Pujols has an *OPS* of 1.055 against left-handed pitchers versus 1.117 against right-handed pitchers.

Based on the 2006 season, one may conclude that Albert Pujols hits better at home and is a better hitter against right-handed pitchers. When looking at his three-year splits for the 2004-2006 seasons, he has a higher *OPS* against left-handed pitchers than right-handed pitchers (1.055 versus 1.041) and a higher *OPS* in away games than home games (1.067 versus 1.021). Looking at a player's *OPS* for one season, just like the problem Albert (1994) addressed when looking at batting averages, it is difficult to conclude that a player is a better hitter at home than on the road or is a better hitter against right-handed pitchers than left-handed pitchers. Fortunately, many players amass a large number of plate appearances during the course of a season. By pooling the data on all of these hitters, one hopes that the detection of significant situational effects on *OPS* can be made with greater ease.

Hitting statistics for the 157 players who had enough plate appearances to

qualify for the 2006 batting titles in each league were investigated. To qualify for the batting title, a player needs to average 3.1 plate appearances per game. This works out to at least 502 plate appearances over the course of a season. A plate appearance is a complete turn of batting and is made up of at bats, walks, times hit by a pitch, sacrifice hits, sacrifice flies, and times reached on defensive interference. These data were found on www.baseballreference.com. The following situations were investigated:

- ahead in the count versus two strikes in the count
- day games versus night games
- groundball pitchers versus flyball pitchers
- home games versus away games
- opposite side versus same side
- games before the All-Star game versus games after the All-Star game
- runners in scoring position versus no runners on base.

For the situation defined as opposite side versus same side, this means a right-handed hitter batting against either a left-handed pitcher or a right-handed pitcher. All switch-hitting players, batters that hit both right-handed and left-handed, were ignored. For the situation defined as ahead in the count versus two strikes in the count, a full count (3 balls and 2 strikes) was considered a two strike count.

In each situation, the goal is to estimate the situational effect on each player's *OPS*; however, it was previously mentioned that the hitting statistics for all the players were going to be pooled together. Previous work suggests that better estimates of these unknown effects can be obtained by shrinking the effects to some common value. Efron and Morris (1975) used Stein's estimator to predict the final batting average for 18 players in 1970 after observing their first 45 bats that season. Morris (1983) applied an empirical Bayes confidence interval with unequal variances to see if Ty Cobb was ever a true .400 hitter. Rosenthal (1996) uses these two papers to illustrate the use of a Gibbs sampler to sample from a "richer" posterior distribution. Albert (1994) used the Gibbs sampler to estimate the effects of the situations previously mentioned on batting average and to detect any hitters that stand out in each of the situations. Casella and Berger (1994) applied a Gibbs sampler, among other techniques, to estimate a hitter's true batting ability when given biased or selected information. Frey (2007) used a joint distribution of at bats, hits, and a hitter's true ability as priors and updated it based on the observed batting averages in order to make an inference on a hitter's true ability,

This paper will focus more on Albert's (1994) results. Section 2 will discuss the model that will be used to attempt to estimate the situational effects and the priors that will be used for the Gibbs sampler. Section 3 will discuss how the Gibbs sampler was run using WinBUGS 1.4.1 and the resulting estimated situational effects on *OPS*. Section 4 will discuss any remaining issues that needs to be addressed with this analysis and possible suggestions for future work.

2 Model

The model that Albert (1994) used to estimate the situational effects on batting average can be used for the on base percentage portion of *OPS*. For each of the 157 players, the number of times a player reaches base and the total number of at bats, walks, times hit by a pitch, and sacrifice flies are recorded. The sum of the at bats, walks, times hit by a pitch, and sacrifice flies will be referred to as plate appearances, even though the baseball definition of plate appearances also includes sacrifice hits and times reached on defensive interference. Albert's (1994) way of presenting the situational breakdown of batting average as a 2×2 contingency table can then be extended to on base percentage. Table 3 shows the *OBP* for the i^{th} player as a contingency table where ob_{i1} and pa_{i1} are the number of times a player reaches base and his total plate appearances for situation A, the first situation. Similarly, ob_{i2} and pa_{i2} are the number of times a player reaches base and his total plate appearances for situation B, the second situation.

Let obp_{i1} and obp_{i2} denote the true on base percentages for the i^{th} player in the two situations. Assuming that each plate appearance is independent with the same probability of reaching base, then the number of times a player reaches base, ob_{i1} and ob_{i2} are distributed as binomial distributions with parameters (pa_{i1}, obp_{i1}) and (pa_{i2}, obp_{i2}) .

Just like Albert (1994), a logit transformation can be employed on the observed on base percentage data. In this case, the observed logits are

$$y_{ij} = \log \left(\frac{ob_{ij}}{pa_{ij} - ob_{ij}} \right), \quad j = 1, 2.$$

Just as Albert (1994) did with the batting averages, for on base percentage the mean of these observed logits, μ_{ij} , is modeled as

$$\mu_{ij} = \log \left(\frac{obp_{ij}}{1 - obp_{ij}} \right) = E(y_{ij}) = \mu_i + \alpha_{ij},$$

where μ_i measures the i^{th} player's ability to get on base and α_{ij} is an effect on the player's ability to get on base because of situation j . Since the model is over-parametrized, $\alpha_{i1} = \alpha_i$ and $\alpha_{i2} = -\alpha_i$.

SITUATION	ON BASE	NOT ON BASE	PLATE APPEARANCE
Situation A	ob_{i1}	$pa_{i1} - ob_{i1}$	pa_{i1}
Situation B	ob_{i2}	$pa_{i1} - ob_{i2}$	pa_{i2}

Table 3: *OBP* as 2×2 Contingency Table for Player i

The goal for on base percentage is to estimate $\alpha_1, \alpha_2, \dots, \alpha_n$, the true situational effect on a player's ability to get on base. Previous work suggests that these effects should be shrunk to some common value. A way to accomplish this shrinkage is to assume that all of the effects are similar in size and are independently distributed from some common population.

Following Albert's (1994) lead, the on base abilities $\mu_1, \mu_2, \dots, \mu_n$ were treated as nuisance parameters. These nuisance parameters are assumed to be independent and are assigned a flat noninformative prior. The situational effects are treated as coming from a t distribution with mean μ_α , standard deviation of σ_α , and a small value of known degrees of freedom, $\nu = 4$. Since the general size of the situational effect is unknown, μ_α is given a flat noninformative prior. The tolerance, $1/\sigma_\alpha^2$, is given a vague prior of a gamma distribution with a shape parameter of 0.5 and a scale/rate parameter of 0.5.

There are slight changes to the model used to estimate situational effects on the slugging percentage portion of a player's *OPS*. Since the slugging percentage can be interpreted as the average number of bases a hitter gets per at bat, the total bases for the i^{th} player in a given situation can be distributed according to a Poisson distribution with a parameter of λ_{ij} where $j = 1, 2$. Furthermore, each λ_{ij} , interpreted as the true count of total bases for the i^{th} player in one of the two situations, can be written as

$$\lambda_{ij} = slg_{ij} \times ab_{ij}, \quad j = 1, 2,$$

where slg_{ij} is the true slugging percentage and ab_{ij} is the number of at bats for the i^{th} player in each of the two situations.

Transforming the observed total bases and at bats for each player into

$$y_{ij} = \log \left(\frac{tb_{ij}}{ab_{ij}} \right), \quad j = 1, 2,$$

then the mean of y_{ij} can be modeled as

$$\mu_{ij} = \log(slg_{ij}) = E(y_{ij}) = \mu_i + \alpha_{ij},$$

where μ_i measures the i^{th} player's ability to hit for power and α_{ij} is an effect on the player's ability to hit for power because of situation j . Just like the on base percentage model, this model is overparametrized, so $\alpha_{i1} = \alpha_i$ and $\alpha_{i2} = -\alpha_i$.

In order to shrink these effects to some common value, priors are placed on the various parameters. Similar assumptions on the parameters for the effects on a player's on base percentage were made on a player's slugging percentage. The $\mu_1, \mu_2, \dots, \mu_n$ were still treated as nuisance parameters and given flat noninformative priors. The $\alpha_1, \alpha_2, \dots, \alpha_n$ are treated as coming from a t distribution with mean μ_α , standard deviation of σ_α , and known degrees of freedom, $\nu = 4$. The μ_α is given a flat noninformative prior while the tolerance, $1/\sigma_\alpha^2$, is given a vague prior of a gamma distribution with a shape parameter of 0.5 and a rate parameter of 0.5.

3 Fitting the Model & Results

In order to obtain estimates from the posterior distribution, a separate Gibbs sampler was run for the *OBP* model and the *SLG* model in each of the seven situations. The following initial values were used for the Gibbs sampler to find the situational effects for the *OBP* model:

$$\begin{aligned}\mu_i^{(0)} &= \frac{y_{i1} + y_{i2}}{2} \\ \alpha_i^{(0)} &= \frac{y_{i1} - y_{i2}}{2} \\ \mu_\alpha^{(0)} &= \frac{\sum_{i=1}^n \alpha_i^{(0)}}{n} \\ \sigma_\alpha^{2(0)} &= \frac{\sum_{i=1}^n (\alpha_i^{(0)} - \mu_\alpha^{(0)})^2}{n - 1},\end{aligned}$$

where y_{ij} is the logit transformation of the i^{th} player's observed on base percentage in the j^{th} situation for the 2006 season. The only difference between these initial values and the initial values used for the Gibbs sampler to find the situational effects for the *SLG* model is that when working with the *SLG* model, y_{ij} is the log transformation of the i^{th} player's observed slugging percentage in the j^{th} situation for the 2006 season.

A burn-in period of 10,000 iterations was used for each metric and situation to approach its stationary posterior distribution. Another 10,000 iterations were run to generate simulated values to estimate the parameters. This entire simulated sample can be thought of as a sample from the parameter's posterior distribution.

The goal in all of this is to obtain estimates of the parameters for the situation effects on *OPS*; however, the estimates of the parameters for the situation effects on *OBP* are on the logit scale and on the log scale for *SLG*. In order to get the

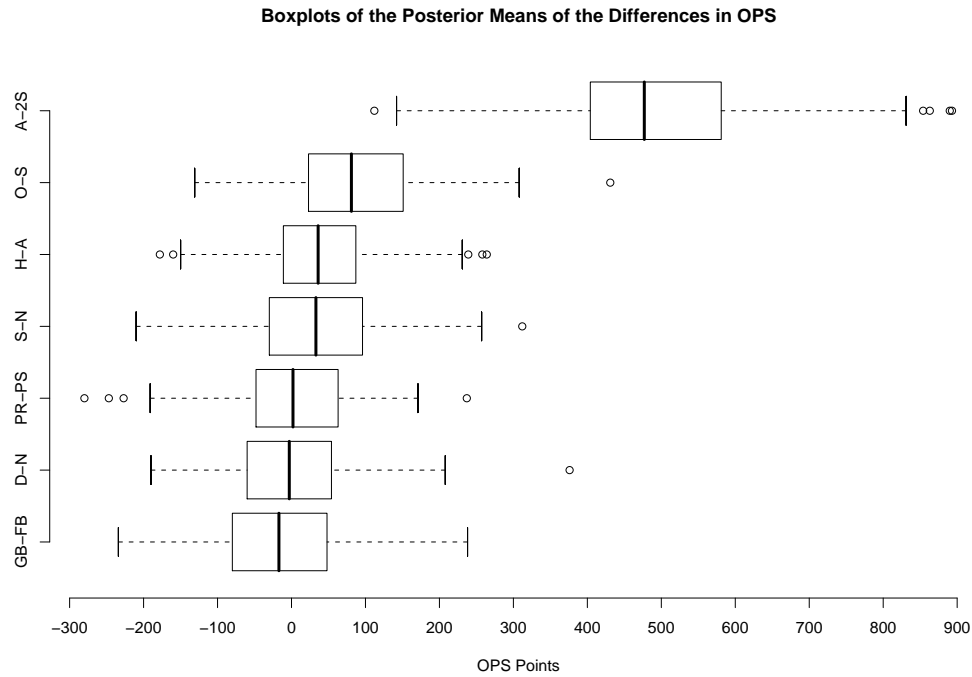


Figure 1: Boxplots of the posterior means of the differences in *OPS* for the seven situational variables.

difference in *OBP* for the i^{th} player, $obp_{i1} - obp_{i2}$ can be computed as

$$obp_{i1} - obp_{i2} = \frac{\exp(\mu_i + \alpha_i)}{1 + \exp(\mu_i + \alpha_i)} - \frac{\exp(\mu_i - \alpha_i)}{1 + \exp(\mu_i - \alpha_i)}.$$

Similarly, in order to get the difference in *SLG* for the i^{th} player, $slg_{i1} - slg_{i2}$ can be computed as

$$slg_{i1} - slg_{i2} = \exp(\mu_i + \alpha_i) - \exp(\mu_i - \alpha_i).$$

Summing these two quantities,

$$obp_{i1} - obp_{i2} + slg_{i1} - slg_{i2} = obp_{i1} + slg_{i1} - (obp_{i2} + slg_{i2}) = ops_{i1} - ops_{i2},$$

results in the i^{th} player's situation effect on *OPS*.

Figure 1 and Table 4 show the results of applying the previously mentioned transformations on the simulated values of μ_i and α_i . Figure 1 plots the boxplots of the differences in *OPS* for the seven different situations. The situations are abbreviated as *A – 2S* for whether a player is ahead in the count versus two strikes;

Situation	OBP		SLG		$E(ops_{i1} - ops_{i2})$ (one unit = 0.001)			
	$E(\mu_\alpha)$	$E(\sigma_\alpha)$	$E(\mu_\alpha)$	$E(\sigma_\alpha)$	Q_1	M	Q_3	$Q_3 - Q_1$
GBALL-FBALL	0.009	0.125	-0.022	0.123	-80	-17	48	128
DAY-NIGHT	0.001	0.121	-0.003	0.117	-60	-3	54	114
PRE/AS-POST/AS	0.003	0.122	0.002	0.114	-48	2	63	111
SCORING-NONE ON	0.071	0.124	0.001	0.125	-30	33	96	126
HOME-AWAY	0.035	0.118	0.024	0.115	-11	36	87	98
OPP-SAME	0.078	0.129	0.052	0.127	23	81	151	128
AHEAD-2 STRIKES	0.458	0.139	0.302	0.131	404	477	581	177

Table 4: Posterior means of the parameters of the situation effects and summary statistics of the posterior means of the OPS differences $ops_{i1} - ops_{i2}$ across all players.

$O - S$ for whether a player is facing a pitcher with the opposite throwing arm or the same throwing arm; $H - A$ for whether a player is hitting in his home ballpark or in an away game; $S - N$ for whether a player is batting with runners in scoring position or no one being on base; $PR - PS$ for whether a player is batting before the All-Star break or after the break; $D - N$ for whether a player is hitting in a day or night game; and $GB - FB$ for whether a player is facing a groundball or flyball pitcher. Table 4 gives the posterior means of the parameters of μ_α and σ_α that describes the population of the situation effects. It also gives the median, quartiles, and interquartile range for the situational effects on OPS.

The one situation that stands out the most is a player being ahead in the count versus having two strikes. The median difference in OPS is 477 points. The next most important situation is when a player is facing a pitcher throwing with the opposite arm versus the same arm. The median difference in OPS in this situation is 81 points. Even though the results shown in Table 4 for OBP are in the logit scale and SLG are in the log scale, they still provide some indication about the significance and the variability for each situation. Again, the pitch count and whether or not a player is facing a pitcher with the opposite throwing arm versus the same throwing arm seem to be the most important comparisons. All of the situations have about the same spread except for the situation dealing with pitch counts.

Table 5 lists any player who could be identified as an outlying situational player based on their large or small differences in OPS compared to all other differences in OPS for a given situation. What stands out the most are the "outliers" for the most significant of variables, whether or not a player is ahead in the count or has two strikes. The players with the largest differences are known for being players prone to striking out a lot over the course of a season. The smallest difference is a

<i>Player</i>	<i>Situation</i>	<i>OPS1</i>	<i>OPS2</i>	<i>Season difference</i>	<i>Est. of $obp_1 - obp_2$</i>
Eric Chavez	ahead-2 strikes	1.482	0.463	1.019	0.893
Adam Dunn	ahead-2 strikes	1.517	0.759	0.758	0.890
Jeff Francouer	ahead-2 strikes	1.353	0.378	0.975	0.863
Manny Ramirez	ahead-2 strikes	1.694	0.759	0.935	0.854
Todd Walker	ahead-2 strikes	0.828	0.824	0.004	0.112
Jim Thome	opposite-same	1.203	0.715	0.488	0.431
Craig Biggio	home-away	0.868	0.541	0.327	0.264
Matt Holliday	home-away	1.132	0.818	0.314	0.258
Vernon Wells	home-away	1.038	0.762	0.276	0.239
Craig Monroe	home-away	0.678	0.897	-0.219	-0.160
Carlos Beltran	home-away	0.855	1.089	-0.234	-0.178
Nomar Garciaparra	pre/AS-post/AS	1.004	0.694	0.310	0.237
Rafael Furcal	pre/AS-post/AS	0.691	0.963	-0.272	-0.227
Richie Sexson	pre/AS-post/AS	0.706	1.012	-0.306	-0.247
Ryan Howard	pre/AS-post/AS	0.923	1.260	-0.337	-0.280
Jason Giambi	day-night	1.294	0.827	0.467	0.376

Table 5: Outlying situational players based on $Q_1 - 1.5 \times IQR$ and $Q_3 + 1.5 \times IQR$ found in Table 4.

player known for being more of a contact hitter.

In Figures 2 and 3, the shrinking of the 2006 season effects to the posterior means is illustrated by the line $y = x$ being plotted on top of each graph. These plots illustrate the "regressing to the mean" phenomenon that James (1986) talked about in his studies of team breakdown statistics. The goal here was to shrink the season effects of *OPS* to some common value. The shrinkage of the seasonal situation differences to the posterior means was anywhere from approximately 23% to a median effect of 477 points in the situation dealing with pitch counts to approximately 29% to a median effect of -17 points in the situation where a batter is facing a groundball pitcher versus a flyball pitcher.

4 Summary & Conclusions

Because of the computational ease of *OPS*, mainstream media, whether broadcasters or print journalists, will be using this metric more often when discussing a baseball player's offensive output. These situational splits with respect to *OPS* will be some of the statistics that may be most often bandied about. The goal of this paper was to quantify these effects using well-established methods in statistical circles. As a fan listens to their favorite team, they now have a benchmark to compare how

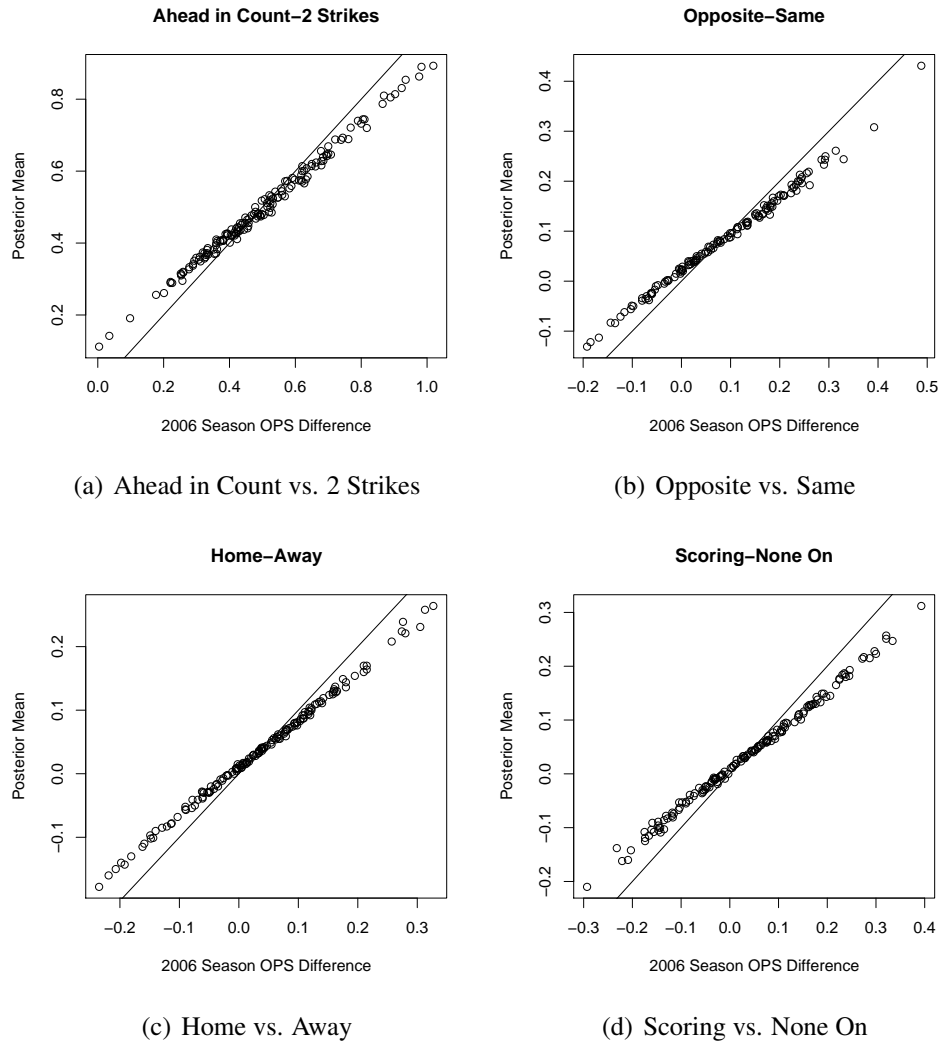
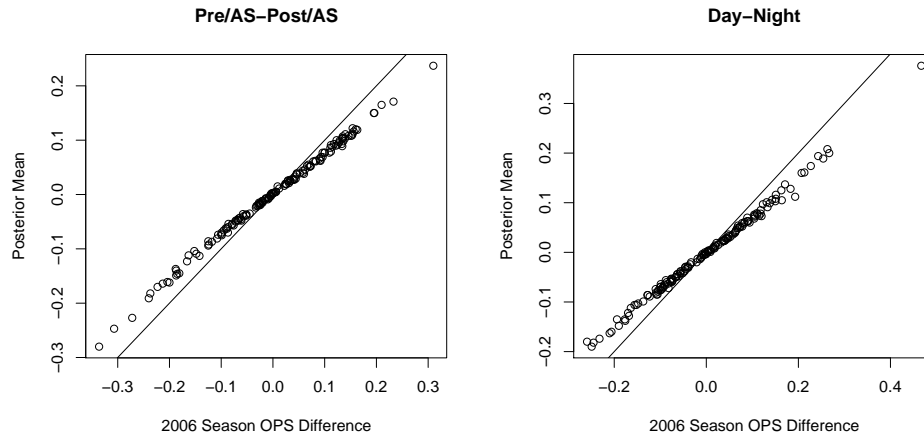
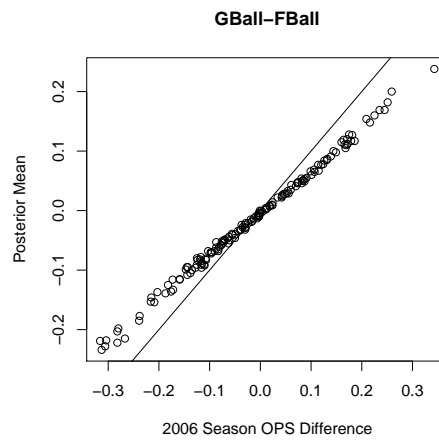


Figure 2: Posterior means of the differences in *OPS* plotted against the 2006 season differences in *OPS*.



(a) Pre-All Star vs. Post-All Star

(b) Day vs. Night



(c) Groundball vs. Flyball

Figure 3: Posterior means of the differences in *OPS* plotted against the 2006 season differences in *OPS*.

well a player performs in a given situation versus what is typically expected.

Looking back at the results, the two situations that stood out the most were whether or not a player is ahead in the count versus having two strikes, which had a median effect of 477 points, and whether a hitter was facing a pitcher with the opposite throwing arm versus the same throwing arm, which had a median effect of 81 points. Any significant differences in a hitter's *OPS* in either of these situations could be argued to be due to one's ability to hit in that situation as opposed to a general bias, which would indicate that the situation had the same effect on all hitters. The same model and priors were used for the 2007 season resulting in a median effect of 505 points when a hitter is ahead in the count versus having two strikes and 101 points when a hitter is facing a pitcher with the opposite throwing arm versus the same throwing arm.

The overall patterns in these situational effects on *OPS* are very similar to Albert's (1994). This probably is not too surprising. One might assume that the same situation has the same overall impact on any type of offensive metric. Future work could entail treating the detection of outliers in a more elegant manner. Due to deadlines for the 2007 New England Symposium on Statistics in Sports (NES-SIS), the players that were identified as potential outliers in Figure 1 and Table 5 were not fully identified as being either "biases" or "ability splits." These were the distinctions in situational statistics that were identified by James (1986) and were approached in a more formal manner by Albert (1994).

The question still remains whether or not *OPS* is the best way to measure how good a hitter is. One reason for using *OPS* is that it is easier to compute than more complex metrics used by sabermetricians. One simply adds the *OBP* and the *SLG* of a hitter together. Unfortunately, *OPS* is a raw metric, meaning that it is not adjusted to control for ball park effects and overall league effects like the differences between the two leagues. A metric that could be of some interest with this type of analysis is *OPS+*, adjusted *OPS*, which adjusts the raw *OPS* for the park and league in which the player played, but not for fielding position. Since this *OPS+* is expressed like a ratio, it might be more difficult to apply the models discussed in this paper to the metric. A better approach could be to analyze a hitter's equivalent average, *EqA*, which is a metric that measures a player's hitting production that is independent of park and league effects.

References

- Albert, J. (1994). Exploring baseball hitting data: What about those breakdown statistics? *Journal of the American Statistical Association*, 89(427), 1066–1074.

- Casella, G., & Berger, R. (1994). Estimation with selected binomial information or do you really believe that Dave Winfield is batting .471? *Journal of the American Statistical Association*, 89(427), 1080–1090.
- Efron, B., & Morris, C. (1975). Data analysis using Stein's estimator and its generalizations. *Journal of the American Statistical Association*, 70(350), 311–319.
- Frey, J. (2007). Is an .833 hitter better than a .338 hitter? *The American Statistician*, 61(2), 105–111.
- James, B. (1986). *The Bill James baseball abstract*. New York: Ballantine Books.
- Morris, C. (1983). Parametric empirical Bayes inference: Theory and applications. *Journal of the American Statistical Association*, 78(381), 47–55.
- Rosenthal, J. (1996). Analysis of the Gibbs sampler for a model related to James-Stein estimators. *Statistics and Computing*, 6, 269–275.
- Schwarz, A. (2004). *The numbers game: Baseball's lifelong fascination with statistics*. New York: Thomas Dunne Books; St. Martin's Press.
- Thorn, J., & Palmer, P. (1984). *The hidden game of baseball: A revolutionary approach to baseball and its statistics*. New York: Doubleday Books.